Tracking By Compound Control

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Abstract : This paper presents a tracking algorithm based on the utilization of the compound control that enables trajectory tracking with one sampling period delay. The main idea is to realize a discrete control with the period Te by an associated system added to the plant. The initial conditions of the associated system are re-computed at each time step in order to enable a trajectory tracking. The control is called compound because it has to be defined at each time step but it is not constant during one sampling period. It is also shown that the necessary conditions enabling the control computation are the plant controllability and the associated system observability. Therefore the compound control is completely defined by the constant coefficients matrix. This enables the real time implementation of the tracking algorithm. Numerical examples are given.

Keywords: Associated system, Compound control

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1. Introduction

There are many interesting technical and theoretical problems in the area of controlled industrial structures that control engineers and researchers are trying to solve. These problems include , for example, in textile industry, dyeing process control, complex chemical process supervision and control. Some of the techniques are the quadratic optimization technique [1], the pole placement technique [2], the virtual passive technique [3,4], the energy dissipation technique [5], and the adaptive control technique [6-13]. Some researchers prefer to work in the frequency domain using the frequency response functions (FRF) while others use the state-space model (SSM) in the time-domain to design controllers. The model-based techniques need a mathematical model (FRF of SSM) within a certain level of accuracy to design a controller. Except for a few simple cases, system identification must be involved in the design process to verify the open-loop model and the closed-loop design as well. As a result, it may take considerable time to iterate the design process until performance requirements are met. For the systems with minimum uncertainties, the iteration procedure would not bother the control engineers, as long as a satisfactory control design can be found. For systems with unknown disturbances and considerable uncertainties, the controller must be able to adapt the unknown changes in real time. Adaptive control techniques are developed for this purpose. The approach is to adjust the control gains to reflect the system changes so as to continuously check and meet the performance requirements. Most adaptive control techniques require the controlled system to be minimum -phase [14-16] in the sense that all the system transmission zeros are stable. The minimum-phase system in the continuous-time domain does not guarantee its minimum phase in the discrete-time domain. In practice, only a few structural systems in the discrete-time domain are minimum -phase. Predictive controller designs [9,12,17] have been developed to particularly address the non minimum-phase problems with the hope that they can be implemented in real time.

Many very interesting papers published in the area of state control [18] investigated the use of generalized sampled-data hold function (GSHF) in the control of linear time-invariant systems. The idea of GSHF is to periodically sample the output of the system, and to generate the control by means of a hold function applied to the resulting sequence. The hold function is chosen based on the dynamics of the system to be controlled. This method has the efficiency of state feedback without the requirement of state estimation.

Several other works have been realized in the field of deadbeat control concerning our developments.

Urikura and Nagata [19] developed the ripple-free deadbeat control method for sampled-data system. The control objective is to settle the error to zero for all time after some finite settling time, i.e. to eliminate the ripples between the sampling instants in deadbeat control of sampled-data systems.

Another interesting work in the field of the control has been realized by Yamamoto [20]. This work handles a new framework for hybrid sampled-data control systems. Instead of considering the state only at sampling instants, Yamamoto introduced a function piece during the sampling period as the state and gave an infinite-dimensional model with such a state space. This gives the advantage of a sampled-data system with built-in intersample behaviour that can be regarded as linear, time-invariant, discrete time system. Tracking problem can be studied in this setting in a simple and unified way, and ripples are completely characterized as a mismatch between the intersample reference signal and transmission zero directions.

In this paper we used the controller initial conditions in order to achieve and to impose desired global control performances. The control method is based on the works on controllability and observability introduced by Kalman. The tracking algorithm developed in our study is obtained by the discrete control with the sampling period Te, and during each sampling period the deadbeat control is applied. This "compound" control is generated by the "associated system" introduced by C. Vasseur [21] and F. Laurent [22]. The control generated by the associated system during a sampling period is considered as deadbeat control from kTe to (k+1)Te. In fact during a sampling period the associated system and a plant make a complex system whose particularity lies in that one part of the initial conditions is not controlled (plant initial conditions at kTe). At the same time the associated initial conditions can be imposed. Because of this we propose to recompute these associated system initial conditions at each sampling instant in order to satisfy performances criteria of a plant. The criterion we exploited in this article is defined by the expression $x_{k+1}=c_k$ (x plant state and c state set points).

In Section 2 the compound control principle is given and examples are given. Finally in Section 3 concluding remarks are made and perspectives are taken.

2. Compound Control

2.1 Plant Description

The continuous model of the plant to be controlled together with sensors and final controller devices (e.g. actuators) is described by $\sum(A,B,C,0)$ (denoted $\sum$ in subsequent notation) where,

$$\frac{dx(t)}{dt}=Ax(t)+Ba(t) \qquad (1)$$

$$y(t)=Cx(t)$$

where t denotes time, $t\in\Re$, the initial time $t_0=0$, the state vector $x\in\Re^n$, the control vector $a\in\Re^r$, and the output vector $y\in\Re^m$. The constant coefficients matrices A,B and C have appropriate dimensions. The solution of Equation (1) for $t\in[kT_e,(k+1)T_e]$ is given below :

$$x((k+1)T_e)=\exp(AT_e)x(kT_e)+ + \int_{kT_e}^{(k+1)T_e}\exp(A((k+1)T_e-\tau)Ba(\tau)d\tau \qquad (2)$$

The control $a(.)$ is generated by the associated system described below.

2.2 Associated System Description and Control Method

Let define the continuous associated system $\sum_a(\alpha,0,\gamma,0)$ (denoted $\sum_a$ in subsequent notation) that is added to the plant by the equation,

$$\frac{dv}{dt}=\alpha v \qquad (3)$$

$$a=\gamma v$$

where $v\in\Re^n$ denotes the state vector of the associated system and $a\in\Re^r$ the output of the associated system. Note that the order of the associated system is the same as the order of the plant. Matrices α and γ have appropriate dimensions. Then the solution of differential Equation (3) for $t\in[kT_e,(k+1)T_e[$ where $k\in N$ is defined by Equation (4),

$$v(t)=\exp(\alpha(t-kT_e))v(kT_e) \qquad (4)$$

$v(kT_e)$ is the initial state of the associated system at the moment kT_e.

and $a(t)=\gamma\exp(\alpha t)v(kT_e)$

Then Equation (2) becomes:

$$x((k+1)T_e)=\exp(AT_e)x(kT_e)+$$
$$+(\int_{kT_e}^{(k+1)T_e}\exp(A((k+1)T_e-B\gamma\exp(\alpha(\tau-kT_e))d\tau)v(kT_e) \qquad (5)$$

Let note

$$M=\int_{kT_e}^{(k+1)T_e}\exp(A((k+1)T_e-\tau)B\gamma\exp(\alpha(\tau-kT_e))d\tau$$

By substituting $\theta=\tau-kT_e$, M becomes

$$M=\int_{kT_e}^{(k+1)T_e}\exp(A(T_e-\theta))B\gamma\exp(\alpha\theta)d\theta \qquad (6)$$

Note that M is independent of time.

The plant state vector response c at the instant kT_e, $x((k+1)T_e)=c(kT_e)$. So, considering that M^{1} exists, the initial state of the associated system is computed from Equation (7). The Initial Condition Generator (ICG) is used in order to compute the associated system initial conditions. Therefore the control architecture is given in Figure 1.

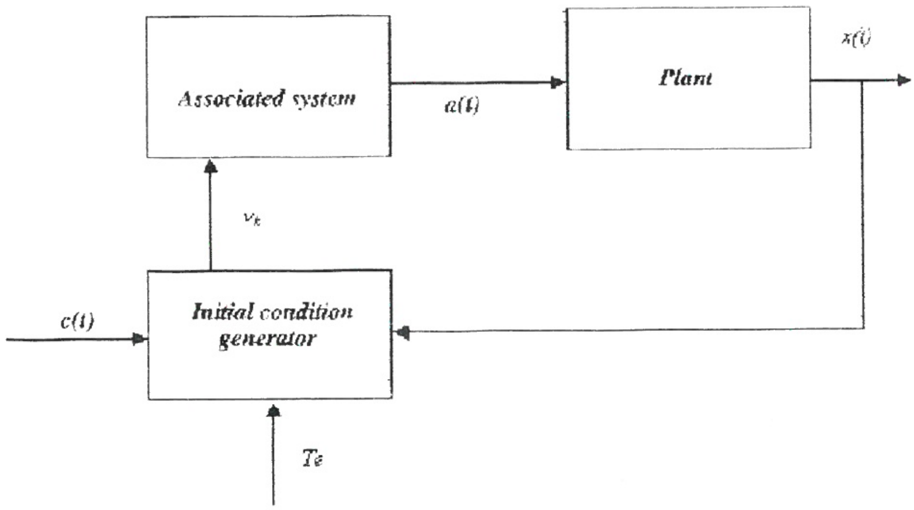


Figure 1. Control Architecture

$$v(kT_e)=M^{-1}(c(kT_e)-\exp(AT_e)x(kT_e)) \qquad (7)$$

The Initial Conditions Generator is shown in Figure 2.

It is important to notice that the choice of associated system model (α,γ matrices) influences the global control system properties.

This is shown in the numerical example of Section 2.4.

2.3 Existence of Associated System Initial Condition Enabling $x((k+1)T_e)=c(kT_e)$

The condition of existence is formulated below.

Theorem

Matrix M^{-1} exists (M is not singular) if and only if:

* $Ker(\widetilde{\Omega})=\{0\}$

$$\gamma=\left[\gamma_1^T\middle|\gamma_2^T\middle|,\ldots\gamma_r^T\right]^T \text{, with } \gamma_i^T\in\Re^{1\times n}$$

$$\widetilde{\Pi}(T_e)=\begin{bmatrix}\Pi & 0 & \ldots & 0\\ 0 & \Pi & \ldots & .\\ . & . & \ldots & .\\ 0 & \ldots & 0 & \Pi\end{bmatrix}\text{, and}$$

$\widetilde{\Pi}(T_e)\in\Re^{nr\times nr}$

$\Pi(T_e)=\int_0^{T_e}PQ^T d\tau$

where $P=[p_0,p_1,\ldots,p_{n-1}]^T$ and

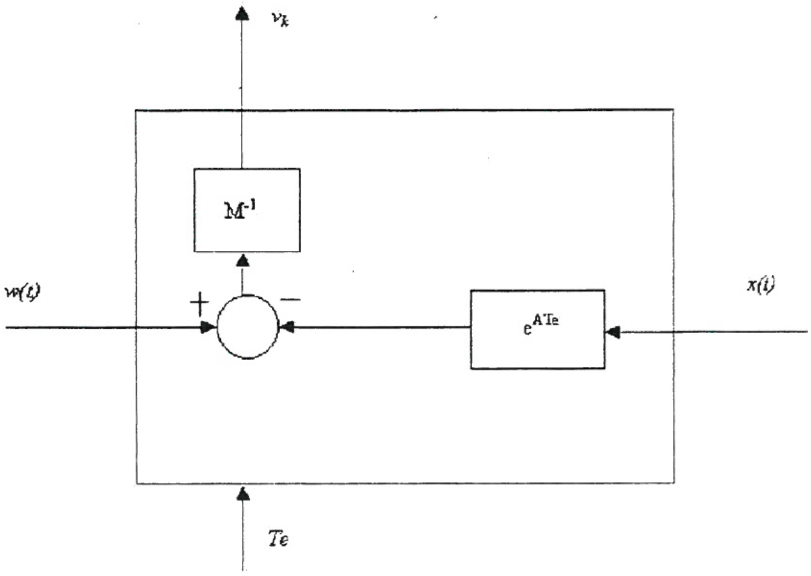


Figure 2. Initial Condition Generator

* $\text{Im}(\widetilde{\Omega})\cap Ker(\widetilde{\Pi}(T_e))=\{0\}$, $\forall T_e$
* $\text{Im}(\widetilde{\Pi}(T_e)\widetilde{\Omega})\cap Ker(\widetilde{K})=\{0\}$, $\forall T_e$

with

$\widetilde{K}=\left[\widetilde{K}_1\middle|\widetilde{K}_2\middle|,\ldots\widetilde{K}_r\right]$,

and

$\widetilde{K}_i=\left[A^0B_i,A^1B_i,\ldots A^{n-1}B_i\right]$,

$B=\left[B_1\middle|B_2\middle|,\ldots B_r\right]$, with $B_i\in\Re^{n\times 1}$

$\widetilde{\Omega}=\left[\widetilde{\Omega}_1\middle|\widetilde{\Omega}_2\middle|,\ldots\widetilde{\Omega}_r\right]$

$\widetilde{\Omega}_i=\left[\gamma_1^T\alpha^0\middle|\gamma_2^T\alpha^1\middle|,\ldots\gamma_r^T\alpha^{n-1}\right]$

$Q=[q_0,q_1,\ldots,q_{n-1}]^T$

and

$\exp(At)=\sum_{j=0}^{n-1}p_j(t)A^j$,

and

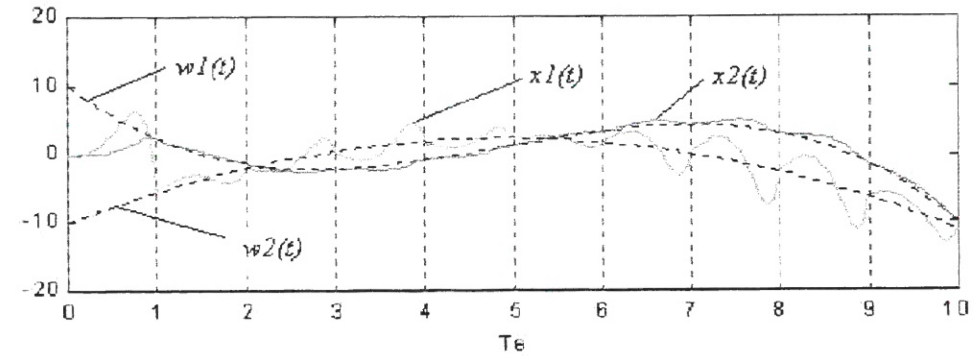
$\exp(\alpha t)=\sum_{j=0}^{n-1}q_j(t)\alpha^j$ □

Proof

Matrix $M(T_e)$ can be represented by

(i) Computation of $M(T_e)$

$$M=\int_0^{T_e}\sum_{i=1}^{r}\exp(A(T_e-\tau))B_i\gamma_i^T\exp(\alpha\tau)d\tau$$



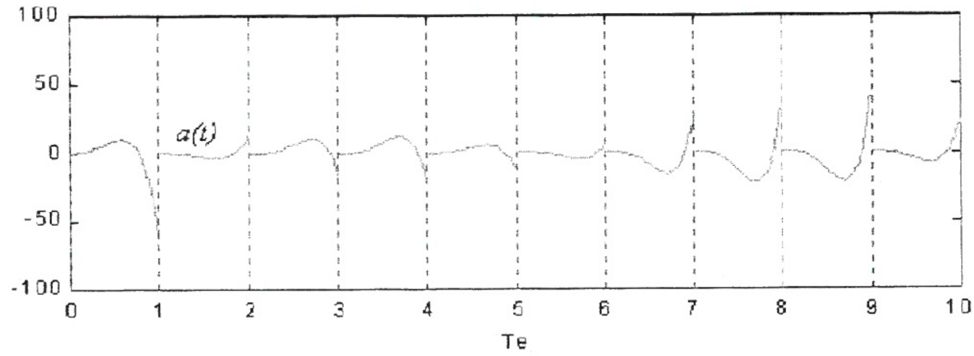


Figure 3a. Tracking Example, State \& Set Points Te=1 sec, 1st Case

Figure 3b. Tracking Example, Control Te=1 sec, 1st Case

$$M=\sum_{i=1}^{r}\int_{0}^{T_e}\left[\sum_{j=0}^{n-1}p_jA^j\right]\left[B_i\gamma_i^T\right]\left[\sum_{j=0}^{m-1}q_j\alpha^j\right]d\tau$$

Therefore : $M(T_e)=\sum_{i=0}^{r}M_i(T_e)$

Let note : $M_i(T_e)=\int_0^{T_e}\widetilde{K}_iP(T_e)Q^T(T_e)\widetilde{\Omega}_id\tau$

With $\widetilde{K}_i\in\Re^{n\times n}$ and $\widetilde{\Omega}_i\in\Re^{n\times n}$ according to notations in theorem

Hence $M_i(T_e)=\widetilde{K}_i\left(\int_0^{T_e}P(T_e)Q^T(T_e)d\tau\right)\widetilde{\Omega}_i$

$M_i(T_e)=\widetilde{K}_i\Pi(T_e)\widetilde{\Omega}_i$ and $M(T_e)=\widetilde{K}_i\widetilde{\Pi}(T_e)\widetilde{\Omega}_i$

and

(ii) Existence condition of M^{-1}

$M(T_e)$ is a square matrix of dimension $n\times n$. Therefore $M(T_e)$ is invertible if and only if :

$$Ker\ (\widetilde{K}.\widetilde{\Pi}.\widetilde{\Omega})=\{0\}.$$

This condition can be evaluated by the expression :

$$\forall x\in Ker\ (\widetilde{K}.\widetilde{\Pi}.\widetilde{\Omega})\Leftrightarrow\left[x\in Ker\ (\widetilde{\Omega})\right]or\left[\widetilde{\Omega}\,x\in Ker\ (\widetilde{\Pi})\right]or\left[\widetilde{\Pi}\,\widetilde{\Omega}\,x\in Ker\ (\widetilde{K})\right]$$

It is known that $Ker\ (\widetilde{K}.\widetilde{\Pi}.\widetilde{\Omega})=\{0\}$, is equivalent, according to previous expression, to

$$\forall x\neq 0\Leftrightarrow\left\{\begin{array}{c}\left[x\notin Ker(\widetilde{\Omega})\right]\Rightarrow Ker(\widetilde{\Omega})=\{0\}\\ and\left[\widetilde{\Omega}x\notin Ker(\widetilde{\Pi})\right]\Rightarrow Ker(\widetilde{\Pi})\cap \mathrm{Im}(\widetilde{\Omega})=\{0\}\\ and\left[\widetilde{\Pi}\widetilde{\Omega}x\notin Ker(\widetilde{K})\right]\Rightarrow Ker(\widetilde{K})\cap \mathrm{Im}(\widetilde{\Pi}\widetilde{\Omega})=\{0\}\end{array}\right\}$$

□

2.4 Numerical Example

Plant state space model

$$A=\begin{bmatrix}0,&1\\2&-1\end{bmatrix},\ B=\begin{bmatrix}0\\2\end{bmatrix},\ \text{state vector } x=\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

2.4.1 PCS Controller State Space Model *1st Case*

$$\alpha=\begin{bmatrix}-2,&8\\-7,&10\end{bmatrix},\ \gamma=[10,10],\ \text{state vector } \lambda=\begin{bmatrix}\lambda_1\\\lambda_2\end{bmatrix}$$

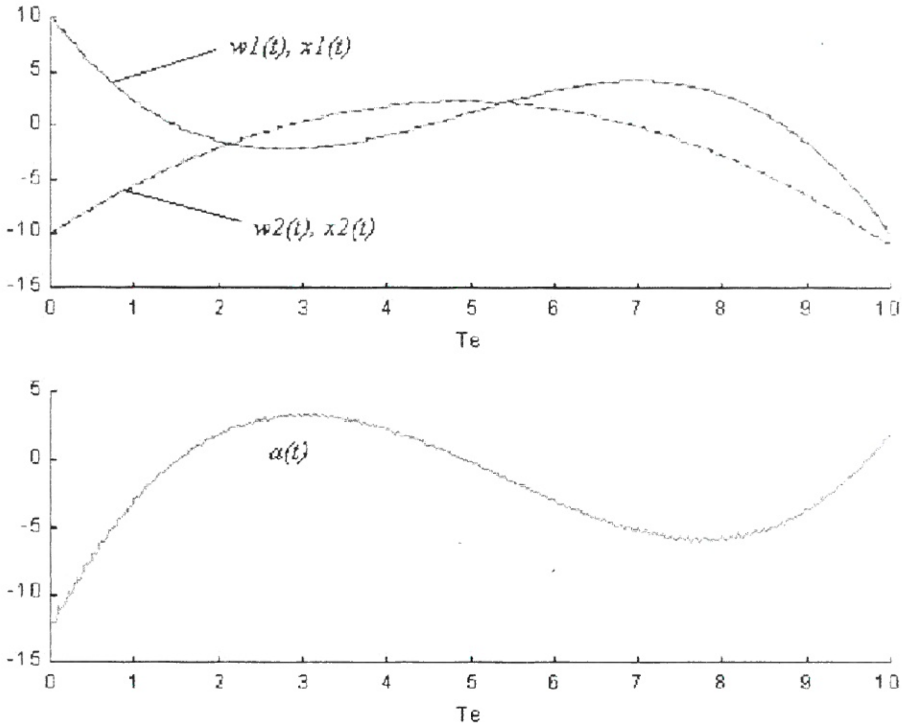


Figure 4a. Tracking Example, State \& Set Points Te=0.1 sec, 1st Case

Figure 4b. Tracking Example, Control Te=0.1 sec, 1st Case

In Figure 3a the plant state evolution from the initial state $\begin{bmatrix}0\\0\end{bmatrix}$ and set points (imposed trajectory) defined as :

$c(t)=\begin{bmatrix}-0.17t^3+2.5t^2-10t+10\\-0.51t^2+5t-10\end{bmatrix}$ is given. The sampling period T_e=lsec.

The control system enables tracking of $c(t)$ with one sampling period delay. Thus, in the following illustrations, $w(t)=c(t-T_e)$ appears instead of $c(t)$, in order to give clear appreciation of control performances.

In Figure 3b the control generated by the PCS controller is given.

In Figure 4 the same example is simulated with the sampling period T_e=lsec. and the plant initial state $\begin{bmatrix}10\\-10\end{bmatrix}$ in order to avoid the phase of transition. The simulation is realized during 10sec. as in the previous example.

2.4.2 PCS Controller State Space Model *2nd Case*

$\alpha=\begin{bmatrix}-2,1\\0.5,1\end{bmatrix}$, $\gamma=[10,10]$

In Figure 5a. the plant state evolution from the initial state $\begin{bmatrix}0\\0\end{bmatrix}$ and set points (imposed trajectory) defined as :

$c(t)=\begin{bmatrix}-0.17t^3+2.5t^2-10t+10\\-0.51t^2+5t-10\end{bmatrix}$ is given. The sampling period T_e=lsec. The control system enables tracking of $c(t)$ with one sampling period delay. Thus, in the following illustrations $w(t)=c(t-T_e)$ appears instead of $c(t)$, in order to give clear appreciation of control performances.

Figure 5b shows the control generated by the PCS controller.

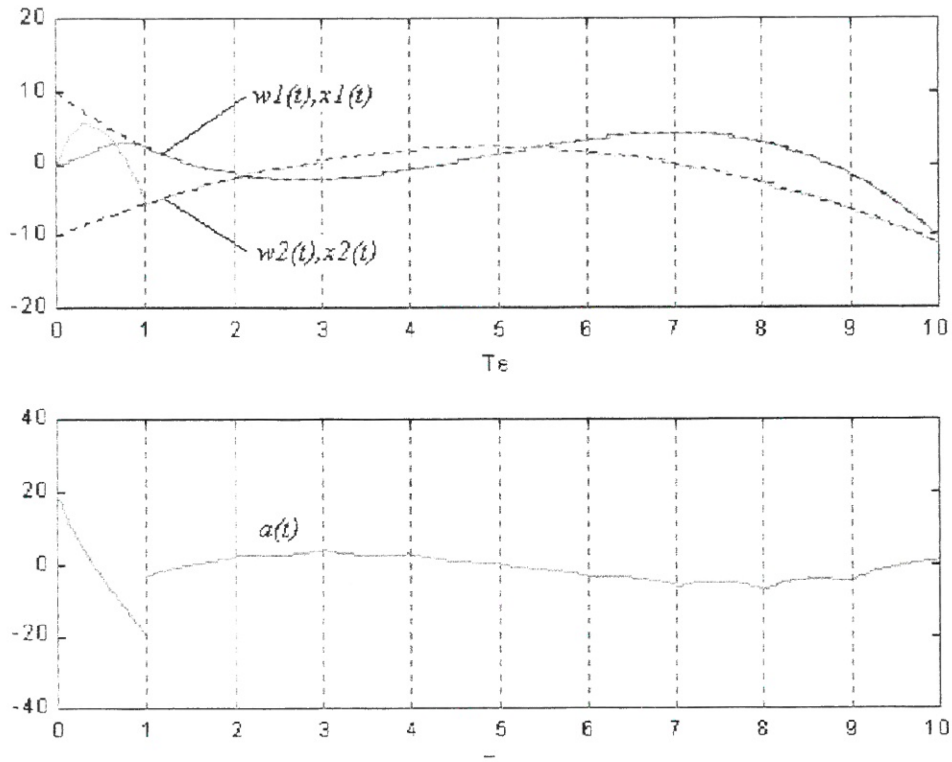


Figure 5a. Tracking Example, State \& Set Points Te=1 sec, 2nd Case

Figure 5b. Tracking Example, Control Te=1 sec, 2nd Case

2.5 Discussion

It is important to notice that the sampling period has great influence on PCS control system behaviour. This influence is generating two kinds of problems.

In the first case bad choice of T_e can induce important intersample ripple and also singular values of T_e can reduce matrix M rank (pathological sampling frequencies [23]). Therefore the control computation becomes impossible. It is also important to respect Shannon's theorem in the choice of T_e.

The second kind of problem is related to a time delay in the trajectory tracking. In fact the sampling period influences time delay and shorter sampling period means shorter time delay. However, in general the control magnitude is increasing with sampling period decreasing. For $T_e \to 0$ the control magnitude, in case of set points discontinuities, is infinite. However for continuous set points in keeping with the plant state we obtain astonishingly good tracking properties.

The influence of the associated system parameters is visible in Figure 5. In fact the intersample ripple is reduced, however this influence is not very important when a sufficiently small sampling period is defined.

3. Concluding Remarks and Perspectives

In this paper a tracking algorithm based on the variable control during one sampling period, called compound control, has been developed. The compound control is completely defined by the constant coefficient matrix defining the Initial Condition Generator (ICG), that enables real time implementation. The `"controller" generating the control is in fact the associated system that has to be observable, which permits, together with the plant controllability, the compound control generation. Theoretically the associated system observability and the plant controllability are sufficient conditions to the control computation (i.e. the associated system initial condition computation), however the choice of an associated system influences the control system performances within a sampling period. The criteria for the choice of an associated system, that is an additional constraint, expected its observability, have not been analysed in this paper. This point will be developed in future works.

The compound control is computed using the control M matrix with constant coefficients called M^{-1} that is determined from the plant model. It is well known that a model is often different from the reality. Besides, the plant is functioning in different conditions and its properties can evolve in time. The unmeasurable disturbance cannot be quantified and included in the model. All these make the computed M^{-1} matrix be different from the ideal M_{ideal}^{-1} matrix and show that the on line plant model identification is necessary. This identification can be done by any least-square recursive algorithm. The classical least-squares method is the most straightforward approach and is also the basis for the others. However that is not the main objective of this paper and because of thi the identification technique is limited to the above statements.

The control action is supposed to bring the state response of a plant to step points for all time steps. That makes theory. In practice, when the system has input and output uncertainties, the control action can only bring the plant state to set points to the level of uncertainties.

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