Optimal Control an Introduction

by Arturo Locatelli

Birkhäuser, Basel, 2001, 294~p.

ISBN 3-7643-6408-4

This book is both a theoretical presentation and a tutorial on optimal control. Basic concepts and ideas are mixed together with many illustrative examples. The style of exposition is clear, therefore suited for the aim of the book to be an introduction to the field.

There are two major parts: one is devoted to global methods (represented by the Hamilton-Jacobi theory), and the other to variational methods (the Maximum Principle and second order variation methods). Moreover, two appendices are filling the information background with some quickly reviewed basic facts in systems and control (from canonical decompositions, poles and zeros, and transfer function, to pole assignment problems).

The first part of the book, **Global methods**, consists of four chapters. At the beginning, general results from **The Hamilton-Jacobi theory** are presented. The first statement of an optimal control problem is given; the most important elements of this problem are described: the controlled system (throughout the book, continuous-time systems are considered), the performance index (as the sum of an integral term and a function of the final event), and the Hamiltonian function. Sufficient conditions for a control function to be optimal are related to the solution of the Hamilton-Jacobi differential equation. The next two chapters make use of this theory, which is applied to particular problems with important physical applications and nice mathematical solutions.

The LQ problem (linear-quadratic problem) is further on discussed in great detail. The problem consists in finding a control that minimizes a quadratic performance index (i.e., both the term under the integral sign and the function of the final event are quadratic functions), when the dynamic system is linear. The well known analytical solution for this problem makes reference to the solution of matrix Riccati equations. In this chapter, this result is proved in several of its variants such as finite or infinite control horizon, time-varying or constant matrix coefficients. The optimal regulator problem (LQ problem over an infinite horizon when both the system and the performance index are time-invariant) is explored thoroughly. Conditions for existence, stability and robustess of the optimal regulator are proved, commented and illustrated with examples.

The LQG problem (linear quadratic gaussian problem) is a well-established approach to optimal control for stochastic linear systems. In order to find an optimal control law that minimizes a certain performance index (which is necessarily taken as an expected value), the Kalman filter is used to estimate the state. Thus, the optimal estimate for the state is taken such as to minimize the variance of the estimation error. The solution for the LQG problem is proved to be a controller that includes in its dynamics a Kalman filter. Mathematically, one is confronted with two differential (or algebraic, in the constant case) Riccati equations.

Because of their important role in linear quadratic optimal control, **The Riccati equations** are studied in a special chapter. For the algebraic equation, necessary and/or sufficient characterizations for the solution are described. Special properties of the stabilizing solution are emphasized.

The second part of the book, **Variational methods**, deals with an alternative approach for optimal control problems. Important necessary conditions of optimization are derived via **The Maximum Principle** of Pontryagin. A specific tool is the auxiliary system derived from a first order partial derivative of the Hamiltonian function. Using also the second order partial derivatives of the Hamiltonian function, the **Second variation methods** provide sufficiency conditions for local optimality. Moreover, similar conditions appear in the problem of neighboring optimal control, where the designed optimal control system preserves its power in the face of uncertainties. It is notable that the methods in the second part are of practical assistance for a wide class of complex problems, unlike the Hamilton-Jacobi theory whose most significant achievements are in the range of linear quadratic problems.

This book is much more than an important reference for the theory and methods in optimal control. Its striking feature is the use of numerous examples (more than one hundred); few of them are purely numerical, and the rest are illustrating applications from mechanics, electricity, etc. The author's teaching experience in the field is visible throughout the book: the style is explanatory, yet very precise; there are no useless remarks.

The book can be used in a control theory course, and the problems sections that follow each chapter seem to be of particular interest in this respect. Another positive feature is that the book is self-contained. People having basic knowledge of systems and control theory and of calculus could form its audience.

Locatelli's book is a very good new presentation of classical optimal control theory. It deepens many aspects of the field and it reveals much information in a very competent exposition.

Diana Maria Sima