

Reliability Analysis Using Bayesian Networks

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Abstract: This paper considers the problem of reliability analysis of systems that consists of many groups of components organized in parallel and/or in serial given the reliability of the basic component. The mathematical tool used to tackle this problem is that of Bayesian networks (BN) that derive from convergence of statistics and Artificial Intelligence (AI). It consists of the representation of probabilistic causal relation between variables of a system.

Keywords: Reliability, Diagnosis, Bayesian Networks

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1. Introduction

Reliability analysis of systems (machines in a factory, different components in computer network, etc.) is very important in order to be able to deliver good products or services or to avoid some catastrophic events due to failure of component(s) and/or subsystem(s). In communication networks, for instance, it is important to have reliable components [4] in order to avoid frequent interruption in communications or unavailability of the communication system for a long period. In some cases the system reliability is a requirement from government agencies for the purpose of security (air-traffic for instance). This is the reason why it gets quite important for the system analyst to be able to quantify the reliability of the system that he/she is dealing with, in order to modify or to add some components, and so to meet the reliability requirements. Another important aspect is the diagnosis, that is, given a system failure, identification of the element(s) likely to be responsible for the failure. This analysis is a way enabling actions on the systems such as:

Reliability Allocation: Reliability allocation is the process of specifying a level of reliability for each subsystem or module in a system so as to achieve a system reliability objective. This process should be performed early in the design cycle to guide designers in choosing components, materials, and a design topology that will meet system objectives.

Reliability Optimization: It is not uncommon for a company to have many engineering projects in progress at any one time, for a given piece of equipment or product line. In this situation it is important that trade-off studies are conducted in order to prioritize potential product modifications. The best approach to this problem is to use optimization methods leading to a combination of modifications in terms of the largest increase in reliability for the lowest cost.

Reliability Prediction: Reliability prediction involves estimation of the reliability of equipment or products previous to their production or modification.

One can easily imagine other possible actions.

It is well known (see formulas (1) and (2)) that given the same components (number and reliability), a system organized in parallel is more reliable than one which is organized in serial. So one way to improve the reliability of a system is to put two or more components in parallel instead of one. But this process will increase the cost of the system. In order to find critical components simulation of the system can be useful because it might improve the efficiency of actions to be considered. But the analysis using formulas (1) and (2) is not useful if the system is composed of a large number of components as it is usually the case and also the problem of diagnosis is not possible in this way. In this paper we will consider the analysis of reliability using Bayesian networks, a tool that permits the representation of probabilistic causal relations. We will demonstrate that it is possible to transform a reliability structure of a system into the framework of a Bayesian network and then use the existing software dedicated to Bayesian networks to analyze or simulate the behavior of the system. Moreover such representation can be used for diagnosis. Existing works on reliability in the literature are more devoted to mathematical modeling of failure and repair processes (see for instance [1] and references therein) rather than easy way of computing reliability of a complex system. This work is on the contrary devoted to reduction of computational difficulties raised by putting reliability structure into the framework of Bayesian networks that allow an interactive simulation process.

The remainder of this paper is organized as follows: Section 2 is dedicated to the presentation and calculation of reliability of a system; Section 3 will consider the presentation of Bayesian networks; Section 4 is the main contribution of this paper and consists in the problem of putting a reliability structure of a system in the framework of a Bayesian network; finally, Section 5 is dedicated to the illustration of the idea of this paper by an example.

2. Reliability: Definitions and Structures

A given system is often composed of many basic components organized in subsystems, whose failure may cause the failure of the whole system or at least reduce its performance. In terms of reliability there are two types of organizations of components/subsystems in subsystems to form the whole system: organization in serial and organization in parallel. These organizations impact on the reliability of the resulting subsystem. The measure of reliability that will be considered in this paper is given in the following definition.

Definition 1: The reliability of a system or a component is the probability that this system or component shall perform its task without failure at an instant t knowing that it was performing well at t_0 ($t \geq t_0$).

The reliability of a given system is calculated by applying the following rules.

If a system consists of n components C_i , $i = 1, \dots, n$ in serial, the system will be performing well if and only if each component is performing well, so if p_i is the reliability of the i^{th} component, then the reliability of the system p_s is given by

$$p_s = \prod_{i=1}^n p_i \quad (1).$$

On the contrary if a system consists of n components C_i , $i = 1, \dots, n$ in parallel, the system will perform well if at least one of these components performs well, so if p_i is the reliability of the i^{th} component, then the reliability of the system p_p is given by

$$p_p = 1 - \prod_{i=1}^n (1 - p_i) \quad (2).$$

Formulas (1) and (2) constitute the fundamental relations for computing the reliability of a system because any system will consist of components or groups of components in serial and/or in parallel.

An example of a system composed of 4 basic components is given in Figure 1. The system has two main parallel branches, one branch consisting only of the components C_4 and the other one consisting of component C_1 in serial with a group formed by components C_2 and C_3 in parallel.

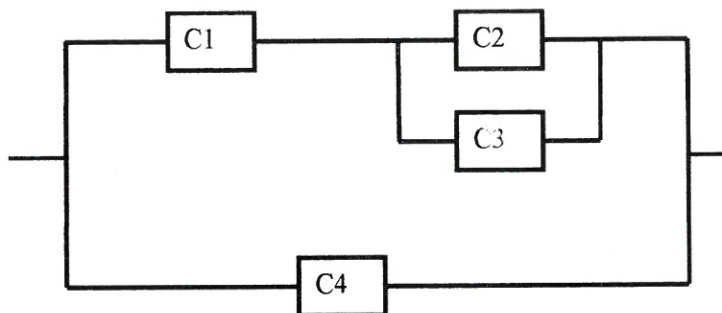


Figure 1. An Example of System With 4 Components

For a complex system with many basic components, manual calculation of the reliability of the system given that of basic elements, may be very demanding. Another problem is the diagnosis, that is finding the element(s) or subsystem(s) likely to be responsible for the failure of the system. This problem is not an easy problem when dealing with a very complex system. So a representation that could make this problem less complicated is necessary. In the following Sections Bayesian networks are used to represent the relationship between components and subsystems. We will show that once the network is defined (structure and parameters), the problem of finding the reliability of a subsystem or of the whole system given the reliability of its basic components becomes an easy problem. Moreover the propagation of an event, failure of a subsystem or the system will permit to identify components most likely to cause this failure, that is to solve the problem of diagnosis. Bayesian networks allow also to easily simulate approximate operation of a subsystem, say, a subsystem composed of elements in serial, the probability that the subsystem fails knowing that one of its elements fails is not 1 but somewhere between 0 and 1, simulating an approximate functioning of the subsystem. The approximate functioning of basic components can also be taken into account by allowing them to have different and not only two states, those of failure and no failure.

3. Introduction to Bayesian Networks

Bayesian Networks (BN) derive from the convergence of statistical methods that permit one to go from information (data) to knowledge (probability laws, relationship between variables, ...) with Artificial Intelligence (AI) that permits computers to deal with knowledge (not only information), (see for example [3]). The terminology BN comes from Thomas Bayes's [2] 18th century work. Its actual development is due to [5]. The main purpose of BN is to integrate uncertainty into expert systems. Indeed, an expert, most of the time, has only approximate knowledge of the system, that he/she formulates in terms like: **A** has an influence on **B** ; if **B** is observed, there is a great chance that **C** occurs and so on. It becomes obvious that this tool is well -suited to deal with the problem concerned, as expert knowledge can be needed to estimate, for instance, the reliability of a component or how components of a subsystem interact in terms of reliability.

BNs consist in a graphical representation of the causality relation between a cause and its effects. Figure 2 shows that **A** is the cause and **B** its effect.



Figure 2. Causality Representation in BN

But as this relation of causality is not strict, the next step is to quantify it by giving the probability of occurrence of **B** when **A** is realized. So an BN consists of an oriented graph where nodes represent variables, and oriented arcs represent the causality relation and a set of probabilities. A rigorous definition of a BN is given below.

Definition 2 (adapted from [3]). Let consider

- an acyclic oriented graph $G = (V, A)$ where V and A represent the set of nodes and the set of arcs, respectively;
- a trial E with whom there is associated a finite probability space (Ω, Z, P) , and n random variables $(X_i)_{1 \leq i \leq n}$; G and E define a Bayesian Network, noted $B = (G, P)$, if and only if
- there exists a bijection between the nodes of G and the variables $(X_i)_{1 \leq i \leq n}$
- the factorization property

$$P(X_1, X_2, \dots, X_n) = \prod_{1 \leq i \leq n} P(X_i / C(X_i))$$

where $C(X_i)$ representing the set of causes (parents) of X_i (notice that because of bijection between V_i and X_i , we will be using interchangeably V_i and X_i) is verified [3] and $P(X_1, X_2, \dots, X_n)$ is the probability of a simultaneous realization of variables X_1, X_2, \dots, X_n and $P(X_i / Y_i)$ is the conditional probability (probability of X_i knowing Y_i).

The main purpose of the BN is to propagate certain knowledge of the state of one or more particular nodes through the network so that one shall learn how the beliefs of the expert in the BN will change. So given an BN, $B = (G, P)$ and the set of n nodes $X = \{X_1, X_2, \dots, X_n\}$ it returns to compute

$$P(X_i / Y) \quad \text{where } Y \subset X, \quad X_i \notin Y.$$

Using the properties of chains, trees, networks and the properties of conditional probabilities, algorithms can be derived to propagate certain knowledge in the BN in terms of modifying the beliefs. For example for a chain of length n , if X_i is downstream of X_j but not a direct relative, then

$$P(X_i / X_j) = \sum_{X_{i-1}} P(X_i / X_{i-1}) P(X_{i-1} / X_j).$$

If X_{i-1} is a direct relative of X_j then stop if not decompose $P(X_{i-1} / X_j)$ as previously shown. For other forms (trees, general networks), this would rather difficult but there exist algorithms, based on other well-known algorithms of networks: maximum flow, short path, maximal weight trees, etc. For more information one may consult specialized literature [3] and the references therein.

The manual calculation of parameters and of prediction, given that some events may occur, can be hard if the BN is large (many variables are interconnected). To overcome this and because of large use of BN in many domains: data mining, diagnostics, planning, banks, finance and defence [3], to name just a few, some software is being developed to aid quick modeling and analysis of BN. The leader in this domain seems to be the Hugin company that has developed a graphical oriented software called Hugin Explorer. A free version -Hugin Lite- is available for download on the website of the company; Microsoft proposes MSBN, a graphical tool for constructing networks. For more information on the developer related to BN, see [3].

In the following Section the formulation of a reliability analysis problem as Bayesian network will be considered.

4. Modeling Reliability Structure as BN

As previously stated, an BN is completely determined by its structure and some parameters, namely a priori probabilities of nodes without parents and conditional probabilities of intermediate nodes for different configurations of states of their parents. The task of transforming a reliability structure into an BN consists therefore in determination of the resulting BN structure and its parameters. This problem is further dealt with.

4.1 Structure of the Resulting BN

The basic element of the system will be the component of which reliability will be available. The components and subsystems are hierarchically regrouped until building the whole system. Two or more components or subsystems will be regrouped into one subsystem if and only if they are organized in serial or in parallel. This process is repeated until the whole system is covered. In the framework of the Bayesian network, each component will be associated with a node N_c that is a node without parents. A subsystem will be associated with an intermediate node N_{int} . The global system is a particular intermediate node S that is a node without descendance. There exist two types of intermediate nodes: nodes $N_{int,s}$ whose parents act in serial and nodes $N_{int,p}$ whose parents act in parallel. The transformation of the reliability structure into an BN structure takes place hierarchically by regrouping basic components into subsystems that constitute intermediate nodes of one of the previous types in so far as the whole system is concerned. We will consider that the resulting BN has n_c nodes of type N_c , n_s nodes of type n_p and N_c nodes of type $N_{int,p}$.

Once defined the structure of the BN, its parameters must be determined: a priori probabilities of states of nodes without parents, that is nodes of type N_c and the conditional probabilities of intermediate nodes, that is nodes of type $N_{int,s}$ and $N_{int,p}$, knowing some configurations of the states of their parents. In the following Section the parameters of the resulting BN will be computed.

4.2 Parameters of the Resulting BN

Though more than two states can be considered for each node, we will consider that each node has only two states: failure (F) or no failure (NF). The generalization to more than two states would not be very difficult. The parameters of the BN consist of two types: a priori probabilities of basic components given by their reliability or the complement of their reliability, and conditional probabilities of the intermediate nodes given the configurations of their parents. A priori probabilities of nodes of type N_c are fully determined by the reliability of the corresponding components. Conditional probabilities of intermediate nodes depend on their type.

4.2.1 Parameters for Nodes of Type N_c

Let suppose that there are n_c basic components in the system and p_i is the reliability of the component i , then parameters of each node of type N_c are given by

$$P(N_c^i = NF) = p_i \quad \text{and} \quad P(N_c^i = F) = 1 - p_i, \quad i = 1, \dots, n_c.$$

4.2.2 Parameters for Nodes of Type $N_{int,p}$

As stated in the previous paragraph, any intermediate node $N_{int,p}^j$, $j = 1, \dots, n_p$, is constituted by elements in parallel so we have the following conditional probabilities:

$$P(N_{int,p}^j = NF / C(N_{int,p}^j)) = 0 \Leftrightarrow \forall N \in C(N_{int,p}^j), N = F$$

else $P(N_{int,p}^j = NF / C(N_{int,p}^j)) = 1$

and

$$P(N_{int,p}^j = F / C(N_{int,p}^j)) = 1 \Leftrightarrow \forall N \in C(N_{int,p}^j), N = F$$

else $P(N_{int,p}^j = F / C(N_{int,p}^j)) = 0$.

4.2.3 Parameters for Nodes of Type $N_{int,s}$

A node $N_{int,s}^k$, $k = 1, \dots, n_s$ is formed by parents acting in serial, resulting that:

$$P(N_{int,s}^k = NF / C(N_{int,s}^k)) = 1 \Leftrightarrow \forall N \in C(N_{int,s}^k), N = NF$$

else $P(N_{int,s}^k = NF / C(N_{int,s}^k)) = 0$

and

$$P(N_{int,s}^k = F / C(N_{int,s}^k)) = 0 \Leftrightarrow \forall N \in C(N_{int,s}^k), N = NF$$

else $P(N_{int,s}^k = F / C(N_{int,s}^k)) = 1$.

These two paragraphs give the fundamental material to completely transform a reliability structure into a BN framework. The following Section applies this process to the system given in Figure 1.

5. Application

We will apply the method described in the previous Section to the system given in Figure 1. Figure 3 shows a representation of the system in Figure 1 in the framework of BN. Let explain how the BN structure is obtained:

- C_1, C_2, C_3 and C_4 are nodes of type N_c ;
- G_2 is a node of type $N_{int,p}$ formed by regrouping the parallel components C_2 and C_3 ;
- G_1 is a node of type $N_{int,s}$ formed by regrouping the component C_1 and the subsystem G_2 ;
- finally the subsystem G_1 and the component C_4 act in parallel on the system so that this one is a node of type $N_{int,p}$.

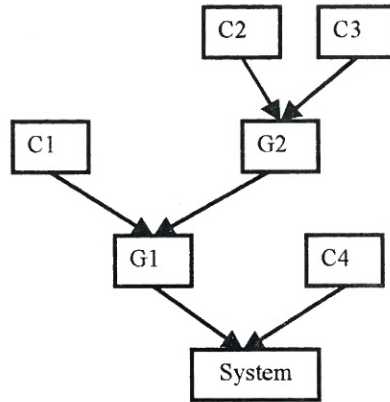


Figure 3. Representation of Figure 1 By BN

To define the parameters of this BN, let suppose that every component has the same reliability 0.9 then the parameters of basic elements are given by

$$P(C_i = NF) = 0.9 \quad \text{and} \quad P(C_i = F) = 0.1, \quad i = 1, \dots, 4$$

and the parameters of intermediate nodes are given in Figure 4 through Figure 6.

G2	C2	F		NF	
	C3	F	NF	F	NF
F		1	0	0	0
NF		0	1	1	1

Figure 4. Conditional Probabilities for Intermediate Node G_2

G1	C1	F		NF	
	G2	F	NF	F	NF
F		1	1	1	0
NF		0	0	0	1

Figure 5. Conditional Probabilities for Intermediate Node G_1

System	C4	F		NF	
	G1	F	NF	F	NF
F		1	0	0	0
NF		0	1	1	1

Figure 6. Conditional Probabilities for Intermediate Node System

This model was introduced in the Hugin Lite software, to simulate the behavior of the system. Figure 7 shows the result of the failure of the whole system (probabilities are given in percentage). As one can see, the failure of the whole system means that component C_4 and subsystem G_1 surely fail. The element responsible for the failure of G_1 is C_1 with 91.74% of chances against G_2 with 9.17% of chances. So the system critical elements, in terms of reliability, are C_1 and C_4 . One can simulate many other configurations.

C1		
<input checked="" type="checkbox"/>	91.74	Failure
<input type="checkbox"/>	8.26	No Failure
C2		
<input type="checkbox"/>	17.43	Failure
<input checked="" type="checkbox"/>	82.57	No Failure
C3		
<input type="checkbox"/>	17.43	Failure
<input checked="" type="checkbox"/>	82.57	No Failure
C4		
<input checked="" type="checkbox"/>	100.00	Failure
	-	No Failure
G1		
<input checked="" type="checkbox"/>	100.00	Failure
	-	No Failure
G2		
<input type="checkbox"/>	9.17	Failure
<input checked="" type="checkbox"/>	90.83	No Failure
System		
<input checked="" type="checkbox"/>	* 100.00	Failure
	-	No Failure

Figure 7. Results of Simulation of the System Failure

6. Conclusion

The problem of analyzing the behavior of a system in terms of reliability using Bayesian networks has been considered in this paper. It is proved that it is always possible to transform a reliability structure of a system into the framework of a Bayesian network and then to use a software dedicated to this mathematical tool to analyze the system. Moreover the usefulness of this transformation for diagnosis making was shown. An example has been considered that shows the efficiency of the proposed approach.

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