Advances in Mathematical Systems Theory

A Volume in Honor of Diederich Hinrichsen

edited by Fritz Colonius, Uwe Helmke, Dieter Prätzel-Wolters and Fabian Wirth
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Important topics and contemporary developments and results in mathematical systems theory and control, associated with the work of Diederich Hinrichsen, are surveyed in 14 articles by leading international researchers. The occasion was the celebration of Dr. Hinrichsen's 60^{th} birthday. The book also includes a Preface by the editors, an Introduction to his biography and work, as well as a complete list of his publications and of his Ph. D students. All of these have contributed to revealing his achievements and fruitful ideas in mathematical systems theory, from which many researchers and practitioners in Germany or in the world have profited.

The Chapters of the book are independent surveys; the topics are on linear and nonlinear systems, on stability and robustness, on nonlinear perturbations, on infinite-dimensional systems; interesting connections are made with notions like multivariate polynomials, convolutional codes, complementarity systems and hybrid systems.

The **Chapters** of the Volume are the following:

1. **Transitory Behavior of Uncertain Systems**, by Anthony J. Pritchard;
2. **Robust Stability of Multivariate Polynomials**, by Vladimir L. Kharitonov;
3. **Robustness of Nonlinear Systems and Their Domains of Attraction**, by Andrew D.B. Paice
4. and Fabian R. Wirth;
5. **On Stability Radii of Slowly Time-Varying Systems**, by Achim Ilchmann and Iven M.Y. Mareels;
6. **An Invariance Radius for Nonlinear Systems**, by Fritz Colonius and Wolfgang Kliemann;
7. **State and Continuity**, by Jan C. Willems;
8. **Parameterization of Conditioned Invariant Subspaces**, by Paul A. Fuhrmann and Uwe Helmke;
9. **Duality Between Multidimensional Convolutional Codes and Systems**, by Heide Gluesing-Luerssen, Joachim Rosenthal and Paul A. Weiner;
10. **Control of Rate-Bounded Hybrid Systems with Liveness Constraints**, by Michael Heymann, Feng Lin and George Meyer;
11. **A General Principle of Marked Extraction**, by Uli Krause
12. **Between Mathematical Programming and Systems Theory: Linear Complementarity Systems**, by Hans Schumacher;
13. **Exact Controllability of C_0-groups with One-Dimensional Input Operators,** by Birgit Jacob and Hans Zwart;
14. **Normalized Coprime Factorization for Strongly Stabilizable Systems**, by Ruth F. Curtain and Job C. Oostveen;
15. **Low-Gain Integral Control of Infinite-Dimensional Regular Linear Systems Subject to Input Hysteresis**, by Hartmut Logemann and Adam D. Mawby.

In *Chapter 1* the transitory behavior of systems is studied. Even though this topic was not treated enough in the literature, Pritchard and Hinrichsen became interested in it around 1993. This Chapter introduces some notions like transitory excursion and stable matrix with normal transitory excursion. The transitory excursion is a measure of the distance from the semigroup generated by a matrix A, $(e^{At})_{t\geq 0}$, to the origin; in some conditions, it also has the meaning of the maximum excursion of a linear system's trajectory $x(\cdot)$ from the origin. Pseudospectra and spectral value sets are measures of the distance of a matrix (or a linear operator) from normality. Therefore, these concepts are linked with the topics of this chapter.

A way of reducing the transient bound by state feedback is considered. Moreover, for a class of uncertain systems of the form $\dot{x} = (A + D\Delta E)x$, where only matrix Δ is unknown, it is shown that the stability radius with respect to the perturbation structure (D,E) has a lower bound, provided that some conditions are satisfied by the solution of certain differential Riccati equations.

In *Chapter 2*, a class of stable multivariate polynomials is introduced; this is the maximal class that preserves stability under small variations of the coefficients. This class of *wide sense stable polynomials* consists of polynomials that do not annihilate when all the variables are taking strictly positive values. Several desirable properties for these polynomials are stated and exemplified.

Moreover, robustness conditions are considered. One of the basic tools for robust stability of univariate polynomials is the Zero Exclusion Principle (ZEP); that is, the origin is excluded from the value set of the polynomial family for every frequency $\omega \in \mathbb{R}$. The ZEP is equivalent to robust stability. In the case of multivariate polynomial families, robust stability can also be guaranteed by a slight modification of the ZEP, which will involve conditions on the subfamilies of main coefficients, as well.

For the class of polytopic families of polynomials, a property that does not require value sets is equivalent to robust stability. This is the condition that the polynomials on the edges of the polytop be stable. Hinrichsen's and Pritchard's stability radius concept [1] can be extended for multivariate polynomials.

The robustness of stability of a class of perturbed nonlinear systems is studied in *Chapter 3*, from both a local and a semiglobal point of view. One of the results from this well-written Chapter is related again to the concept of stability radius, that will illustrate a local property. It is shown that the time-varying stability radius of the studied perturbed nonlinear system generically coincides with the linear stability radii of the linearization in 0, associated with the nonlinear system. Moreover, a method for approximate computation of the time-varying stability radius, by means of optimal control problems, is presented.

The corresponding semiglobal concept is the robust domain of attraction. As suggested by the terminology, it represents the set of initial states that guarantee the trajectories convergence to the equilibrium point zero. Again, the linearization theory can be used to determine a ball contained inside the domain of attraction; an optimal control approach can give a method to iteratively approximate this set. An algorithm is sketched and its correctness proven.

Chapter 4 contains interesting results on time-varying linear systems subject to perturbations. Three types of perturbation classes are presented: time-varying linear perturbations, time-varying nonlinear perturbations, and dynamical perturbations. For the third and widest class of dynamical perturbations, several types of globally uniform stability are introduced and sufficient conditions for their fulfilment are proven in detail.

The concluding result of this Chapter turns again to the stability radius concept. This time, a (structured) stability radius of an exponentially stable time-varying linear system with respect to dynamic perturbation is defined in a natural manner. It is shown that, as long as the stability radii of "shifted frozen" (time-invariant) systems are uniformly larger than a constant ρ, then the stability radius of the time-varying system is also bounded by ρ, provided that the time variations in the systems' matrices are sufficiently slow.

The main concept of *Chapter 5* is the invariant control set of a nonlinear system. A set C is an invariant control set if and only if every limit point of C is a limit point for the set of the perturbed system trajectories, with the same initial state. This notion is a generalization of asymptotically stable equilibrium for the perturbed system. In a straightforward way, an invariant radius of an asymptotically stable equilibrium x_0 for the perturbed system is introduced. It is the supreme value for which the control set surrounding the equilibrium x_0 still retains the invariance property. The purpose of this Chapter is to investigate what happens when this value is attained. It is shown that at the invariance radius r the invariance control set merges with another variant control set and itself becomes variant. A model of a continuous flow stirred tank reactor is used, to explain some of the specific features.

Chapter 6 addresses the problem of linear differential systems representation. A special representation type consists of state representations; it is the case when the latent variables of the system (called the state variables) separate the past and the future, given a present value. This value of the state variable is a common point both from any past leading to it and for any future emanating from it.

The main results concern the minimal realization of a state representation. It is shown that necessary and sufficient conditions for a state representation to be minimal are state trimness, observability and a smoothing property of the state map. The novelty consists in the third condition. (The first two are also present in the case of discrete-time systems). The state map $X(d/dt)$ is smoothing if it yields continuous image $x(t) = X(d/dt)w(t)$ for any vector-valued function $w(\cdot)$. The conclusion is that the minimal state $x(\cdot)$ of a linear system consists exactly of the system functionals that evolve continuously in time.

Chapter 7 is a detailed survey on a subject having its anchor in a pioneering paper of Hinrichsen et al, from 1981, *Parameterization of (C,A)-invariant subspaces.* In the context of polynomial and rational models, conditioned invariant subspaces are related to kernels of associated Toeplitz operators. An important result is a co-dimension formula for conditioned invariant subspaces in terms of Wiener-Hopf indices.

The problem of parameterization of the set of conditioned invariant subspaces is expressed as the reduction of a polynomial matrix of full column rank to a modified Kronecker-Hermite canonical form. This guarantees that the number of free parameters is minimal.

The topics of this Chapter belong to the geometric control theory; this fact is underlined by the study of the topology of tight conditioned invariant subspaces and the Brunovsky strata for conditioned invariant subspaces.

Chapter 8 gives an insight into some unexpected connections between coding theory and systems theory. The multidimensional convolutional codes are powerful encoding devices for the transmission of data over noisy channels. Formally, an m-dimensional code is a submodule of the polynomial ring $F^n[z_1,...,z_m]$. Every code has a generator matrix G. On the set of power series in m variables, the backward shift L_i along the ith variable is defined for all i. $G(L_1,...,L_m)$ becomes a linear partial difference operator. This is one reason to seek connections with discrete-time systems theory. In fact, it is proven that codes and behaviors are dual objects.

Some interesting representations are deduced; they are very useful in the case of one-dimensional codes. Also, the duality shows that algebraic theory of linear systems can be successfully applied to convolutional codes, too.

The purpose of *Chapter 9* is the synthesis of a controller, which guarantees that a set of legal specifications is satisfied at any run of a given system. The reader is introduced into the world of hybrid systems, in which both discrete and continuous behaviors exist. The main concept, the elementary hybrid machine, is presented in great detail. Elementary hybrid machines running in parallel form a composite hybrid machine that allows both signal sharing (continuous aspect) and event synchronization (discrete aspect).

The legal specifications are treated in this Chapter are liveness specifications; they state what the system is required to do in order for its tasks to be completed.

The controller will run in parallel with the open hybrid machine, interfering in the transitions of the machine to ensure that every configuration is "live", i.e. a good final configuration is reachable from it. An algorithm is presented, that synthesises such a controller in the case of bounded-time liveness. Moreover, the controller will be a minimally interventive one.

Chapter 10 starts with an example from economy on joint production; for this model, a nonsubstitution theorem and a (more general) substitution theorem are stated.

The results of this Chapter are in part extensions of Hinrichsen's and Krause's work in [2]. The topic is the unique representation of an element as combination of other special elements. Two are the main directions that are analysed in detail: convexity and algebraic rings. The proposed method for deriving such representations is a marked extraction algorithm. This algorithm works by iteratively constructing finitely many elements that depend on the so-called marking μ. In the area of convexity, the extraction algorithm is applied to convex cones, bounded convex sets or polyhedra (where an element will be represented by a convex combination of extreme points). In the algebraic context, the extraction algorithm yields representations as products of finitely many irreducible elements. Special attention is paid to Krull monoids as examples of the notion of „extraction monoids"; they are important for the problem of restoring unique factorization of a nonfactorial monoid.

Chapter 11 addresses the problems arising from connecting an input-output dynamical system with a set of complementarity conditions. Several motivating examples are presented; they arise naturally in problems from mechanics, electronics, optimal control, piecewise linear systems, equilibrium theory, economics (with the discretization of the differential operator from a Black-Scholes equation), and so on.

A complete analysis is done for linear complementarity systems, which have the form: $\dot{x} = Ax + Bu$, $y = Cx + Du$, $u, y \geq 0$, $u_i y_i = 0$, for all i.

Complementarity systems are in fact hybrid systems. This characterization is due to the fact that the active index set (the set I where $y_i = 0$, for $i \in I$) changes in discrete time, to prevent violation of the inequality constraints.

The well-posedness problem for linear complementarity systems is discussed; it involves the existence, uniqueness and smoothness of solutions for all initial states. Sufficient and necessary conditions are given in terms of the transfer matrix, but also in terms of the rational complementarity problem.

For an infinite-dimensional linear system of the form $\dot{x}(t) = Ax(t) + bu(t)$, where the operator A generates a C_0-group $T(t)$, the problem of exact controllability is investigated in *Chapter 12*. Necessary conditions for exact controllability are given in terms of the spectrum of A. Equivalent conditions to exact controllability involve the solvability of a certain interpolation problem in $\boldsymbol{H}_2(\mathbb{C}^+)$. Further equivalent conditions using the concepts of Bessel sequence and Riesz basis are given. As subsidiary results, properties of exact controllable systems are proven and used in the proof of the main results.

Strong stabilizability, a central concept in *Chapter 13*, is a weaker concept than exponential stabilizability. Let $\Sigma(A,B,C,D)$ be an infinite dimensional system, where A generates a C_0-semigroup $T(t)$ and B, C, D are bounded operators. The semigroup $T(t)$ is strongly stable if $lim_{t\to\infty}T(t)z = 0$. The system $\Sigma(A,B,C,D)$ is strongly stabilizable if there exists an operator F such that $(C \ \ F)\,(sI - A - BF)^{-1}\, z \in \boldsymbol{H}_2$ and $A - BF$ generates a strongly stable semigroup.

For this type of systems, the authors have previously developed a rather complete theory. This chapter adds to the picture the subject of normalized coprime factorization. The reason why the usual approach to the formulae of the doubly coprime factorization cannot be used is that conditions of holomorphism and boundedness (that automatically hold for exponential stabilizability) do not hold for strong stabilizability. Therefore other explicit formulae for normalized coprime factorization are proven, involving solutions of some Riccati equations. This result completes the theory of robust stabilization with respect to normalized coprime factor perturbation for this class of systems.

In *Chapter 14*, the integral control of exponentially stable, single-input, single-output, infinite-dimensional, regular, linear systems subject to input dynamical nonlinearities is studied. The main results show that the output trajectory converges to an arbitrary reference signal provided that the gain parameter is smaller than a certain constant value.

The wide class of nonlinear operators considered in this Chapter includes a large number of hysteresis nonlinearities. This class, denoted by $N(\lambda)$, where λ is a Lipschitz constant, is defined as the set of operators that satisfy certain assumptions; among others, it is assumed that the operators satisfy causality and monotonicity. Several examples prove that the considered class includes static nonlinearities, relay hysteresis, backlash hysteresis, elastic-plastic hysteresis, Preisach and Prandl operators.

The book is a valuable source of information on recent advances and current research topics in mathematical systems theory. Its audience should be composed of researchers, professionals and graduate students with a background in systems and control theory or in applied mathematics.

The influential role of Diederich Hinrichsen in the area of systems theory is very transparent from the Chapters of this book. He opened the way to many research directions and he (co-)authored several important new concepts. The solid mathematical basis for topics like algebraic systems theory or stability analysis, which he took part in constructing, offers long term possibilities for future developments.

This book is a fine tribute to Diederich Hinrichsen.

1. Hinrichsen, D. and Pritchard, A. J., *Stability Radii of Linear Systems,* SYST. CONTROL LETTERS, 7, 1986, pp. 1-10.
2. Hinrichsen, D. and Krause, U., *Unique Representation in convex sets by extraction of marked components*, LINEAR ALGEBRA APPL., 51, 1983, pp 73-96.

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