Matrix Diagonal Stability in Systems and Computation

by Eugenius Kaszkurewicz and Amit Bhaya
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This book is a well-structured monograph combining theoretical results, applications, and examples, in order to emphasize the importance of diagonal stability in various areas of the systems theory.

The purpose of the book is ilustrated by the fact that all the stability results in applications from either continuous-time or discrete-time dynamical systems are derived by means of appropriate diagonal matrix stability.

Matrix stability is related to the stability of the zero solution (i.e. the convergence to zero of any other solution) of the differential linear system $dx/dt = Ax$ (in the continuous-time case) or of the difference linear system $x(k+1) = Ax(k)$ (in the discrete-time case).

For a matrix A to be continuous-time or Hurwitz stable, there must be positive definite matrices P and Q such that $A^TP + PA = -Q$ (the Liapunov equation). If P is a diagonal matrix, A is said to be Hurwitz diagonally stable. Similarly, a matrix A is discrete-time or Schur stable, if there are positive definite matrices P and Q such that $A^TPA - P = -Q$ (the Stein equation). If P is a diagonal matrix, A is said to be Schur diagonally stable.

Another important concept is the one of D-stability, referring to the matrices that maintain the property of Hurwitz (respectively, Schur) stability under multiplication by any diagonal matrix.

After an **introductory Chapter**, where main subjects of the book are briefly presented. The **second Chapter** that offers valuable information on the algebraic properties of matrix diagonal and D-stability, with all the studies of system stability, to be described later, based on matrix diagonal stability issues. Therefore, a special attention is due to the way of how to check whether a given matrix is diagonally stable or not. There are some classes of matrices known to exhibit diagonal or D-stability properties, possibly under additional simple conditions. Let us mention that results are proven on triangular, quasidominant, M-matrices, sign-stable, tridiagonal matrices and many others. Moreover, testing for diagonal stability can be done either algebraically (as in the case of 2 by 2 or 3 by 3 matrices), or numerically (utilizing linear matrix inequalities (LMI) characterizations and, perhaps, the Matlab's specialized toolbox, LMI Lab).

A most useful tool in stability problems is represented by the Liapunov functions. The theme of the book implies that the diagonal-type Liapunov functions are to be used. In fact, these diagonal-type Liapunov functions benefit from simplicity and an easy to handle way. which account for the systems diagonal stability study, in the first place. As an example, the simplest form of this type of function is $V(x) = x^TPx$, with P a positive diagonal matrix.

All of the **third Chapter**, *Mathematical Models Admitting Diagonal-Type Liapunov Functions*, is devoted to proving stability results for systems with nonlinearities, making use of the Liapunov theory of stabilization. Both continuous and discrete-time systems are treated, and the derived conditions that ensure stability are of the type of Hurwitz or Schur diagonally or D-stability on a certain matrix.

The **last three Chapters** are, in fact, the ones describing the application areas. In *Convergence of Asynchronous Iterative Methods*, it is shown that the system that models the absolute error of an iterative numerical method, implemented on a parallel computer, for finding a fixed point (provided existence and uniqueness) of a function, admits the zero solution, which is globally asymptotically stable. In an iterative method, every single processor on a parallel computer must wait to receive data from the other processors. There are two cases, both treated in this Chapter: the synchronous case (there are no delays in data transmission between processors, so that at the k+1st iteration, the information is from the *kth* iteration, everywhere) and the asynchronous case (there are delays in the data transmissions, so that at the *kth*+1st iteration, a processor is working with possibly older data than the *k* iterates, from the other processors).

In the **fifth Chapter**, continuous or discrete-time applications from „neural networks, circuits, and systems" are presented in the light of the results on diagonal stability, that includes Hoppfield-Tank model for artificial neural networks, discrete neural networks useful in the design of analog-to-digital convertors (digital filtering), and, also the equilibrium state in trophic chains models.

Some topics from the **last Chapter,** *Interconnected Systems: Stability and Stabilization*, are: *the large scale systems approach*, where the construction of a Liapunov function for the whole system (by either a vectorial approach or a scalar approach, using the diagonal-type Liapunov functions associated with the subsystems) leads to the conclusion that diagonal stability has an important role in this area; *the stabilization by linear feedback*, together with the interesting problems of discrete interval systems (where the linear system matrix *A(k)* varies within a matrix interval): finding a feedback matrix that guarantees stability to the widest possible class of systems, when this is done by an optimization problem involving Schur diagonal matrix stability, leads to the computation of better bounds on the matrix interval than those already found by other methods in the literature, as numerical examples; *decentralized power-frequency control of power systems*, for those familiarized with power systems, relations with Persidskii-type systems and diagonal-type Liapunov functions are derived.

Each of the **six Chapters** of the book ends with valuable notes and references, and appendices presenting stand-alone theoretical results. A comprehensive bibliography, of over 300 titles, and a ten- page index are conclude the book.

Reading this book, from the beginning to the end, makes you get thorough knowledge of several topics concerning diagonal stability, even to a novice in the field. The book was so conceived, so that to meet consulting purposes as well, because when looking for a specialized topic in the applications Section, one receives a complete image, as models and results from previous Sections are restated.

Diana Maria Sima