Some Comments About the Insurance Of Accidents in Construction

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Abstract: We can define the probability that an insured person adopts measures for preventing accidents and p_a the probability of the same type of accident taking place. Similarly we can define the p_m as the probability that an accident occurs by the measures imposed by the insurer and the dual probability $p_{m/a}$ which means the probability that an insurer adopts measures for prevention of the accidents conditioned by the event of producing an accident. From such reasoning there results a link between these probabilities. Another important relation can be obtained from the condition of an equilibrium of the expenses of the insurer- the compensations and the whole sum paid by the insured person. The similarities between an insurance system and a prey – predator system are shown. To determine the elements of a conditional probability matrix, the method of open rectangles is used. Some heuristic rules, which apply to an insurance system, are stated. Based on these rules a simulation of the learning process of the insurance system may be thought of. To solve the problem of disasters the insurance of the insurer is suggested.

Keywords: insurance, risk, random variable, square mean deviation, conditional probability, prey-predator system, heuristic rules, learning process, coefficient of verisimilitude, problem of disaster

1. Introduction

In construction area accidents may occur during the execution and the exploitation of buildings. These accidents can largely vary. They may be fires, explosions, earthquakes, inundation, loss of stability, fall downs, sabotages etc. The causes of these accidents can also be various: faults committed during the activity of projection, errors in execution, neglectedness in exploitation etc. To prevent the occurrence of such types of accidents, companies of insurance were set up. During the years of centralised economy in Romania there was a unique company named ADAS, which represented a state monopoly. Under these circumstances, the insurance policies practiced by this company were extremely simplified. After 1989 in Romania started operate several insurance companies (ASIT TIRIAC, ASIROM, ASTRA), which if contrasted with the years preceding the turning point of 1989 could seem diversified, but which were still few compared with the necessities of a modern economy. At present insurance companies change their perspective, trying to take the insured person and the insurer on a real contract of co-operation instead of a formal contract. This new type of contract sees each partner has a well-defined role, with obligations, duties, and common objectives all to lead to the improvement of the resistance of the construction, the installation or the devices being insured.

In these new conditions, the principle underlying the conclusion of an insurance contract of a new type is that of ethics and equity. Considering this principle, the following turns necessary:

- partner's risks to be shared;
- partner's payments and expenditures to be shared;
- either partner share the advantages of each other.

Obviously, it is practically impossible in the short- term to realise all these. In the long- term, supposing the existence of a fair competition between the insurance companies, it could be foreseen a tendency of balance between the expenditures of the insured person and of the insurer.

To approach such problems one should use the theory of probabilities and the ethical principles in business. Apart of the independent probabilities, it is also necessary to use conditional probabilities, especially those conditioned by the application of a program for preventing from accidents.

Suppose an insurance contract is concluded between the insured person and the insurer for a given type of goods (tools, building furniture, computers, etc.) for a period of time *d* in the following conditions:

- the whole sum updated to a given moment t_0 for a period *d* paid by the insured person is P_a;
- provided an accident, the insurer pays a compensa.'on D equal to the value updated at the moment t_0 for the goods of the insured person;
- during the time period *d* the insured person will stick to the measures for preventing from accidents, taking into account the instructions received from the specialists named by the insurer;
- if one side does not observe the provisions of the contract, the side which considers to have been damaged may sue the other side and ask for penalties;

Under these circumstances the insurer has the following advantages:

- the redeeming of the damages provoked by the accident if he complied with the conditions of the contract;
- the reduction of the accident occurrence probability as a result of the observance of the measures for preventing from accidents as indicated by the insurer experts;
- the determination of higher confidence of persons who use the goods of the insured person that these goods are protected from accidents.

The observance of the measures indicated by the insurer induces some expenses from the insurer C_{ha} updated at the moment t_0.

The insurer has in his turn the obligation to engage the most qualified persons (on a full-time or part-time basis) into conceiving a most rigorous program of accidents' prevention.

The advantage of the insurer consists in his possibility of receiving an insurance premium and gaining the prestige for his good results.

Noting that $p_{a/m}$ is defined as the probability to produce an accident conditioned by the measures imposed by the insurer. Taking into account the Bayes theory of probabilities conditioned, it is necessary also to consider the dual probability $p_{m/a}$ that means the probability that an insurer adopts measures of prevention of accidents conditioned by the event of producing an accident. Note p_m is the probability that the insured person adopts measures for preventing from accidents and p_a the probability that the same type of accident as the accident considered, takes place. According to the Bayes theory the relation between the probabilities defined above is the following:

$$p_m \, p_{a/m} = p_a \, p_{m/a} \qquad (1)$$

Suppose the sum paid by the insured person p_a includes the insurer's staff costs with the management expenses, etc. and also the compensation he pays, adding a reasonable profit.

By hypotheses one may suppose that in the long- term the insurer's costs and the expenses made by the insured person referring to the contract of insurance will get balanced with respect to an ethical and equitable principle accepted by the great majority of specialists. An analogy with the equilibrium of the forces in mechanics comes naturally. Consider that at one end of the arm of lever, the insurer's compensations D and costs C_{ha} act as some forces. Analogously at the other end a force P_a acts, representing the whole sum paid by the insured person. The arm of lever represents the probability that an accident occurs, conditioned by the measures for preventing against the accident $p_{a/m}$. The moment of these forces is balanced by the moment produced by a force P_a representing the whole sum paid by the insured person. This moment is updated at time t_0 and the arm of lever is $(1-p_{a/m})$, in accordance with Figure 1.

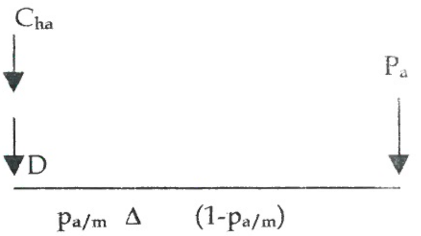

Figure 1

From the equilibrium condition of the moments shown in Figure 1 results Equations:

$$p_{a/m}(D + C_{ha}) = (1 - p_{a/m})\, P_a \qquad (2)$$

$$p_{a/m} = \frac{Pa}{Cha + D + Pa} \qquad (3)$$

Using the notation

$$r = \frac{Cha + D}{Pa} \qquad (4)$$

will result in:

$$p_{a/m} = \frac{1}{1 + r} \qquad (5)$$

This conditional probability is smaller than the probability p_a in the case in which measures for preventing from accidents have not been adopted. That means:

$$p_{a/m} < p_a \qquad (6)$$

Given this inequality an indicator may be defined, to measure cost-effectiveness in preventing accidents, noted with E_{cp}. This indicator will measure the reduction of the probability an accident occurs for one unit of value spent ($C_{ha} + P_a$).

$$E_{cp} = \frac{pa - pa/m}{Cha + Pa} \qquad (7)$$

The conclusions of this Section are both theoretical and practical. From a theoretical point of view the calculation of the probabilities resulted to be possible not only as frequency but also using relations between expenses as shown in Equation (5). Of course, this conclusion sounds unexpected and apparently it can be considered farfetched. In the long -term, the surveys of some statistical data made by insurance companies (the ethical and the equitable conditions being met), show that the relation given by this formula tends to be correct. If the statistical data are insufficient and refer to a short period of time the information provided by Equation (5) has only an orientative character.

From a practical point of view the action of insuring a construction turns beneficial if supported by a program including measures for preventing accidents, to be observed by both the insured person and the insurer.

2. Similarities Between An Insurance System and A Prey-Predator System

We may consider that the action of preventing accidents by different anticipatory measures is similar to the action when a prey is caught by a predator. Using preventing measures a possible accident occurrence is fully or at least partially removed. If we note with y the probability of accidents occurrence and with x the costs needed to undertake different measures for preventing these accidents, here results a graph of events, having the form as shown in Figure 2.

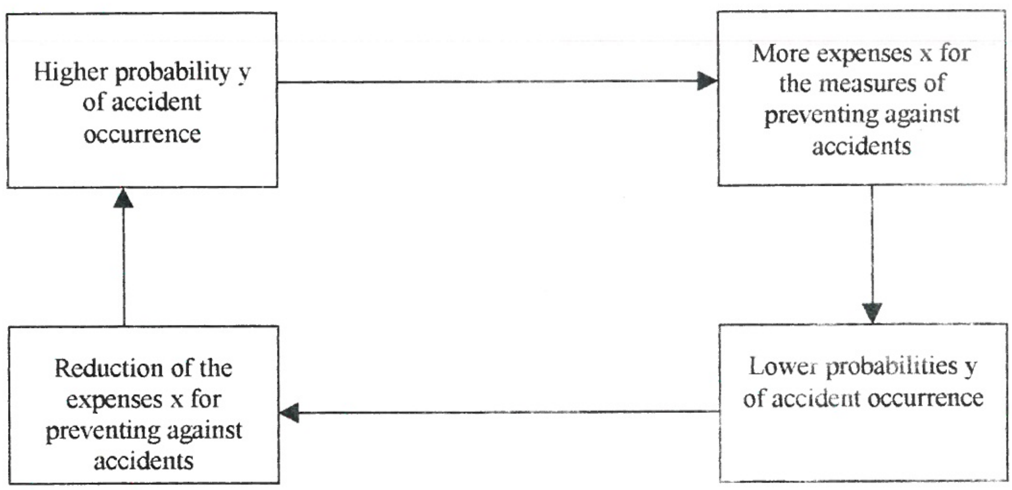

Figure 2

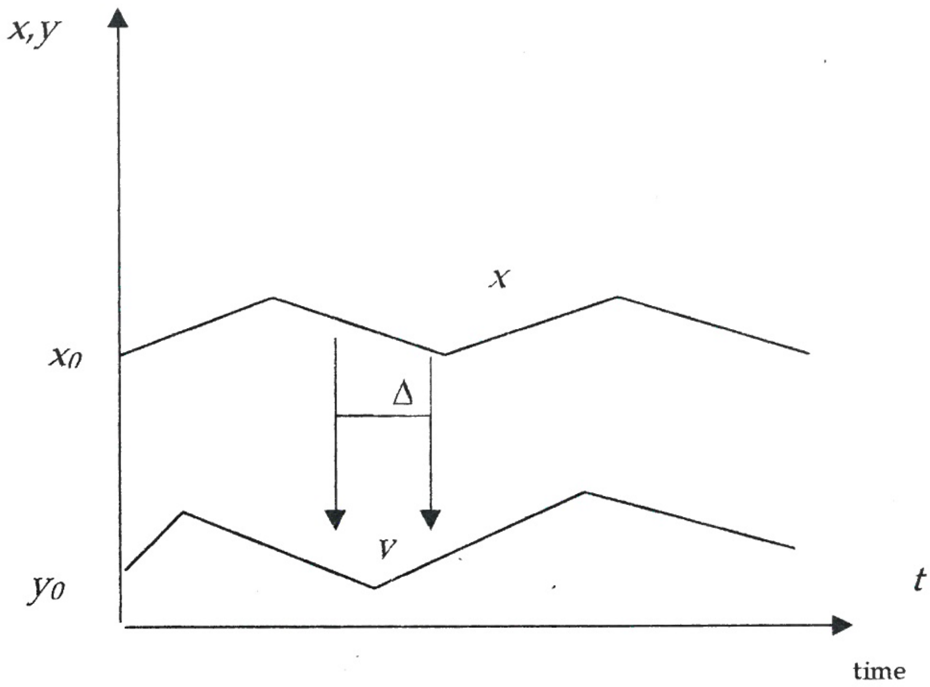

Figure 3

Suppose the number of accidents increases, it results that the probability y also increases. In this case measures will be adopted to get new funds for preventing against accidents (using the national insurance system). The effect will be a smaller number of accidents, which will in turn lead to decisions for reducing the accidents prevention costs. As a result the probability y of accidents occurrence will be higher once more and the cycle will be resumed indefinitely.

This process will be expressed graphically as in Figure 3.

Figure 3 presents the evolution of the probabilities of accidents occurrence and the costs evolution. The two trajectories have the same evolution as a lag noted with Δ, which represents the delay in reaction from the insurance beneficiaries. These must initially realise the proliferation of accidents and then conclude insurance contracts with the specialised companies. Delay is also perceivable in case of fewer accidents.

From an analytical point of view, differential equations may be written, to express the trajectories in Figure 3.

If we note the time with t, the rate of higher costs linearly depends on the probability y, as follows:

$$dx/dt = Ly \qquad (8)$$

where L represents the rate of the costs push up to be considered constantly in time.

Analogously, the rhythm of reducing the probability linearly depends on the level of costs (x) for preventing against an accident:

$$dy/dt = -Kx \qquad (9)$$

Differentiating from Equation (9) and making the replacements in (9) we obtain a differential equation at the second level, having the shape:

$$d^2y/dx^2 = -KLy \qquad (10)$$

We note $a^2 = KL$ and obtain:

$$d^2y/dt^2 + a^2 y = 0 \qquad (11)$$

Analogously, for the costs x for preventing accidents, the differential equation:

$$d^2x/dt^2 + a^2 x = 0 \qquad (12)$$

may be obtained.

The solutions of the differential equations (11) and (12) are sinusoids, having the shape of those shown in Figure 3, with a delay Δ between the maximum point and the minimum point.

The delay Δ and the coefficients K and L can be determined from statistical data.

Similarly with the relation (12) a differential equation may be identified to express the probability y of accidents occurrence. Its expression, under the same previous hypotheses is:

$$d^2y/dt^2 + b^2 y = 0 \quad (13)$$

where b is a coefficient which can be determined statistically.

To solve the differential equation (13) the following restriction should be considered:

$$0 \leq y \leq 1 \quad (14)$$

The hypothesis of coefficients L and K from Equations (8) and (9) is a restrictive one, and could be farfetched in practical applications. That is why, we consider that Equations (12), (13), (14) are theoretically more important. A more appropriate prey-predator system could be obtained if considering that parameters L and K are functions of x and y.

It would also be necessary to give another definition to variables, to avoid the restriction (14). The new variables will be:

x=the expenses on the insurances and on the measures for preventing accidents;

y=the compensations given by insurance companies, in the case an accident occurs;

Δx= the expenses made on insurance and on the measures for preventing accidents in a period of time equal to a unit;

Δy= the expenses assumed by the insurance companies, when accidents happen in a period of time equal to a unit (a year, usually).

Taking into account the new hypothesis and being knowledgeable of the values x_t, y_t,, and Δy for a period of time, we have:

$$\Delta x_t = a_1 x_t^2 + b_1 x_t y_t + c_1 y_t^2 + e_1 x_t + f_1 y_t + g_1 \quad (15)$$

$$\Delta y_t = a_2 x_t^2 + b_2 x_t y_t + c_2 y_t^2 + e_2 x_t + f_2 y_t + g_2 \quad (16)$$

where $t \in \{1, 2, \ldots T\}$

Using the method of the smallest rectangles we can determine the coefficients a_1, b_1, c_1, e_1, f_1, g_1, a_2, b_2, c_2, f_2, and g_2. These coefficients help us imagine a system of two differential equations, having the shape:

$$dx/dt = a_1 x^2 + b_1 xy + c_1 y^2 + e_1 x + f_1 y + g_1 \quad (17)$$

$$dy/dt = a_2 x^2 + b_2 xy + c_2 y^2 + e_2 x + f_2 y + g_2 \quad (18)$$

Starting from an initial situation, which is well-known (x_0, y_0), each of these equations can be solved using the Runge-Kutta algorithm.

3. The Determination of the Elements of the Conditional Probabilities Matrix

Stimulation of an insurance system for getting effective in construction makes it necessary to determine the probability that conditioned accidents take place, aiming at adopting some preventive methods (the dual, that means the probabilities of adoption of some measures when some sorts of accidents happen). To enhance the precision of the simulation results of the simulation results precision should apply in defining the type of accidents and the quality of the precaution measures. The types of accidents can be classified as minor, medium and severe. The quality of the preventive measures for accidents can be superficial, temperate and radical. In both cases we can see that the level of costs is ever higher. Generally, we admitted that m types of accidents and n qualities of preventive measures had been taken into account. Thus two matrices of conditional probabilities are obtained:

- the matrix A of the Bayes probabilities $p_{ai/mj}$ of occurrence of accidents of type i for adopting of some quality measures j;
- the matrix B of the Bayes probabilities $p_{ai/mj}$ of adopting the quality measure h when accidents of type j happen.

Using the notations mentioned above the relation (1) becomes:

$$p_{mj}\, p_{a/mj} = p_{ai}\, p_{mj/ai} \quad (19)$$

where

p_{ai} the probability of occurrence of an accident of type a_i;

p_{mj} the probability of adopting the quality measures m_j.

The probabilities p_{ai} and p_{mj} are independent and can be determined statistically.

To determine the elements of matrices A and B we can use relations having the shape (13) and the method of open polygons. Further this method is described. Based on statistical data we can determine (m+n-1) elements of the matrix situated on an open polygon, the other points being determined using proportionality relations. The method exemplification is in the case of a matrix type A, having 4 rows and 4 columns.

quality of adopted measures / type of accident	m_1	m_2	m_3	m_4
a_1	p_{11}	$p_{12\bullet}$	p_{13}	$p_{14\bullet}$
a_2	$p_{21\bullet}$	$p_{22\bullet}$	p_{23}	p_{24}
a_3	p_{31}	p_{32}	$p_{33\bullet}$	p_{34}
a_4	p_{41}	p_{42}	$p_{43\bullet}$	$p_{44\bullet}$

It is supposed in this matrix that the type of accident ranks from less severe to the severest accidents and from radical to superficial measures. Given this hypothesis, the probabilities multiply on rows and columns. If this multiplication is almost in geometrical progression, we give a simple rule (having a heuristic nature) to fill up the matrix when knowing about (m+n-1) elements. We illustrate this rule for the square matrix (4×4) from above.

An open polygon has been chosen in the matrix considered above. This polygon must have (m+n-1) = 4+4-1 = 7 tops. For instance, we consider: $p_{21\bullet}$, $p_{22\bullet}$, $p_{12\bullet}$, $p_{14\bullet}$, $p_{44\bullet}$, $p_{43\bullet}$, $p_{33\bullet}$.

Using statistical methods suppose that the seven probabilities p_{ij} have been evaluated. In this case all the other probabilities (16-7) = 9 can be determined using methods which observe the proportion, to be established in a triangle with the property that the probabilities are known in three points. In the fourth point two variation indices are estimated and their mean is calculated. For example, in the square (1,1), (1,2), (2,2), (2,1) only p_{11} is unknown, the other three probabilities are supposed to be known. We calculate a variation index having the following expression:

$$I_1 = p_{22\bullet} < p_{21\bullet} \quad (20)$$

To calculate the p_{11} probability we apply the relation:

$$p_{11} = p_{12} < I_1 \quad (21)$$

Similarly a variation index I_2 is calculated having the following shape:

$$I_2 = p_{22\bullet} < p_{12\bullet} \quad (22)$$

In this case the probability p_{11} could also be calculated using Equation:

$$p'_{11} = p_{21} < I_2 \quad (23)$$

If the probabilities p_{11} and p'_{11} obtained using Equations (15) and (17) respectively, have equal values, that means there are no errors. If $p_{11} \neq p'_{11}$, that means that there are errors, and, to reduce them, an arithmetic mean of the two values should be calculated, thus obtaining a better estimation.

4. Heuristic Rules Which Can Be Applied in An Insurance System

An insurance system asks for some efficient rules of calculation of the total of the insurance premiums P^*_a to be known. This should be like this because in the long -term (T) it is absolutely necessary to have covered the costs C_{ha} and the compensation D, updated to the moment t_0 with a certain probability. There are many rules having a heuristic and fuzzy nature for selecting some functions and parameters that could answer to the mandate expressed above.

Further some such types of rules having a heuristic and fuzzy nature are presented.

R_1) The experts can appreciate a safety coefficient:

$$P^*_a = c_s P_a \quad (24)$$

If we divide P^*_a to the months of validity of the insurance contract, the monthly insurance premium updated P_{ha} is obtained:

$$P_{ha} = P^*_a/T \quad (25)$$

R_2) An estimation of a degree of affiliation μ_R of the property of preventive measures shows a radical quality. In this case Equation (24) becomes:

$$P^*_a = P_a/\mu_R \quad (26)$$

This means that between the degree of affiliation μ_R and the safety coefficient c_s there is the relation:

$$\mu_R = 1/c_s \quad (27)$$

R_3) Given the statistical data an estimation of the mean square deviation of the whole sum of the insurance premium is possible. The whole sum of the insurance premium P_a is added to the mean square deviation multiplied by a probability coefficient and a different shape of the relations (24) and (26) is obtained:

$$P^*_a = P_a + K\sigma_{Pa} \quad (28)$$

R_4) The level of compensation D^*_t, updated at the moment t_0 in the case of a given type of accident occurred at the moment t is a function of:

- the type of accident;
- the costs of rehabilitation of the construction to its initial state;
- other provisions of the contract between the insured person and the insurer (the construction maintenance costs affected by repairing period, by expenses due to the degradation of some materials, etc.).

R_5) The level of the insurer expenses C_{ha} (updated at the moment t_0) can be determined as a percentage α deduced from the total sum of the insurance premium brought to the day:

$$C_{ha} = \alpha P_a \quad (29)$$

This percentage α can result from statistical data or from anticipatory calculations of some development costs, the staff training expenses ensued by the co-operation with experts, etc.).

A lot of rules can be set up following the choice an insured person made for a special package offered by the insurer.

Suppose σ_{ha} the level of costs (updated at the moment t_0) of the measures taken by the insured person upon the insurer's request to accept the conclusion of the insurance contract. The insurer can offer the insured person the following options:

If the conditions required by the insured person are met, the total sum of the updated insurance premium is P^*_a:

I. Provided the insured person cannot meet these conditions, but the conditions can be technically overlooked according to the present knowledge, the total sum of the insurance premium will increase with ΔP^*_a (calculated in accordance with a contract provision). ΔP^*_a generally depends on the quality level stipulated in the insurance document. The total costs of the insurer are equivalent in both options of the insured person. To realise that, it is necessary to observe the equilibrium relation:

$$p_a P^*_a = p_{am} (P^*_a + \Delta P^*_a) \quad (30)$$

because $p_{am} < p_a$

From the insurer point of view it is necessary that:

$$(P^*_a + \gamma_m) p_{am} = p_a P^*_a \quad (31)$$

where γ_m represents the preventive measures application costs.

5. Business Ethical Problems and the Market of Insurance

Call for an insurance package can be satisfied following the option made by the insured person on:

- the quality of the insurance measures (radical, moderate or superficial);
- the insurance company.

Competition makes that the insurance company not to ask too high commissions and not to harden the insurance conditions.

Due to the competition rules the temptation of an unethical conduct from an insurer is suppressed. The unethical conduct of an insured person can be prevented if using measures for a central acquisition of the information in the insurers' data banks. These data banks store information about:

- types of accidents;
- the adopted measures quality;
- accidents' origin;
- the place where these accidents occurred;
- persons and companies involved, etc.

Thus one may find out if an insured person has cashed an insurance premium many times under similar conditions, and all types of frauds can be prevented.

6. Simulation of the Learning Process of the Insurance System

To simulate a learning program of the insurance system the (financial) reserve REZ of this system is calculated using the following equation, obtainable from Equation (2) corrected with the relation (18) or (20) or (22):

$$REZ=\sum_{ij} N_i(1 - p_{ai/mj})P^{*}_{ai} - \sum_{ij} N_i\, p_{ai/mj}(D_i + C_{hai}) \quad (32)$$

where

N_i=n the number of the insured person for the accidents of type *i*.

If we calculate the financial reserve REZ for different periods of time: 1, 2, ...,t we obtain the results REZ_1, REZ_2, ..., REZ_t, allowing to calculate a mean value REZ and a mean square deviation σ_{REZ} , that means an estimation of the parameters of this random variable.

Considering an admissible value of the deviation Σ, a minimum probability η which must be the insured person and a coefficient of verisimilitude K, the criterion of appreciating whether the learning process of the insurance system has managed to reach acceptable performances, will be that of a system functioning "probably approximately correct":

$$P\,[\,|\,REZ - K\sigma_{REZ}\,| \leq \varepsilon\,] \geq \eta \quad (33)$$

7.The Problem of Disasters

In the case of some rare events such as earthquakes, inundation, which follow the Poisson Law, the sums paid by the insurer can surpass the reserve accumulated by the insurer. For this reason, the insured person must conclude his/her contract of insurance with a reputed insurance company. In this case, the same company has a double quality (insurer and insured person). When the company acts as an insured person the reserve REZ′ has the following shape:

$$REZ' = \sum_{i,j} N_i'\, p'_{ai/mj}(D_i + \gamma'_{mj/ai}) - \sum_{i,j} N_i'(1 - p'_{ai/mj})\, P'^{*}_{ai} \quad (34)$$

$p'_{ai/mj}$=the probability of a disaster taking place.

The total target function is the total reserve, which has the following shape:

$$REZ_T = REZ + REZ' - K'\sigma' \quad (35)$$

in which:

K′= coefficient of verisimilitude

σ′=the mean square deviation of the sum of function

This kind of make sums could sometimes lead to some paradoxes, in the sense given by the Romanian mathematician Gheorghe Paun. We can demonstrate, using the Lebesgue measure, the framing in some pre-established limits of the degree of compatibility. That means that "in fuzzy sense", the paradoxes can be eliminated.

The diversification of the types of insurance packages should be considered in the future. First of all, new types of business insurance contracts are to be envisaged. In this case we may consider that the bankruptcy actually is an "accident" in the activity of enterprises. As concerns persons insurance contracts for unemployment situations (especially in the case of unemployment induced by technical progress) can be entered.

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