Project Portfolio Selection Models and Decision Support

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Abstract: The problem considered in this paper is that of selecting, from a larger set of project proposals, a subset of projects (a portfolio of projects) to be implemented. The set of project proposals is divided into several subsets of equivalent projects.

The project proposals within these subsets may present different levels of performance, the cost for their implementation may be different and they may use resources at different levels. It is desired to find a portfolio of projects from the set of competing projects proposals which contains only one project from each subset, meets all the requirements concerning the resources constraints, maximize the performance (benefits) and minimize the risk. In second section we present several approaches to the project selection problem.

In the third section we present an original zero-one mathematical programming model for project selection problem under risk and limited resources which represents a version of a previous model from (Radulescu and Radulescu, 2001). Our model includes several resources and experts' opinions which generate the risk. The project risk is greater if experts' opinions have a greater degree of dispersion. Several versions of the model are discussed. In the fourth section is presented a decision support system (DSS), which we have named PROSEL (PROject analysis and SELection system) to assist managers in making high quality project portfolio selections.

Keywords: project selection, mathematical programming, decision theory, decision support systems, risk.

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1. Introduction

Many organizations have been making serious efforts to analyze a large set of project proposals in order to choose project portfolios which maximize the performance, meet the resource constraints and minimize the risk. The project proposals may be intended for strategic R&D planning (selection of directions, topics or projects), the development of new commercial products, the management and the implementation of organizational change, the management, the development and the implementation of information technology etc.

In the process of project portfolio selection, decision makers must cope with significant uncertainties concerning the required investment, the necessary time to complete the project, the availability of resources when required, and the likelihood of successful project completion. These may depend on project size, complexity, and project team experience. In addition, there may be multiple criteria to be satisfied, and the choice of projects typically is made by a committee that represents different organizations or companies that may be involved in the project. Selecting a project portfolio is a semi-structured decision. This task differs from choosing a financial portfolio, where stocks or bonds usually have a market history.

The problem considered in this paper is that of selecting, from a larger set of project proposals, a subset of projects (a portfolio of projects) to be implemented.

The set of project proposals is divided into several subsets of equivalent projects. The project proposals within these subsets may present different levels of performance, the cost for their implementation may be different and they may use resources at different levels. The problem is to choose a portfolio of projects from the set of competing projects proposals which contains only one project from each subset, maximize the performance, meet the resource constraints and minimize the risk. It is clear that our problem is a multicriteria decision problem and our purpose is to find a compromise between the conflicting criteria.

In the third section we present an original zero-one mathematical programming model for project selection problem under risk and limited resources which represent a version of a a previous model presented in (Radulescu and Radulescu, 2001). Our model includes several resources and experts' opinions which generate the risk. The project risk is greater if experts' opinions have a greater degree of dispersion. Several versions of the model are discussed.

Since the projects do not have the same impact under every criterion and the relative importance of the criteria is vague definite, at least at the start of the decision process the solution of the real problem is not an easy task.

The prioritization problem, in various forms, has received substantial attention over the past several decades (see the references at the end of the paper).

In second section we present several approaches to the project selection problem.

In the fourth section is presented a decision support system (DSS), which we have named PROSEL (PROject analysis and SELection system), intended to assist managers in making high quality project portfolio selections. Some features of the module RISKSEL of PROSEL, which is based on the model, presented in the third section, is discussed.

2. Approaches to the Project Selection Problem

There is a large literature dedicated to the project selection problem. It includes several approaches which take into account various aspects of the problem.

In (Baker, 1974; Danila, 1989; Shpak and Zaporojan, 1996) are surveyed some of the project selection methodologies and several multicriteria aspects of the problem are discussed.

In (Mehrez and Sinuany-Stern, 1983) the project selection problem is formulated as a multicriteria decision making problem. The approach in this paper is based on the search of utility functions. In a little different context, but close enough, in (Khorramshahgole and Steiner, 1988) is used "Goal programming" associated to a DELPHI process for finding the utility map. It is necessary to mention that both approaches need great efforts in order to generate a series of questions, which must be put to the decision maker so that his preferences can be captured.

Another approach to the project selection problem uses the fuzzy logic and a unique objective map whose construction reflects the multiple objectives of the project selection process. These objectives include the traditional objectives of the projects as profitability and risk. In (Chu *et al.* 1996; Coffin and Taylor, 1996a) a heuristic method based on fuzzy logic is used for ranking projects. The problem for optimal project funding implies decisions on the new projects and on the projects to be continued. The decision of how to be allocated the financial resources between these two types of projects is very important. It is studied in (Baker and Freeland, 1975; Souder, 1975).

In (Lockett and Stratford, 1987; Petersen, 1967; Regan and Holtzman, 1995) are studied several 0-1 mathematical programming models which take into account the hierarchical decisions and the fund allocation problem between independent projects. The goal is to maximize the anticipated benefits taking into account the resource constraints. The great majority of project selection models use mathematical programming methods. A main critique addressed to these models is that they ignore the stochastic nature of the problem since they use the mean and variance instead of probability distributions. In (Locket and Freeman, 1970) the authors tried to improve the above mentioned methods, taking into account the stochastic nature of the resource requirements and the project benefits. But the stochastic models have a great computational complexity. Consequently they are solved by simulation.

Another important problem is the scheduling of the projects. It can be introduced in the selection process if it is used a heuristic method called "filter beam search". See (Coffin and Taylor, 1996b; Heidenberger, 1996; Kyparisis, *et al.*, 1996).

The problems connected with the formal utilization of multiattribute utility functions were studied in (Borcherding and Eppel, 1991; Shoemaker and Waid, 1982). A slightly different approach is suggested in (Mandakovic and Souder, 1985) where the project selection problem is treated as a classical operations research problem "the knapsack problem". The implementation of the above mentioned approach is discussed in (Shpak and Zaporojan, 1996; Eilon and Williamson, 1988).

A different approach is based on the reference point and reference level (see (Lewandowski and Grauer, 1982; Wierzbicki, 1980)). The reference level is represented by a set of performance measures, which are associated to each attribute. The basic idea of the method is to find, for the corresponding optimization problem, the nearest feasible nondominated solution from the point defined by reference levels. The above method was successfully applied to the optimal selection of a portfolio of projects.

A hierarchical approach to the project selection problem, similar to the approach in which each project is associated with a set of scores, may be found in (Cooper, *et al.*, 1980). In this approach instead to associate to each project a weight for each criterion, several rounds are used.

In (Ghasemzadeh, *et al.*, 1999) a zero-one integer linear programming model is proposed for selecting and scheduling an optimal project portfolio, based on the organization's objectives and constraints such as resource limitations and interdependence among projects. The proposed model not only suggests projects that should be incorporated in the optimal portfolio, but it also determines the starting period for each project. Scheduling consideration can have a major impact on the combination of projects that can be incorporated in the portfolio, and may allow the addition of certain projects to the portfolio that could not have been selected otherwise.

Another model of the same type is discussed in (Ghasemzadeh and Archer, 2000). The model is integrated in a decision support system.

3. A Zero-one Mathematical Programming Model for Project Selection Problem Under Risk and Limited Resources

In this section we present an original zero-one mathematical programming model for project selection problem under risk and limited resources which represents a version of a previous model from (Radulescu and Radulescu, 2001). Our model includes several resources and experts' opinions which generate the risk. The project risk is greater if experts' opinions have a greater degree of dispersion.

Consider a set of n project proposals which can be implemented and a partition of this set in q subsets $F_1, F_2, \dots, F_q$. For every $i=1,2,\dots,q$ denote by n_i the cardinal number of F_i and by $P_{i1}, P_{i2}, \dots, P_{in_i}$ the projects contained in the subset F_i. Consequently $n = n_1 + n_2 + \dots + n_q$.

We consider that all the projects from F_i are equivalent; consequently only one project from each subset F_i must be selected. Suppose that the projects are evaluated by m experts $E_1, E_2, \dots, E_m$ which assign scores to each project. Of course instead of experts one can consider m criteria. Denote by a_{sij} the score given by expert E_s to project P_{ij}. Suppose that for the project implementation are available k resources $R_1, R_2, \dots, R_k$. We shall denote by b_{sij} the quantity from the resource R_s necessary to carry out the project P_{ij}. For every resource R_s, we shall denote by c_s the upper limit that it is available. Let x_{ij} $i = 1,2,\dots,q$, $j = 1,2,\dots,n_i$ be the decision variables of the model. $x_{ij} = 1$ if the project P_{ij} is selected for funding, $x_{ij} = 0$ if project P_{ij} is not selected for funding. The benefit or performance of the portfolio $x = (x_{11}, x_{12}, \dots, x_{qn_q})$ of projects is defined by:

$$\sum_{s=1}^{m}\sum_{i=1}^{q}\sum_{j=1}^{n_i} a_{sij}x_{ij} \tag{1}$$

One can easily see that

$$y_s = \sum_{i=1}^{q}\sum_{j=1}^{n_i} a_{sij}x_{ij} \tag{2}$$

is the score of the project portfolio x given by the expert E_s. We define the risk of the project portfolio x as the variance of the scores given by the m experts $E_1, E_2, \ldots, E_m$. Consequently the risk of the project portfolio x is:

$$R(x) = \frac{\sum_{s=1}^{m}\left(y_s - \frac{1}{m}\sum_{r=1}^{m} y_r\right)^2}{m} = \frac{\sum_{s=1}^{m}\left(\sum_{i=1}^{q}\sum_{j=1}^{n_i} a_{sij}x_{ij} - \frac{1}{m}\sum_{r=1}^{m}\sum_{i=1}^{q}\sum_{j=1}^{n_i} a_{rij}x_{ij}\right)^2}{m} \tag{3}$$

The selection problem of a portfolio of projects for funding is a multicriteria optimization problem:

$$\max\left(\sum_{s=1}^{m}\sum_{i=1}^{q}\sum_{j=1}^{n_i} a_{sij}x_{ij}\right)$$

$$\min\left[\sum_{s=1}^{m}\left(\sum_{i=1}^{q}\sum_{j=1}^{n_i} a_{sij}x_{ij} - \frac{1}{m}\sum_{r=1}^{m}\sum_{i=1}^{q}\sum_{j=1}^{n_i} a_{rij}x_{ij}\right)^2\right]$$

subject to:

$$\sum_{i=1}^{q}\sum_{j=1}^{n_i} b_{sij}x_{ij} \le c_s, \qquad s = 1,2,\ldots,k$$

$$\sum_{j=1}^{n_i} x_{ij} = 1 \qquad i = 1,2,\ldots,q$$

$$x_{ij} \in \{0,1\},\ i = 1,2,\ldots,q\ ,\ j = 1,2,\ldots,n_i \tag{4}$$

One can easily see that in the above multicriteria problem one looks for the maximization of the performance (the benefit) and the risk minimization of the portfolio of projects that meet the constraints on the existing resources. The optimization problem is of type zero-one with two objective functions: one is linear and the other is quadratic. The constraints are linear. One can easily see that a project is considered more risky if the experts' opinions about the project have a greater degree of dispersion. Denote by θ the risk aversion coefficient of the decision maker. We shall suppose that parameter θ

takes values in interval [0,1]. The decision makers characterized by small values of θ (near zero) are risk averse. For them the first thing is the safety degree of their return and after that the amount of the return. The decision makers characterized by large values of θ (near one) are interested first by the amount of return and after that by the safety degree of their return. By using the risk aversion coefficient θ we can transform the above bicriterial problem in a zero-one quadratic programming problem with a single objective function.

$$\min\left[(1-\theta)R(x) - \theta\sum_{s=1}^{m}\sum_{i=1}^{q}\sum_{j=1}^{n_i} a_{sij}x_{ij}\right]$$

$$\sum_{i=1}^{q}\sum_{j=1}^{n_i} b_{sij}x_{ij} \le c_s,\ s = 1,2,\ldots,k$$

$$\sum_{j=1}^{n_i} x_{ij} = 1 \qquad i = 1,2,\ldots,q$$

$$x_{ij} \in \{0,1\},\ i = 1,2,\ldots,q\ ,\ j = 1,2,\ldots,n_i \tag{5}$$

We may reduce the number of variables in the above problems by removing the subsets F_i which have a single element. Thus we may suppose $n_i \geq 2$ for every $i=1,2,\ldots,q$.

We may reduce the number restrictions connected with resources by removing the restrictions

$$\sum_{i=1}^{q}\sum_{j=1}^{n_i} b_{sij}x_{ij} \leq c_s$$

in case that

$$c_s \leq \sum_{i=1}^{q} \max_{1\leq j\leq n_i} b_{sij}$$

One can easily see that if

$$c_s < \sum_{i=1}^{q} \min_{1\leq j\leq n_i} b_{sij} \quad \text{for some } s \in \{1,2,\ldots,k\}$$

then there are no feasible solutions.

One can derive the following optimization problems from the above bicriterial problem.

3.1. The risk minimization problem.

The decision maker looks for a project portfolio so that it can minimize the portfolio risk, satisfy the resource constraints and have a performance greater than a given level M. The mathematical model for the risk minimization problem is the following:

$$\begin{cases} \min R(x) \\ \sum_{s=1}^{m}\sum_{i=1}^{q}\sum_{j=1}^{n_i} a_{sij}x_{ij} \geq M \\ \sum_{i=1}^{q}\sum_{j=1}^{n_i} b_{sij}x_{ij} \leq c_s,\ s = 1,2,\ldots,k \\ \sum_{j=1}^{n_i} x_{ij} = 1 \quad i = 1,2,\ldots,q \\ x_{ij} \in \{0,1\},\ i = 1,2,\ldots,q\ ,\ j = 1,2,\ldots,n_i \end{cases} \tag{6}$$

Denote

$$S_0 = \left\{x \in \{0,1\}^n : \sum_{j=1}^{n_i} x_{ij} = 1 \quad i = 1,2,\ldots,q\right\}$$

$$S_1 = \left\{x \in S_0 : \sum_{i=1}^{q}\sum_{j=1}^{n_i} b_{sij}x_{ij} \leq c_s,\ s = 1,2,\ldots,k\right\} \tag{7}$$

$$M_1 = \inf\left\{\sum_{s=1}^{m}\sum_{i=1}^{q}\sum_{j=1}^{n_i} a_{sij}x_{ij} : x \in S_1\right\}, \tag{8}$$

$$M_2 = \sup\left\{\sum_{s=1}^{m}\sum_{i=1}^{q}\sum_{j=1}^{n_i} a_{sij}x_{ij} : x \in S_1\right\} \tag{9}$$

The decision maker has to choose the level M in the interval $[M_1, M_2]$.

3.2. The performance maximization problem.

The decision maker looks for a project portfolio so that it can maximize the portfolio performance, satisfy the resource constraints and have a risk smaller than a given level r. The mathematical model for the performance maximization problem is the following:

$$\begin{cases} \max\left(\sum_{s=1}^{m}\sum_{i=1}^{q}\sum_{j=1}^{n_i} a_{sij}x_{ij}\right) \\ R(x) \le r \\ \sum_{i=1}^{q}\sum_{j=1}^{n_i} b_{sij}x_{ij} \le c_s,\ s = 1,2,\ldots,k \\ \sum_{j=1}^{n_i} x_{ij} = 1 \quad i = 1,2,\ldots,q \\ x_{ij} \in \{0,1\},\ i = 1,2,\ldots,q\ ,\ j = 1,2,\ldots,n_i \end{cases} \tag{10}$$

Denote

$$r_1 = \inf\{R(x) : x \in S_1\} \tag{11}$$

$$r_2 = \sup\{R(x) : x \in S_1\} \tag{12}$$

The decision maker has to choose the level r in the interval $[r_1, r_2]$.

Our models are nonlinear and consequently they are difficult to be solved. Obtaining the optimal solution is very expensive even for small values of n. Since the problem of finding the optimal solution is intractable starting from small number of projects we need to develop heuristic algorithms which provides near-optimal solutions but take nonexponential computing time. The class of nonlinear zero one integer programming problems has received a considerable amount of attention by researchers. A detailed survey on the subject can be found in (Hansen, 1979). We refer readers to this survey for extensive references on nonlinear zero one algorithms. For more recent references on the subject see (Thiel and Voss, 1994).

4. A DSS for Project Selection

Tools for portfolio project selection and management are a recognized need in research, development, production and marketing activities for manufacturing firms and in other sectors such as engineering, construction and software development. They are also used in the public sector, in government, health care and military. As a result of such a diversity of applications were developed several methods and decision support systems for the portfolio project selection: (Ghasemzadeh and Archer, 2000), (De Maio et al., 1994), (Hall and Nauda, 1990), (Kira et al., 1990) etc.

In 1996 at the Institute for Research in Informatics in Bucharest started some research on the decision analysis for competition systems. As a result was designed and realized a software package, MULTICRIT for the multicriteria decision analysis.

The package was intended to help the decision makers for the management of research and development projects. A module of MULTICRIT was dedicated to the project selection problem for the competition systems. The selection problem from the module was based on multiattribute decision theory. MULTICRIT was applied successfully to the management of research project proposals at the Ministry of Research and Technology and for the selection of research grant proposals at the Romanian Academy. For a detailed description of MULTICRIT see (Radulescu 1997a, 1997b, 1997c, 1999a, 1999b).

In 2000 at the Institute for Research in Informatics in Bucharest had started a research on the design and realization of a software package for project and product selection under risk and limited resources. As a result was designed a software package, named PROSEL (PROject analysis and SELection system) intended to help managers in making high quality project portfolio selections. The design of PROSEL was based on the experience acquired with MULTICRIT. The potential users of such a software product may be represented by managers from various domains: decision makers from research-development institutions, marketing and human resources departments, army, financial institutions such as banks, mutual funds or pension funds etc. More generally the software product is intended for users who act in domains in which occur the problem of selecting from a set of actions (projects, proposals, products) a subset taking into account the risk and the resource constraints.

PROSEL is designed as an interactive system, helping the decision makers to make optimal decisions. The software product has a high degree of generality and allows the decision makers to define a wide range of project selection problems. The first step in the decision making process is the problem statement. PROSEL helps the decision maker to get a better understanding of the decision process and to obtain higher performance. When humans make decisions usually they do not start the process by defining a hierarchy of goals, alternatives and criteria. They have in mind a goal and they look for making a decision to attain it. The software product facilitates the decision maker the appropriate structuration and the analysis of the problem.

In the "Decision Analysis" language, PROSEL is a "prescriptive" software instrument and not a descriptive or a normative one. A "prescriptive" instrument is an instrument that tries to amplify the human ability in making decisions by structuring and analyzing the information. The essence of decision analysis is to break complicated decisions down into small pieces that the decision maker can deal with individually and then recombine them logically. A key goal of decision analysis is to make a clear distinction between the choices that can be made (the alternatives), the characteristics of the alternatives (quantified by the measures) and the relative desirability of different sets of characteristics (preferences). These distinctions let the decision maker clearly separate the objective and subjective parts of his decision. The alternatives and the way they are quantified using the measures are relatively objective. Even if there are uncertainties in the levels of the measures, it is usually possible to come to an agreement about how to characterize those uncertainties. On the other hand, the relative importance (weights) of the different measures, the interactions between them, and attitudes toward risk are inherently subjective. Reasonable people can have wide disagreements on these subjects. One can't generally eliminate these subjective parts of a decision. PROSEL provides methods for logically dealing with both the objective and subjective parts of a decision while keeping them well separated.

PROSEL is based on a set of multicriteria decision methods, which is organized in a collection. The methods are of two types: multiattribute decision methods and multiple objective methods. The multiattribute decision methods used in PROSEL are adaptations of some methods that can be found in the Decision Analysis literature. After the definition and validation of the selected problem, the decision maker may select one or several multicriteria methods in order to solve his problem. By using these methods the decision maker can build several selection scenarios. The prioritization which is obtained as an application of the selected method and taking into account various user parameters become a decision support especially for conflicting criteria.

Some of the methods take into account a risk aversion coefficient which is provided by the decision maker. The coefficient takes values in the interval [0,1]. In the case it is equal to 0 then the project portfolio risk is small, but this implies that the project portfolio performance is small. In case it is equal to 1 then the project portfolio risk is great, but this implies that the project portfolio performance may be great. If the decision maker selects a coefficient in the interior of the interval [0,1] then the obtained prioritization signifies a specific trade-off between risk and performance.

The design of PROSEL takes into account the following steps of the decision analysis:

1) Identification of the alternatives to be ranked.
2) Clarification of the goals and objectives that should be met by choosing the top-ranking alternative.
3) Identification of the measures that quantify how well the alternatives meet the goals and objectives.
4) Quantification of the level for each measure for each alternative.
5) Quantification of the preferences about different levels of the measures.
6) Ranking the alternatives by combining information from steps (4) and (5).
7) Performing the "sensitivity analysis" to see the effects on the results of changes in measure levels or preferences.

PROSEL incorporates a module, named RISKSEL which facilitates the selection decision under risk and limited resources. RISKSEL is based on the model presented in the third section. To identify optimal solutions RISKSEL employs techniques based on genetic algorithms. Our experience has been that the techniques based on genetic algorithms are computationally more efficient (by an order of magnitude) and computationally more accurate in comparison to dynamic programming. In special cases, when the number of feasible solution is small RISKSEL provides optimal solutions by using smart enumeration procedures, which avoid unnecessary computations. The above procedure is computationally efficient. RISKSEL using the smart enumeration procedure, obtains the optimal project portfolio for n=10 and for 11 different values of m and k in less than 20 seconds of CPU time on a Pentium II system. The computer time goes up only linearly with respect to the number of resources.

A detailed description of a first version of PROSEL will appear elsewhere.

A numerical example for model (5) is presented in tables. In the columns of table 3 corresponding to the values of θ are given the optimal project portfolios. In the last row of table 4 are given the upper limits for resource levels that is the vector $c = (c_1, c_2, \dots, c_k)$.

Table 1. Scores given by experts to projects

Subset number	Project number in the subset	Scores given by experts					Project risk
		expert 1	expert 2	expert 3	expert 4	Mean score	
1	1	85	87	76	72	80.000	51.333
	2	97	90	91	89	91.750	12.917
	3	78	76	70	76	75.000	12.000
	4	76	80	82	86	81.000	17.333
	5	75	79	85	80	79.750	16.917
2	1	70	67	60	65	65.500	17.667
	2	60	65	55	57	59.250	18.917
	3	88	90	93	91	90.500	4.333
	4	69	62	64	67	65.500	9.667
3	1	59	50	54	53	54.000	14.000
	2	55	57	60	63	58.750	12.250
	3	78	74	75	75	75.500	3.000
	4	89	97	95	93	93.500	11.667
	5	56	54	56	55	55.250	0.917
4	1	67	69	56	70	65.500	41.667
	2	78	79	86	87	82.500	21.667
	3	77	68	78	70	73.250	24.917
	4	45	50	54	53	50.500	16.333
5	1	90	89	82	81	85.500	21.667
	2	89	87	83	81	85.000	13.333
	3	49	56	50	51	51.500	9.667
	4	77	79	80	82	79.500	4.333
	5	95	94	95	90	93.500	5.667
Experts average scores:		74.00	73.87	73.04	73.35		

Table 2. Amounts of resources used by projects

Subset number	Project number in the subset	Amounts of resources used by projects				
		Resource 1	Resource 2	Resource 3	Resource 4	Resource 5
1	1	0.7	5	2.500	600	1000
	2	0.4	4	4.500	200	2000
	3	0.5	3	3.000	266	5800
	4	0.3	2	4.700	300	4999
	5	0.5	1	3.800	160	5999
2	1	0.2	7	3.560	170	3000
	2	0.6	1	2.500	280	2999
	3	0.8	3	1.500	400	1888
	4	0.4	2	2.000	199	2000
3	1	0.7	2	2.500	200	2500
	2	0.6	4	3.600	260	2700
	3	0.4	3	4.800	277	1000
	4	0.2	4	4.700	300	4000
	5	0.5	6	1.600	400	1500
4	1	0.2	5	1.500	266	2800
	2	0.5	3	1.900	250	3800
	3	0.6	5	2.900	100	2700
	4	0.4	3	3.000	160	2500
5	1	0.1	6	4.800	270	3600
	2	0.3	4	3.700	240	3400
	3	0.5	2	4.500	250	4700
	4	0.2	1	2.800	170	5000
	5	0.2	4	4.800	180	3700

Table 3. Optimal project portfolio for various risk aversion coefficients

Subset number	Project number in the subset	theta=0	theta=0.1	theta=0.2	theta=0.4	theta=0.5	theta=0.6	theta=0.8	theta=1
1	1	0	0	0	0	0	0	0	0
	2	0	0	1	1	1	1	1	1
	3	0	0	0	0	0	0	0	0
	4	1	0	0	0	0	0	0	0
	5	0	1	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0
	2	1	0	0	0	0	0	0	0
	3	0	1	1	1	1	1	1	1
	4	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0
	2	1	0	0	0	0	0	0	0
	3	0	1	0	0	0	0	0	0
	4	0	0	1	1	1	1	1	1
	5	0	0	0	0	0	0	0	0
4	1	0	1	1	0	0	0	0	0
	2	0	0	0	1	1	1	1	1
	3	1	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0
5	1	0	0	0	1	0	0	0	0
	2	1	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0
	5	0	1	1	0	1	1	1	1

Table 4. **Amounts of resources used by optimal portfolio projects for various risk aversion coefficients**

	Amounts of resources used by optimal portfolio projects				
	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5
theta=0	2.4	16	17.4	1180	16798
theta=0.1	2.1	16	16.4	1283	15387
theta=0.2	1.8	20	17	1346	14388
theta=0.4	2	20	17.4	1420	15288
theta=0.5	2.1	18	17.4	1330	15388
theta=0.6	2.1	18	17.4	1330	15388
theta=0.7	2.1	18	17.4	1330	15388
theta=0.8	2.1	18	17.4	1330	15388
theta=0.9	2.1	18	17.4	1330	15388
theta=1	2.1	18	17.4	1330	15388
vector c	2.1	18	17.4	1330	15388

5. Conclusions

We have presented a zero-one mathematical programming model for project selection problem under risk and limited resources. It may be easily extended to include the inter-dependencies with other projects.

In view of the easily proven NP-hardness of the project portfolio selection models, it is attractive to search for fast approximation algorithms which solve the models. Such heuristics should be easier to implement than the fairly complex enumeration approach which can be applied only for the case the projects number is small. It would be of interest to investigate dynamic extensions of models presented above. This is left to future research.

In the paper was presented the workstation version of PROSEL. A Web-based version of PROSEL is intended to be realized in the near future. It will provide access to a large class of Internet users.

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