

# Analysis of the Simple WISP Method Results Using Different Normalization Procedures

Edmundas Kazimieras ZAVADSKAS<sup>1\*</sup>, Dragisa STANUJKIC<sup>2</sup>,  
Darjan KARABASEVIC<sup>3</sup>, Zenonas TURSKIS<sup>1</sup>

<sup>1</sup> Institute of Sustainable Construction, Vilnius Gediminas Technical University  
Saulėtekio al. 11, LT-10223 Vilnius, Lithuania

edmundas.zavadskas@vilniustech.lt (\*Corresponding author), zenonas.turskis@vilniustech.lt

<sup>2</sup> University of Belgrade, Technical Faculty in Bor, Serbia Vojske Jugoslavije 12, Bor 19210, Serbia  
dstanujkic@tfbor.bg.ac.rs

<sup>3</sup> Faculty of Applied Management, Economics and Finance, University Business Academy in  
Novi Sad Jevrejska 24/1, Belgrade 11000, Serbia  
darjan.karabasevic@mef.edu.rs

**Abstract:** The standard WISP method uses the max normalization procedure. In order to check its robustness, the use of the WISP method with square root and sum normalization procedures is discussed in this article. In order to check the similarity of the obtained results, several analyzes were performed using the Python programming language, where the similarity of the obtained results was checked using the cosine similarity measure. The similarity was also checked concerning the results obtained using prominent MCDM methods. The achieved results confirm the significant similarity of the results obtained using the WISP method with the results obtained using the selected MCDM methods, even in the case of using square root or sum normalization procedures.

**Keywords:** Simple WISP, MCDM, Normalization procedure.

## 1. Introduction

The use of multiple criteria decision-making (MCDM) methods for solving various decision-making problems in many research areas is still an actual domain of exploration. As a result of previous researches, many well-known MCDM methods have been proposed, such as the Simple Additive Weighting (SAW) method (MacCrimmon, 1968), Elimination Et Choix Traduisant la REalité (ELECTRE) method (Roy, 1968), Compromise Programming (CP) method (Zeleny, 1973), Analytic Hierarchy Process (AHP) method (Saaty, 1977), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method (Hwang & Yoon, 1981), Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) method (Brans, 1982), Multi-criteria Optimization and Compromise Solution (VIKOR) method (Opricovic, 1998), Additive Ratio Assessment (ARAS) method (Zavadskas & Turskis, 2010) and the Preference Selection Index (PCI) method (Maniya & Bhatt, 2010).

Besides, some new MCDM methods have also been proposed, such as the Weighted Aggregated Sum Product Assessment (WASPAS) method (Zavadskas et al., 2012), Combined Compromise Solution (CoCoSo) method (Yazdani et al., 2019), the Multiple Criteria Ranking by Alternative Trace (MCRAT) method (Urošević et al., 2021), and the MULTIMOOSRAL method (Ulutaş et al., 2021).

In addition to using existing and new MCDM methods, normalization procedures have also been the subject of numerous studies. A comprehensive review of normalization procedures can be found in Jahan and Edwards (Jahan & Edwards, 2015). After a detailed literature review, the authors identified thirty-one normalization procedures, classified them, and made recommendations for their use. Vafaei et al. (2019) and Ersoy (2021) suggest frameworks for selecting normalization procedures.

In numerous articles, Opricovic and Tzeng (2004), Acuña-Soto et al. (2018), Peldschus (2018), Vinogradova (2019), Sařabun et al. (2020) and Jafaryeganeh et al. (2020) have analyzed the impact of the use of different normalization procedures on the results obtained using different MCDM methods.

Numerous other approaches can also be found in the literature. For example, Tsaur (2011) uses the TOPSIS method with a new normalization procedure, Serrai et al. (2017) combines four MCDM methods, VIKOR, SAW, TOPSIS, and COPRAS, i.e. four normalization procedures, to select the dominant alternative, Zhou et al. (2020) uses the VIKOR method with the hybrid normalization technique, while Ataei et al. (2020) consider the use of a new Ordinal Priority Approach (OPA) method, based on linear programming,

which does not require normalization, while Yue (2021) proposed new normalization procedures adapted for group decision making. Stanujkic et al. (2017) considered the use of target-based normalization procedures with MULTIMOORA, ARCAS, and WASPAS methods.

Also evident are articles that analyze the effects of different normalization procedures with selected MCDM methods.

Stanujkic et al. (2021) proposed a new MCDM method that integrates some approaches implemented in the ARAS, WASPAS, CoCoSo, and MULTIMOORA methods, Simple Weighted Sum-Product (WISP) method. The Simple WISP method uses the max normalization procedure. This article considers the possibility of using the Simple WISP method with square root and sum normalization procedures. In order to confirm the similarity of the obtained results, several simulations were performed using the Python programming language, where the similarity of the obtained results was checked using cosine similarity measures (CSM). The obtained results were also compared with those obtained by applying some well-known MCDM methods. Finally, the similarity of the obtained results was checked on an example of investment projects evaluation.

Therefore, this article is organized as follows. In section 2, the Simple WISP method is presented in detail and uses normalization procedures and CSM. Section 3 presents the results of five simulations performed to determine the similarity between the results obtained using the standard Simple WISP method and those obtained by using the Simple WISP method with square root and sum normalization procedures. The similarity of the obtained results was also checked concerning some well-known MCDM methods. In section 4, an example of investment projects evaluation is considered to recheck the similarity of the obtained results. Finally, conclusions are given at the end of the article, in section 5.

## 2. Preliminaries

### 2.1 The WISP Method

The Simple Weighted Sum Product (WISP) method integrates four relationships between beneficial and non-beneficial criteria to determine the overall utility of an alternative. The process of ranking and selecting the best alternative

using this method for a problem containing  $m$  alternatives and  $n$  beneficial and non-beneficial criteria can be shown using the following steps:

Step 1. Construct a decision-making matrix and determine criteria weights.

Step 2. Construct a normalized decision-making matrix as follows:

$$r_{ij} = \frac{x_{ij}}{\max_k x_{kj}}, \quad (1)$$

where  $r_{ij}$  denotes a dimensionless number representing a normalized rating of alternative  $i$  regarding criterion  $j$ .

Step 3. Calculate the values of four utility measures, as follows:

$$u_i^{sd} = \sum_{j \in \Omega_{\max}} r_{ij} w_j - \sum_{j \in \Omega_{\min}} r_{ij} w_j, \quad (2)$$

$$u_i^{pd} = \prod_{j \in \Omega_{\max}} r_{ij} w_j - \prod_{j \in \Omega_{\min}} r_{ij} w_j, \quad (3)$$

$$u_i^{sr} = \frac{\sum_{j \in \Omega_{\max}} r_{ij} w_j}{\sum_{j \in \Omega_{\min}} r_{ij} w_j}, \text{ and} \quad (4)$$

$$u_i^{pd} = \frac{\prod_{j \in \Omega_{\max}} r_{ij} w_j}{\prod_{j \in \Omega_{\min}} r_{ij} w_j} \quad (5)$$

where:  $u_i^{sd}$  and  $u_i^{pd}$  denote the differences between the weighted sum and the weighted product of normalized ratings of alternative  $i$ , respectively, and  $\Omega_{\max}$  and  $\Omega_{\min}$  denote a set of beneficial and a set of non-beneficial criteria, respectively. Similar to the previous one,  $u_i^{sr}$  and  $u_i^{pr}$  denote the ratios between the weighted sum and the weighted product of normalized ratings of alternative  $i$ , respectively.

Step 4. Recalculate values of four utility measures, as follows:

$$\bar{u}_i^{sd} = \frac{1 + u_i^{sd}}{1 + \max_k u_k^{sd}}, \quad (6)$$

$$\bar{u}_i^{pd} = \frac{1 + u_i^{pd}}{1 + \max_k u_k^{pd}}, \quad (7)$$

$$\bar{u}_i^{sr} = \frac{1 + u_i^{sr}}{1 + \max_k u_k^{sr}}, \text{ and} \quad (8)$$

$$\bar{u}_i^{pr} = \frac{1 + u_i^{pr}}{1 + \max_k u_k^{pr}}, \quad (9)$$

where:  $\bar{u}_i^{sd}$ ,  $\bar{u}_i^{pd}$ ,  $\bar{u}_i^{sr}$  and  $\bar{u}_i^{pr}$  denote the recalculated values of  $u_i^{sd}$ ,  $u_i^{pd}$ ,  $u_i^{sr}$  and  $u_i^{pr}$ .

Step 5. Determine the overall utility  $u_i$  of each alternative as follows:

$$u_i = \frac{1}{4}(\bar{u}_i^{sd} + \bar{u}_i^{pd} + \bar{u}_i^{sr} + \bar{u}_i^{pr}). \quad (10)$$

Step 6. Rank the alternatives and select the most suitable one. The alternatives are ranked in descending order, and the alternative with the highest value of  $u_i$  is the most preferred one.

## 2.2 Normalization Procedures

Different MCDM methods use different normalization procedures, such as square root, sum, min, and logarithmic (Zavadskas & Turskis, 2008) normalization.

Using min normalization has already been shown using Equation (1).

The normalization procedure of the square root normalization is as follows:

$$r_{ij}^{sq} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^m x_{kj}^2}}. \quad (11)$$

The normalization procedure of the sum normalization is as follows:

$$r_{ij}^s = \frac{x_{ij}}{\sum_{k=1}^m x_{kj}}. \quad (12)$$

The computational procedure of the Simple WISP method with square root and sum normalization procedures can be implemented using the Python and NumPy library.

## 2.3 The Cosine Similarity Measure

Cosine similarity is an angle-based measure (*csm*) of similarity between two  $m$ -dimensional vectors in  $m$ -dimensional space (Candan & Sapino, 2010). The cosine similarity measure for two vectors  $\vec{a} = (a_1, a_2, \dots, a_n)$  and  $\vec{b} = (b_1, b_2, \dots, b_n)$  is as follows:

$$csm(\vec{a}, \vec{b}) = \frac{\sum_{i=1}^m a_i b_i}{\sqrt{\sum_{i=1}^m a_i^2} \sqrt{\sum_{i=1}^m b_i^2}} \quad (13)$$

where  $m$  denotes the number of vector elements.

## 3. Comparison of Results Obtained Using WISP Method with Those of Three Normalization Procedures

In order to check the similarity of the results achieved using the standard WISP method and those obtained through the recalculated WISP method with the use of vector or sum normalization procedures, several simulations were conducted. In these simulations, decision matrices were generated using random numbers. The simulations were performed using Python and its NumPy library.

Based on the generated decision matrices, the calculation was performed, and the obtained results were placed in appropriate vectors to check the similarity by applying the cosine similarity measure. In the performed simulations, the matching of the best-placed alternative and the matching of the rank of all the considered alternatives were performed by applying three selected normalization procedures.

### 3.1 The First Simulation

The first simulation was based on the generation of 10 decision matrices used to form the vectors based on the calculated similarity. Matrix generation was performed five times cyclically to avoid the influence of randomly generated numbers on similarity. For the same reason, this simulation was repeated three times.

In this simulation, 5 x 4 matrices were used, with the first two criteria being beneficial and the next two non-beneficial. The following weight vector  $w^i = \{0.30, 0.20, 0.20, 0.30\}$  was used in this simulation. The obtained results, i.e., the similarity of matching the first-placed alternative using the WISP method and the alternatives using the WISP method with vector and sum normalization procedures, are shown in Table 1.

The results shown in Tables 1 and 2 indicate high similarity between the calculation results obtained using the WISP method and those obtained using the WISP method with the considered normalization procedures (*In this and subsequent experiments, the seed () function is not set to a specific value. Therefore, certain, but not significant, differences may occur in the case of repeated experiments*).

**Table 1.** The similarity between best-placed alternatives using the WISP method and different normalization procedures achieved based on the first simulation

| Repetition Cycle | Vector normalization |       |       | Sum normalization |              |       |
|------------------|----------------------|-------|-------|-------------------|--------------|-------|
|                  | 1                    | 2     | 3     | 1                 | 2            | 3     |
| 1                | 1.000                | 1.000 | 1.000 | 0.994             | 0.984        | 1.000 |
| 2                | 0.983                | 1.000 | 1.000 | 0.991             | 0.993        | 1.000 |
| 3                | 1.000                | 1.000 | 1.000 | 1.000             | 1.000        | 1.000 |
| 4                | 0.960                | 1.000 | 1.000 | 0.995             | 1.000        | 1.000 |
| 5                | 1.000                | 0.996 | 1.000 | 0.997             | 0.992        | 1.000 |
| min              | <b>0.960</b>         | 0.996 | 1.000 | 0.991             | <b>0.984</b> | 1.000 |

**Table 2.** The similarity between best-placed alternatives using the WISP method and WISP method and different normalization procedures achieved based on the first simulation

| Repetition Cycle | Vector normalization |       |       | Sum normalization |       |              |
|------------------|----------------------|-------|-------|-------------------|-------|--------------|
|                  | 1                    | 2     | 3     | 1                 | 2     | 3            |
| 1                | 0.994                | 1.000 | 0.996 | 0.995             | 0.998 | 0.996        |
| 2                | 0.991                | 0.992 | 0.994 | 0.993             | 0.995 | 0.992        |
| 3                | 1.000                | 0.996 | 0.999 | 0.997             | 0.993 | 0.996        |
| 4                | 0.995                | 0.998 | 0.999 | 0.996             | 0.998 | 0.996        |
| 5                | 0.997                | 0.996 | 0.996 | 0.997             | 0.994 | 0.993        |
| min              | <b>0.991</b>         | 0.992 | 0.994 | 0.993             | 0.993 | <b>0.992</b> |

### 3.2 The Second Simulation

The second conducted simulation was very similar to the first simulation, with the difference that in this case, the vectors were generated based on 50 randomly generated matrices, as opposed to 10 in the first simulation. Similar to the first simulation, the achieved results are shown in Tables 3 and 4.

**Table 3.** The similarity between best-placed alternatives using the WISP method and different normalization procedures achieved based on the second simulation

| Repetition Cycle | Vector normalization |       |              | Sum normalization |       |              |
|------------------|----------------------|-------|--------------|-------------------|-------|--------------|
|                  | 1                    | 2     | 3            | 1                 | 2     | 3            |
| 1                | 1.000                | 1.000 | 1.000        | 0.999             | 1.000 | 0.993        |
| 2                | 0.999                | 1.000 | 1.000        | 1.000             | 1.000 | 0.999        |
| 3                | 1.000                | 1.000 | 0.999        | 0.999             | 0.996 | 0.999        |
| 4                | 1.000                | 0.999 | 1.000        | 1.000             | 0.998 | 0.999        |
| 5                | 1.000                | 0.989 | 0.996        | 0.986             | 0.989 | 0.996        |
| min              | 0.999                | 0.989 | <b>0.996</b> | 0.986             | 0.989 | <b>0.993</b> |

**Table 4.** The similarity between best-placed alternatives using the WISP method and WISP method and different normalization procedures achieved based on the second simulation

| Repetition Cycle | Vector normalization |       |              | Sum normalization |       |       |
|------------------|----------------------|-------|--------------|-------------------|-------|-------|
|                  | 1                    | 2     | 3            | 1                 | 2     | 3     |
| 1                | 0.999                | 0.998 | 0.998        | 0.999             | 0.997 | 0.997 |
| 2                | 0.995                | 0.996 | 0.999        | 0.993             | 0.995 | 0.999 |
| 3                | 0.997                | 0.998 | 0.995        | 0.997             | 0.998 | 0.994 |
| 4                | 0.998                | 0.997 | 0.997        | 0.998             | 0.996 | 0.997 |
| 5                | 0.998                | 0.997 | 0.998        | 0.996             | 0.996 | 0.997 |
| min              | <b>0.995</b>         | 0.996 | <b>0.995</b> | <b>0.993</b>      | 0.995 | 0.994 |

As in the first simulation, the results shown in Tables 3 and 4 indicate high similarity between the results obtained using the WISP method and those obtained using the WISP method with the considered normalization procedures. In addition, the increase in the number of cycles in which matrices were generated affected the increase of the cosine similarity measures, instead of their decreasing.

### 3.3 The Third Simulation

The third simulation was similar to the second, except that the weights of the criteria were also generated as random numbers. The obtained results are shown in Tables 5 and 6.

From Tables 5 and 6, it can be seen that the variations in the criteria weight did not significantly influence a change in the similarity of the results obtained using the standard WISP method and those obtained by using the modified WISP method with vector and sum normalization.

**Table 5.** The similarity between best-placed alternatives using the WISP method and different normalization procedures achieved based on the third simulation

| Repetition Cycle | Vector normalization |       |       | Sum normalization |       |       |
|------------------|----------------------|-------|-------|-------------------|-------|-------|
|                  | 1                    | 2     | 3     | 1                 | 2     | 3     |
| 1                | 1.000                | 0.991 | 0.999 | 0.997             | 0.991 | 0.991 |
| 2                | 1.000                | 0.999 | 0.997 | 0.986             | 0.999 | 0.997 |
| 3                | 1.000                | 0.993 | 0.989 | 0.999             | 0.993 | 0.982 |
| 4                | 0.982                | 0.982 | 0.999 | 0.982             | 0.982 | 0.999 |
| 5                | 0.999                | 0.996 | 0.985 | 0.999             | 0.995 | 0.993 |
| min              | 0.982                | 0.982 | 0.985 | 0.982             | 0.982 | 0.982 |

**Table 6.** The similarity between best-placed alternatives using the WISP method and WISP method and different normalization procedures achieved based on the third simulation

| Repetition Cycle | Vector normalization |       |       | Sum normalization |       |       |
|------------------|----------------------|-------|-------|-------------------|-------|-------|
|                  | 1                    | 2     | 3     | 1                 | 2     | 3     |
| 1                | 0.997                | 0.996 | 0.997 | 0.991             | 0.995 | 0.995 |
| 2                | 0.998                | 0.997 | 0.998 | 0.995             | 0.995 | 0.995 |
| 3                | 0.995                | 0.997 | 0.996 | 0.994             | 0.996 | 0.995 |
| 4                | 0.996                | 0.997 | 0.997 | 0.993             | 0.995 | 0.996 |
| 5                | 0.997                | 0.995 | 0.996 | 0.996             | 0.995 | 0.995 |
| min              | 0.995                | 0.995 | 0.996 | 0.991             | 0.995 | 0.995 |

### 3.4 The Fourth Simulation

In the following simulation, the influence of the type of optimization was checked, due to which the number of criteria was increased to five. To ensure that the decision matrix contains at least one beneficial and non-beneficial criteria, criterion  $C_1$  is set to max type while criterion  $C_2$  to min type. The optimization type of the remaining three criteria was generated based on random numbers. As in the previous simulation, decision matrices and criterion weights were also generated using random numbers.

The results obtained based on this simulation are shown in Tables 7 and 8.

**Table 7.** The similarity between best-placed alternatives using the WISP method and different normalization procedures achieved based on the fourth simulation

| Repetition Cycle | Vector normalization |       |       | Sum normalization |       |       |
|------------------|----------------------|-------|-------|-------------------|-------|-------|
|                  | 1                    | 2     | 3     | 1                 | 2     | 3     |
| 1                | 1.000                | 0.999 | 0.982 | 0.999             | 0.999 | 0.982 |
| 2                | 1.000                | 1.000 | 0.999 | 0.999             | 0.999 | 0.991 |
| 3                | 0.985                | 0.983 | 0.971 | 0.982             | 0.983 | 0.974 |
| 4                | 0.996                | 0.995 | 0.999 | 0.996             | 0.994 | 0.991 |
| 5                | 1.000                | 1.000 | 0.992 | 1.000             | 0.999 | 0.992 |
| min              | 0.985                | 0.983 | 0.971 | 0.982             | 0.983 | 0.974 |

**Table 8.** The similarity between best-placed alternatives using the WISP method and WISP method and different normalization procedures achieved based on the fourth simulation

| Repetition Cycle | Vector normalization |       |       | Sum normalization |       |       |
|------------------|----------------------|-------|-------|-------------------|-------|-------|
|                  | 1                    | 2     | 3     | 1                 | 2     | 3     |
| 1                | 0.997                | 0.997 | 0.998 | 0.995             | 0.995 | 0.997 |
| 2                | 0.999                | 0.997 | 0.996 | 0.997             | 0.994 | 0.994 |
| 3                | 0.996                | 0.994 | 0.998 | 0.993             | 0.993 | 0.996 |
| 4                | 0.994                | 0.996 | 0.996 | 0.992             | 0.995 | 0.995 |
| 5                | 0.995                | 0.996 | 0.994 | 0.993             | 0.994 | 0.993 |
| min              | 0.994                | 0.994 | 0.994 | 0.992             | 0.993 | 0.993 |

The obtained results indicate a high similarity between the results achieved using the WISP method and those achieved by different normalization procedures.

### 3.5 The Fifth Simulation

The fifth simulation is also similar to the previous one, except that in this simulation, the number of generated matrices has been increased from 50 to 500. The results obtained based on this simulation are shown in Tables 9 and 10.

**Table 9.** The similarity between best-placed alternatives using the WISP method and different normalization procedures achieved based on the fifth simulation

| Repetition Cycle | Vector normalization |       |       | Sum normalization |       |       |
|------------------|----------------------|-------|-------|-------------------|-------|-------|
|                  | 1                    | 2     | 3     | 1                 | 2     | 3     |
| 1                | 0.993                | 0.990 | 0.994 | 0.990             | 0.987 | 0.994 |
| 2                | 0.994                | 0.996 | 0.993 | 0.993             | 0.992 | 0.988 |
| 3                | 0.994                | 0.992 | 0.990 | 0.988             | 0.991 | 0.992 |
| 4                | 0.998                | 0.998 | 0.989 | 0.996             | 0.994 | 0.982 |
| 5                | 0.995                | 0.995 | 0.993 | 0.993             | 0.991 | 0.988 |
| min              | 0.993                | 0.990 | 0.989 | 0.988             | 0.987 | 0.982 |

**Table 10.** The similarity between best-placed alternatives using the WISP method and WISP method and different normalization procedures achieved based on the fifth simulation

| Repetition Cycle | Vector normalization |       |       | Sum normalization |       |       |
|------------------|----------------------|-------|-------|-------------------|-------|-------|
|                  | 1                    | 2     | 3     | 1                 | 2     | 3     |
| 1                | 0.995                | 0.996 | 0.996 | 0.993             | 0.993 | 0.994 |
| 2                | 0.996                | 0.996 | 0.996 | 0.994             | 0.995 | 0.993 |
| 3                | 0.997                | 0.995 | 0.996 | 0.994             | 0.992 | 0.995 |
| 4                | 0.996                | 0.997 | 0.996 | 0.994             | 0.994 | 0.993 |
| 5                | 0.996                | 0.996 | 0.996 | 0.994             | 0.994 | 0.994 |
| min              | 0.995                | 0.995 | 0.996 | 0.993             | 0.992 | 0.993 |

As in previous simulations, the obtained results confirm a high correlation between the results obtained by the WISP method and those obtained by using three different normalization procedures.

## 4. A Numerical Illustration

In order to further consider the possibility of applying the WISP method with vector and sum normalization procedure, a numerical illustration regarding the selection of investment projects is presented in this section. In addition to the WISP method, with different normalization procedures,

**Table 11.** The evolutionary criteria and criteria weights

| Criteria |                               | Optimization | Criteria weights |
|----------|-------------------------------|--------------|------------------|
| $C_1$    | Net Present Value – NPV       | max          | 0.23             |
| $C_2$    | Internal Rate of Return – IRR | max          | 0.20             |
| $C_3$    | Profitability Index – PI      | max          | 0.16             |
| $C_4$    | Pay Back Period – PBP         | min          | 0.20             |
| $C_5$    | Risk – R                      | min          | 0.20             |

**Table 12.** The characteristics of investments projects

|  | Project $A_1$ | Project $A_2$ | Project $A_3$ | Project $A_4$ |
|--|---------------|---------------|---------------|---------------|
| Project costs (Project costs and Average annual profits are shown in thousands of euros) | 150           | 160           | 170           | 180           |
| Average annual profit  | 40            | 40            | 40            | 45            |
| Years of project duration  | 5             | 6             | 7             | 7             |
| Risk ( <i>Project risk is expressed using a scale of 1 - 10.</i> )                       | 5             | 6             | 7             | 8             |

the evaluation was also performed using several prominent MCDM methods.

The evaluation criteria and their weights are shown in Table 11, while the initial decision matrix on which the evaluation was performed is shown in Table 12.

Economic indicators of these investment projects, calculated based on Table 12, are shown in Table 13.

**Table 13.** The economic indicators of the four investments projects

|       | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|-------|-------|-------|-------|-------|-------|
|       | NPV   | IRR   | PI    | PBP   | R     |
| $A_1$ | 23.18 | 10.4% | 1.15  | 3.8   | 5     |
| $A_2$ | 43.03 | 13.0% | 1.27  | 4.0   | 6     |
| $A_3$ | 61.45 | 14.3% | 1.36  | 4.3   | 7     |
| $A_4$ | 80.39 | 16.3% | 1.45  | 4.0   | 8     |

In the above calculation, the required rate of return was of 5.0%. The normalized decision matrix, calculated using Equation (1), is shown in Table 14.

**Table 14.** Normalized decision matrix applying max normalization

|       | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|-------|-------|-------|-------|-------|-------|
| $A_1$ | 0.29  | 0.64  | 0.80  | 0.88  | 0.63  |
| $A_2$ | 0.54  | 0.79  | 0.88  | 0.94  | 0.75  |
| $A_3$ | 0.76  | 0.88  | 0.94  | 1.00  | 0.88  |
| $A_4$ | 1.00  | 1.00  | 1.00  | 0.94  | 1.00  |

Table 18 shows calculated values of four utility measures using Equations (2) - (5) and their recalculated values using Equations (6) - (9).

Table 15 shows the overall utilities of the alternatives and the rank of each alternative. From

the mentioned table, it can be seen that alternative  $A_4$  is more acceptable.

Calculation details obtained using the WISP method and square root normalization are shown in Table 16. The calculation details obtained using the WISP method and sum normalization are shown in Table 17.

From Tables 16 and 17, it can be seen that, in this case, the use of different normalization procedures did not affect the change in the ranking orders of alternatives.

Table 18 demonstrates the comparison of the results obtained using several well-known MCDM methods and confirms the obtained result.

As can be seen from Table 18, the TOPSIS, SAW, ARAS, and WASPAS methods gave the same ranking orders as the WISP method.

## 5. Conclusion

This article considers the use of the Simple WISP method with different normalization procedures in order to emphasize the robustness of this method.

The conducted simulations and the considered numerical illustration indicated minor deviations in the results obtained by applying the standard Simple WISP method, which uses max normalization, and the Simple WISP method with square root and sum normalization.

The comparison with the selected MCDM methods also confirmed the similarity between the results obtained using the Simple WISP method and those obtained using the used MCDM methods.

**Table 15.** Calculation details using the WISP method

|       | $u_i^{sd}$ | $u_i^{pd}$ | $u_i^{sr}$ | $u_i^{pr}$ | $\bar{u}_i^{sd}$ | $\bar{u}_i^{pd}$ | $\bar{u}_i^{sr}$ | $\bar{u}_i^{pr}$ | $u_i$ | Rank |
|-------|------------|------------|------------|------------|------------------|------------------|------------------|------------------|-------|------|
| $A_1$ | 0.02       | -0.021     | 1.07       | 0.049      | 0.849            | 1.000            | 0.820            | 0.877            | 0.887 | 4    |
| $A_2$ | 0.08       | -0.025     | 1.25       | 0.097      | 0.902            | 0.995            | 0.893            | 0.918            | 0.927 | 3    |
| $A_3$ | 0.13       | -0.030     | 1.34       | 0.132      | 0.937            | 0.990            | 0.928            | 0.947            | 0.951 | 2    |
| $A_4$ | 0.20       | -0.030     | 1.52       | 0.196      | 1.000            | 0.990            | 1.000            | 1.000            | 0.998 | 1    |
| max   | 0.02       | -0.021     | 1.76       | 0.0175     |                  |                  |                  |                  |       |      |

**Table 16.** Calculation details using the WISP method and square root normalization

|       | $u_i^{sd}$ | $u_i^{pd}$ | $u_i^{sr}$ | $u_i^{pr}$ | $\bar{u}_i^{sd}$ | $\bar{u}_i^{pd}$ | $\bar{u}_i^{sr}$ | $\bar{u}_i^{pr}$ | $u_i$ | Rank |
|-------|------------|------------|------------|------------|------------------|------------------|------------------|------------------|-------|------|
| $A_1$ | 0.02       | -0.007     | 1.15       | 0.036      | 0.890            | 1.000            | 0.800            | 0.906            | 0.899 | 4    |
| $A_2$ | 0.07       | -0.008     | 1.36       | 0.071      | 0.929            | 0.998            | 0.881            | 0.937            | 0.937 | 3    |
| $A_3$ | 0.10       | -0.010     | 1.48       | 0.097      | 0.957            | 0.997            | 0.923            | 0.960            | 0.959 | 2    |
| $A_4$ | 0.15       | -0.010     | 1.68       | 0.143      | 1.000            | 0.996            | 1.000            | 1.000            | 0.999 | 1    |
| max   | 0.15       | -0.007     | 1.68       | 0.143      |                  |                  |                  |                  |       |      |

**Table 17.** Calculation details using the WISP method and sum normalization

|       | $u_i^{sd}$ | $u_i^{pd}$ | $u_i^{sr}$ | $u_i^{pr}$ | $\bar{u}_i^{sd}$ | $\bar{u}_i^{pd}$ | $\bar{u}_i^{sr}$ | $\bar{u}_i^{pr}$ | $u_i$ | Rank |
|-------|------------|------------|------------|------------|------------------|------------------|------------------|------------------|-------|------|
| $A_1$ | 0.01       | -0.002     | 1.17       | 0.019      | 0.937            | 1.000            | 0.792            | 0.946            | 0.919 | 4    |
| $A_2$ | 0.04       | -0.002     | 1.40       | 0.038      | 0.960            | 1.000            | 0.877            | 0.964            | 0.950 | 3    |
| $A_3$ | 0.06       | -0.003     | 1.52       | 0.052      | 0.976            | 0.999            | 0.921            | 0.977            | 0.968 | 2    |
| $A_4$ | 0.08       | -0.003     | 1.74       | 0.077      | 1.000            | 0.999            | 1.000            | 1.000            | 1.000 | 1    |
| max   | 0.08       | -0.002     | 1.74       | 0.077      |                  |                  |                  |                  |       |      |

**Table 18.** Ranking results using selected MCDM methods

|       | TOPSIS |      | VIKOR |      | SAW   |      | ARAS  |      | WASPAS |      | CoCoSo |      |
|-------|--------|------|-------|------|-------|------|-------|------|--------|------|--------|------|
|       | $S_i$  | Rank | $Q_i$ | Rank | $S_i$ | Rank | $Q_i$ | Rank | $Q_i$  | Rank | $k_i$  | Rank |
| $A_1$ | 0.272  | 4    | 1.000 | 4    | 0.722 | 4    | 0.705 | 4    | 0.692  | 4    | 1.407  | 4    |
| $A_2$ | 0.399  | 3    | 0.393 | 2    | 0.777 | 3    | 0.770 | 3    | 0.774  | 3    | 2.500  | 2    |
| $A_3$ | 0.617  | 2    | 0.700 | 3    | 0.821 | 2    | 0.823 | 2    | 0.824  | 2    | 2.192  | 3    |
| $A_4$ | 0.733  | 1    | 0.312 | 1    | 0.903 | 1    | 0.912 | 1    | 0.901  | 1    | 2.613  | 1    |

Finally, the analysis of the use of the WISP method with a logarithmic normalization procedure can be mentioned as one of the future research directions.

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