Contribution to Multimodel Analysis and Control

Abdelkader El Kamel, Pierre Borne

Ecole Centrale de Lille, LAIL
F59651 Villeneuve d'Ascq Cedex
FRANCE

E-mails: {kamel, p.borne}@ec-lille.fr

Moufida Ksouri-Lahmari

Ecole Nationale d'Ingenieurs de Tunis
Laboratoire de Recherche en Automatique
Le Belvédère, 1002 Tunis
TUNISIA

Abstract: The use of the multimodel approach in the modelling, analysis and control of non-linear complex and/or ill-defined systems was advocated by many researchers. This approach supposes the definition of a set of local models valid in a given region or domain. Different strategies exist in the literature and are generally based on a partitioning of the non-linear system's full range of operations into multiple smaller operating regimes, each of which is associated with a locally valid model or controller. However, most of these strategies, which suppose the determination of these local models as well as their validity domain, remain arbitrary and are generally fixed thanks to certain a priori knowledge of the system. Meanwhile, the transition problem between the different models, which may use either a simple commutation or a fusion technique, is still pending.

The purpose of this paper is to propose a new approach to derive a multimodel base allowing us to limit the number of models in the base to four models. Besides, a fuzzy fusion technique is presented and has the following main advantages: (i) use of a fuzzy partitioning in order to determine the validity of each model which enhances the robustness of the solution; (ii) introduction, besides the four extreme models, of another model, called average model, determined as an average of the boundary models.

The theoretical study is validated by simulation and a robustness analysis is carried out under particularly severe conditions.

Keywords: multimodel approach, validity domain, fuzzy fusion, Kharitonov criteria

Dr. **Abdelkader El Kamel** is an Associate Professor at the Ecole Centrale de Lille, France, where he is in charge of the Automatic Control Teaching Laboratory, and a Lecturer at the Ecole Centrale de Paris. He is elected member of the National Scientific Council (CNU) in France. In 1995 Dr. El Kamel was appointed a Permanent Visiting Professor at different Tunisian Engineering and Business Schools (Ecole Polytechnique de Tunisie, Ecole Supérieure de Communications, ENIT, IHEC Carthage). He is involved in several Franco-Tunisian research projects and Ph.D co-supervision. Born in Nabeul, Tunisia in 1965, he obtained in 1990 the Engineering Diploma from the Ecole Centrale de Lille and the same year a Master Degree in industrial engineering. He received his Ph.D in Automatic Control from the University of Lille I in February 1994. Dr. El Kamel received an IEEE Outstanding Contribution Award in October 1998 for "leadership in organisation of the SMC sponsored CESA '98 Conference in Tunisia and for many contributions to research and scholarship". He was the Co-organizer and Vice-General Chairman of the IMACS/IEEE SMC Multi-conference CESA'98 held at Hammamet, Tunisia in April 1998. He was also Chairman and Co-chairman of several workshops held in Tunisia and France (MIAD '2000, MIAD' 99, Bond Graph'97, DSS & Uncertainty'96...). Dr. El Kamel is Chair of the Technical Committee on Uncertain Systems for IEEE SMC, Chair of the Student Activities Committee and was member of the Panel of Judges for the Best Student Paper Award for SMC. He has been on the Program Committee for the SMC Conferences in San Diego 1998, Tokyo 1999 and Nashville 2000 as well as for the WAC Conference in Hawaii 2000, CESA' 98 at Hammamet and CESA'96 in Lille. He has organized several invited sessions and served as sessions chair at different international conferences. Dr. El Kamel will be the Organizer and the Co-general Chairman of the IEEE SMC Conference in 2002 to be held in Tunisia. His research interests include modeling, analysis and control of complex systems using the multimodel/multicontrol approaches, fuzzy logic and neural networks, and integrated design of physical automated systems using bond graphs.

Professor **Pierre Borne** received the MSc. degree in Physics in 1967 and the MSc. degree in Electronics, Mechanics and Applied Mathematics in 1968. He graduated from Institut Industriel du Nord (French Grande Ecole d'Ingénieurs) in 1968 and received Ph.D and DSc. in Control Engineering from the University of Lille (France) in 1970 and 1976, respectively. He is the author and co-author of more than 200 journal articles, book chapters and communications at conferences. He is the author of 6 books in automatic control, the co-editor of the Concise Encyclopedia of Modelling and Simulation (Pergamon, NY) and has organised 7 international conferences and symposia. He received an IEEE Outstanding Award in 1992 and the Kuhlmann Prize in 1994. Professor Borne was IMACS Vice-President from 1988 to 1994. He has been Fellow of the IEEE since 1996. Since 1999 he has been IEEE System, Man and Cybernetics President. He is currently Professor at the Ecole Centrale de Lille, France where he is Director of the Research and Head of the Department of Automatic Control. His activities concern automatic control and robust control, including the implementation of fuzzy controllers and neural networks.

1. Introduction

The multimodel approach appears as a powerful technique to deal with complex, non-linear and/or ill-defined systems represented using a set of simple linear or smoothly non-linear models, each of them allowing the generation of a partial controller. Different approaches exist in the literature [1, 2] to cope with the two key points in the multimodel/multicontrol strategy: the model basis determination and the use of partial controllers in order to derive a global one. Hence, it seems obvious that one of the main objects to focus on in the multimodel/multicontrol approach is the model base reduction for a practical representation of the complex system. Our aim in this paper is to propose a new approach to deriving a multimodel base in which the number of models be limited to four. This allows, at a practical stage, for considering different control laws, easier- to-evolve and -implement. Indeed, the simplicity of the four models M_i involves an easy control law which can be implemented in real -time. However, another major point appears to be that of decision-making at each time concerning the choice and the value of model M_i validity permitting to evolve the control law.

2. Validity Issue

The validity issue in the multimodel approach can be expressed as an estimation problem concerning the relevance of each model in the model base. Hence, it is possible to consider a supervisor whose aim is to estimate at each time the validity of each model M_i characterized by an index v_i the value of which belongs to the interval [0, 1]. The value 1 means that the corresponding model M_i perfectly describes, with respect to the associated criterion, the process at that time. On the contrary, the value 0 denotes that the model M_i is fully inadequate.

The determination of these indexes appears therefore to be of great importance, if realising their influence on the performance of the global control law.

The literature [3, 4] considers different methods dealing with this issue and being generally founded on the determination of the residues.

In the geometrical approach, the residues are assimilated to the distances d_i between the models and the process. A standardised distance d'_i can for instance be defined by:

$$d'_i = \frac{d_i}{\sum d_i}$$

and the validity by:

$$v_i = 1 - d'_i.$$

In the general case, however, the model of the process is not available, and the residues are determined via the measured variables on the process and compared to the estimated outputs of the models M_i. It is possible for instance to write:

$$R_i = | y_i - y |$$

where y is the output of the process while y_i the output of the model M_i. The validity can then be expressed by:

$$v_i = 1 - R'_i$$

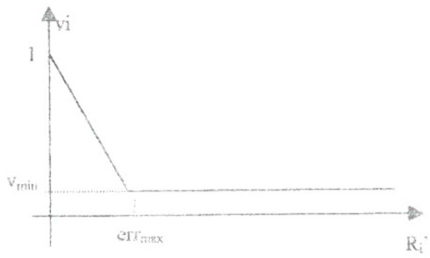
where R'_i is a standardised distance given by:

$$R'_i = \frac{R_i}{\sum R_i}$$

Also, in order to express the validity, more sophisticated formulas are likely to be considered, such as:

$$v_i = \max[v_{min}, 1 - R'_i / err_{max}]$$

which permits modulating the velocity of the validity decrease with respect to the model errors as illustrated in the following Figure.



In practice, it appears important to proceed on overweighing the validity indexes in order to get rid of perturbations induced by "inconvenient" models over the satisfactory ones. For instance, we can write the new validity as follows:

$$v_i^{renf} = v_i \prod_{j \neq i} (1 - v_j)$$

Once the validity indexes determined, the global control law u to be applied to the system can be processed by fusion of the local control laws u_i derived from the models M_i in the base. The easiest way of carrying out this task is to use a linear fusion expressed by:

$$u = \frac{\sum v_i u_i}{\sum v_i}$$

However, this technique is a limited one and it cannot be adopted when dealing with complex non-linear and/or ill-defined systems. A new approach is then proposed in the following Section.

3. Validity Estimation Using Fuzzy Logic

In this Section, we propose the use of the fuzzy logic as a new approach to cope with the validity issue. Indeed, this approach will overcome the previous limitations and improve the robustness of the results with respect to either the definition of the residue itself or the way of its determination.

Let us consider a process represented by the following non-linear model:

$$\begin{cases} \dot{x} = f(x,u,t) \\ y = g(x,t) \end{cases}$$

It is possible, as we will see in Section 4, to define four linear models, considered to be the extreme models, in the base described by the equations:

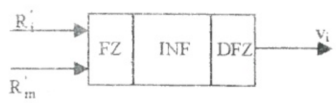
$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i \\ y_i = C_i x_i \\ i = 1 \text{ to } 4. \end{cases}$$

Besides, we define an average model in the base serving as a reference which the extreme models are compared with and which is defined by:

$$\begin{cases} \dot{x}_m = A_m x_m + B_m u_m \\ y_m = C_m x_m \end{cases}$$

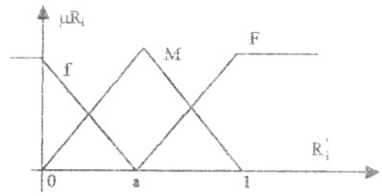
The determination of the standardised residues can be done by using one of the previously explained methods. We then obtain 5 coefficients R'_i and R'_m.

The validity management through fuzzy logic is based on the following scheme:



Four identical fuzzy moduli have to be realised and derive separately the validity indexes v_i. Each of the moduli has two inputs and one output and includes three stages:

1. The first one, FZ, is a fuzzification stage allowing the fuzzification of the numerical values of R'_i and R'_m. The membership functions may be set to be triangular while the universe of discourse has three dimensions {f, M, F}. We can have for instance the following description:



2. The INF stage permits the validity determination by applying the inference rules. Different inference tables can be proposed via expertise. In the case of a universe of discourse with three classes {f, M, F}, we can consider the following Table:

R'i / R'm	f	M	F
f	M	f	f
M	F	M	f
F	F	F	M

The Table expresses, for instance, the correlation between models M_i with respect to the average model M_m. Indeed, when the validity of the average model v_m is weak (i.e. the residue R'_m is high) the global validity v_i is strengthened whereas when the validity of the average model v_m is high (i.e. the residue R'_m is weak) the global validity v_i is weakened.

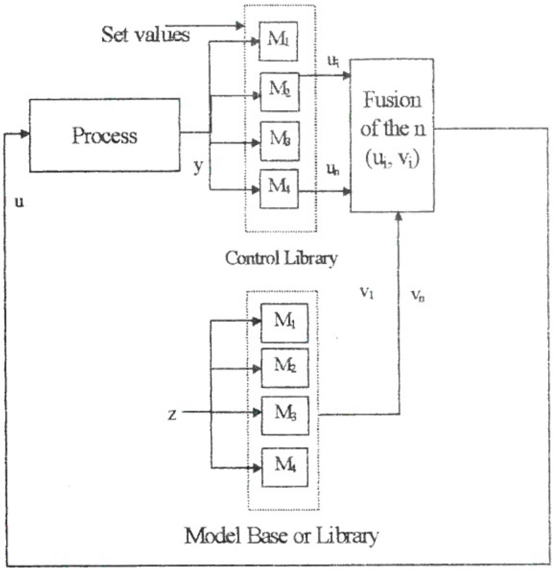
3. The defuzzification stage DFZ derives the numerical values of the validity indexes. We can for instance use the gravity centre method.

4. Model Base Determination

The multimodel approach is extremely interesting when dealing with complex systems, hence, it is represented using a set of simpler models each of which allowing to derive a partial control law. Different control structures exist in the literature [1, 5] and are generally based on the control scheme shown on the next page.

The process output y is considered as the input of control models defined using the local models in the model base (or library). The partial control variables u_i are used to derive by fusion the global control variable u to be applied to the process. The fusion is piloted by the validity indexes v_i associated with the partial models and any other input vector z, by the internal variables, the environmental parameters and/or expert data.

The drawback with the proposed approach is the



use of a large set of local models, the number of which may increase exponentially. Besides, there exists no systematic method for partial model determination, which supposes several preliminary tests before its choice.

In this Section, we propose a new strategy for the model base construction. This strategy main advantage is that of being systematic and of limiting the models in the base to four or five.

4.1 Model Base Definition for Linear Ill-defined Systems

The stability analysis of linear continuous systems of which parameters are known via their localisation intervals, can be carried out through considering four particular systems [6, 7]. Indeed, let us consider a polynomial with real coefficients in the form:

$$P(\lambda) = \sum_{k=0}^{n} a_k \lambda^k$$

with the following supplementary information:

$a_k \in \left[\underline{a_k}, \overline{a_k}\right]$ where $\underline{a_k} = \min(a_k)$; and $\overline{a_k} = \max(a_k)$.

Considering the coefficients a_k as constant, Kharitonov [6] demonstrated that the roots of $P(\lambda)$ had their real part strictly negative

$$\forall a_k \in \left[\underline{a_k}, \overline{a_k}\right] \text{ and } \forall k$$

if and only if the following four polynomials have also their roots with negative real part:

Polynomial	Coefficients classified in an increasing power order						
$P_1(\lambda)$	$\underline{a_0}$	$\overline{a_1}$	$\overline{a_2}$	$\underline{a_3}$	$\underline{a_4}$	$\overline{a_5}$	$\overline{a_6}$	...
$P_2(\lambda)$	$\underline{a_0}$	$\underline{a_1}$	$\overline{a_2}$	$\overline{a_3}$	$\underline{a_4}$	$\underline{a_5}$	$\overline{a_6}$	...
$P_3(\lambda)$	$\overline{a_0}$	$\overline{a_1}$	$\underline{a_2}$	$\underline{a_3}$	$\overline{a_4}$	$\overline{a_5}$	$\underline{a_6}$	...
$P_4(\lambda)$	$\overline{a_0}$	$\underline{a_1}$	$\underline{a_2}$	$\overline{a_3}$	$\overline{a_4}$	$\underline{a_5}$	$\underline{a_6}$	...

This property appears interesting in the stability analysis of systems having constant but uncertain parameters. For instance, if we consider a system characterised by its classical transfer function for which we have:

$$a_i \in \left[\underline{a_i}, \overline{a_i}\right],\ b_i \in \left[\underline{b_i}, \overline{b_i}\right] \quad \forall\ i = 0, 1, 2, ..., n-1$$

the transfer function for the closed loop system verifies:

$$\frac{Y}{Y^c} = \frac{b_0 + b_1 s + b_2 s^2 + \cdots}{(a_0 + b_0) + (a_1 + b_1)s + (a_2 + b_2)s^2 + \cdots s^n}$$

The stability condition is so that the Routh criteria be satisfied for the following four polynomials:

$$P_1(s) = \left(\underline{a_0} + \underline{b_0}\right) + \left(\overline{a_1} + \overline{b_1}\right)s + \left(\overline{a_2} + \overline{b_2}\right)s^2 + \cdots$$

$$P_2(s) = \left(\underline{a_0} + \underline{b_0}\right) + \left(\underline{a_1} + \underline{b_1}\right)s + \left(\overline{a_2} + \overline{b_2}\right)s^2 + \cdots$$

$$P_3(s) = \left(\overline{a_0} + \overline{b_0}\right) + \left(\overline{a_1} + \overline{b_1}\right)s + \left(\underline{a_2} + \underline{b_2}\right)s^2 + \cdots$$

$$P_4(s) = \left(\overline{a_0} + \overline{b_0}\right) + \left(\underline{a_1} + \underline{b_1}\right)s + \left(\underline{a_2} + \underline{b_2}\right)s^2 + \cdots$$

In practice, the stability analysis of a process with uncertain and/or ill-defined parameters and under the transfer function form:

$$F(s) = \frac{b_0 + b_1 s + b_2 s^2 + \cdots}{a_0 + a_1 s + a_2 s^2 + \cdots s^n}$$

can be carried out through the following four processes:

$$F_1(s) = \frac{\underline{b_0} + \overline{b_1}s + \overline{b_2}s^2 + \cdots}{\underline{a_0} + \overline{a_1}s + \overline{a_2}s^2 + \cdots s^n},$$

$$F_2(s) = \frac{\underline{b_0} + \underline{b_1}s + \overline{b_2}s^2 + \cdots}{\underline{a_0} + \underline{a_1}s + \overline{a_2}s^2 + \cdots s^n},$$

$$F_3(s) = \frac{\overline{b_0} + \overline{b_1}s + \underline{b_2}s^2 + \cdots}{\overline{a_0} + \overline{a_1}s + \underline{a_2}s^2 + \cdots s^n},$$

$$F_4(s) = \frac{\overline{b_0} + \underline{b_1}s + \underline{b_2}s^2 + \cdots}{\overline{a_0} + \underline{a_1}s + \underline{a_2}s^2 + \cdots s^n}.$$

These four models, defined using the extreme values of definition intervals of the coefficients, will then be considered as the extreme models for the process. Hence, the extreme models will constitute the model base for the process and have the main advantage of limiting to four models.

Thanks to this procedure, on the one hand the problem associated with the multiplicity of the partial models is solved by limiting the library to just four models whatever the order and the complexity of the system, and on the other hand, a systematic approach of their determination is proposed.

Moreover, while looking for a multimodel control law, it seems interesting to associate in the previous model base, the average model defined by:

$$a_{im} = \frac{\underline{a_i} + \overline{a_i}}{2}, \text{ and } b_{im} = \frac{\underline{b_i} + \overline{b_i}}{2}$$
$$\forall i = 0, 1, 2, \ldots, n-1$$

The associated transfer function is defined by:

$$F_m(s) = \frac{b_{0m} + b_{1m}s + b_{2m}s^2 + \cdots}{a_{0m} + a_{1m}s + a_{2m}s^2 + \cdots s^n}$$

We can remark that this model is indeed generally used in the mono-model control for a process with uncertain parameters.

4.2 Model Base Definition for Non-linear and/or Non-stationary and/or Ill-defined Systems

In this case, the process evolution can be described by:

$$a_0(.)y + a_1(.)y^{(1)} + a_2(.)y^{(2)} + \cdots + a_{n-1}(.)y^{(n-1)} + y^{(n)} =$$
$$b_0(.)u + b_1(.)u^{(1)} + b_2(.)u^{(2)} + \cdots + b_{n-1}(.)u^{(n-1)}$$

where (.) represents all kinds of variables, uncertainties, noise or perturbations affecting the process coefficients.

If we have the evolution domain for each parameter under localisation intervals:

$$a_i \in \left[\underline{a_i}, \overline{a_i}\right],\ b_i \in \left[\underline{b_i}, \overline{b_i}\right] \quad \forall\ i = 0, 1, 2, \ldots, n-1$$

by analogy with the linear case, we consider in the model base the extreme models as well as the average one.

Besides, if each coefficient $\gamma_i(.)$ depends explicitly on the state $\gamma_i(.) = \gamma_i(x)$, it seems interesting to add to the base made up of the five previous models, the local model of which coefficients are those defined at the set point x_c.

$$a_{ic} = a_i(x_c),\ b_{ic} = b_i(x_c), \quad \forall i = 0, 1, 2, \ldots, n-1,$$

$$F_c(s) = \frac{b_{0c} + b_{1c}s + b_{2c}s^2 + \cdots}{a_{0c} + a_{1c}s + a_{2c}s^2 + \cdots s^n}$$

when $\gamma_i(.) = \gamma_i(x, t, \ldots)$; with the unique information:

$$\underline{\gamma_{ic}} \le \gamma_i(x_c, t,.) \le \overline{\gamma_{ic}}$$

it is convenient to choose for $F_c(s)$ the coefficients:

$$\gamma_{ic} = \frac{\underline{\gamma_{ic}} + \overline{\gamma_{ic}}}{2}$$

hence:

$$a_{ic} = \frac{\underline{a_{ic}} + \overline{a_{ic}}}{2}, \text{ and } b_{ic} = \frac{\underline{b_{ic}} + \overline{b_{ic}}}{2}$$
$$\forall i = 0, 1, 2, \ldots, n-1$$

with

$$\underline{a_{ic}} \le a_i(x_c, t,.) \le \overline{a_{ic}}$$

and

$$\underline{b_{ic}} \le b_i(x_c, t,.) \le \overline{b_{ic}}$$

5. Control Law Structure

The proposed idea to be developed is a generalisation of the LQ method over systems with uncertainty. In this control law, the classical gains ll and KK are derived from the gains l_i and K_i of the four extreme models, plus the average model, when needed, weighted by the validity indexes v_i.

The multimodel control uses the five models in the base as follows:

1. We determine the output y_i associated with the model M_i by :

$$\dot{x}_i = A_i x + B_i u$$

$$y_i = C_i x_i$$

2. We determine the output y_m associated with the model M_m by :

$$\dot{x}_m = A_m x + B_m u$$

$$y_m = C_m x_m$$

3. The state vector x_i is used to generate the control variable u_i defined in the linear quadratic optimisation method LQ by:

$$u_i = l_i\, y_i^c - K_i\, x_i$$

which supposes the use of the criteria

$$J = \int \left(y^2 + u^2\right) dt$$

The gains l_i are determined so as to cancel the static error in position for the model M_i and then these are defined by :

$$l_i = -[C_i(A_i - B_i K_i)^{-1} B_i]^{-1}$$

4. The validity indexes are determined using the previously presented fuzzy approach. Note that a parameter "a" was purposely introduced in the description of the membership functions in order to illustrate the influence of their choice on the overall behaviour of the system.

The validity indexes are used to determine the global control law as follows:

$$ll = \sum_{j=1}^{4} v_j\, l_j$$

The state feedback vector KK is determined by:

$$KK = \sum_{j=1}^{4} v_j K_j$$

The global control is then expressed by:

$$u = ll\, y_c - KK\, x$$

6. Robustness Analysis

In order to study the robustness of the proposed approach, let us consider a strongly unstable non-linear non-stationary process represented by the following model in the general form of:

$$\dot{x} = f(x, u, t) = A(t)x + Bu$$

$$y = C(t)x$$

where the state matrices are in the controllable form and are expressed by:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_1 & a_2 & a_3 \end{pmatrix};\quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix};\quad C = \begin{pmatrix} c_1 & c_2 & 2 \end{pmatrix};$$

the different parameters of the model are supposed to be ill-defined and can be expressed by the following relations:

$$a_1 = -3 + 3q_1 - q_2 + q_1 q_2$$

$$a_2 = -1 - 2q_1 - q_2$$

$$a_3 = 1 + q_1$$

$$c_1 = q_1 + q_2 - q_1 q_2 - 2q_1 q_2 q_3 - 1 - 2q_1 q_3$$

$$c_2 = 1 - 2q_1 q_3 + q_2 - 2q_1$$

the coefficients q_1 , q_2 and q_3 are supposed to be bounded by:

$$-0.1 \le q_1 \le 0.1$$

$$-0.1 \le q_2 \le 0.1$$

$$-1 \le q_3 \le 1$$

We can for instance choose the following expressions:

$$q_1 = 0.1\sin(10t)$$

$$q_2 = 0.1\sin(10t + 0.05)$$

$$q_3 = 0.1\sin(10t + 0.01)$$

In order to derive the four extreme models to consider in the model base, we use the results presented at Section 4. We obtain:

$$A_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \underline{a_1} & \overline{a_2} & \overline{a_3} \end{pmatrix}; \qquad C_1 = \begin{pmatrix} \underline{c_1} & \overline{c_2} & 2 \end{pmatrix};$$

$$A_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \underline{a_1} & \underline{a_2} & \overline{a_3} \end{pmatrix}; \qquad C_2 = \begin{pmatrix} \underline{c_1} & \underline{c_2} & 2 \end{pmatrix};$$

$$A_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \overline{a_1} & \overline{a_2} & \underline{a_3} \end{pmatrix}; \qquad C_3 = \begin{pmatrix} \overline{c_1} & \overline{c_2} & 2 \end{pmatrix};$$

$$A_4 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \overline{a_1} & \underline{a_2} & \underline{a_3} \end{pmatrix}; \qquad C_4 = \begin{pmatrix} \overline{c_1} & \underline{c_2} & 2 \end{pmatrix};$$

where the upper and lower arrow notations designate, respectively the maximum (max) and minimum (min) values of the ill-defined parameters.

If we consider the transfer functions to be associated with these models in the classical form, with a numerator and a denominator using the Laplace transformation, we obtain the numerical values of the coefficients in these transfer functions, as follows:

$$\overline{a_1} = -2.79; \quad \overline{a_2} = -0.7; \quad \overline{a_3} = 1.1$$

$$\underline{a_1} = -3.19; \quad \underline{a_2} = -1.3; \quad \underline{a_3} = 0.9;$$

$$\overline{b_1} = -0.95; \quad \overline{b_2} = 1.02;$$

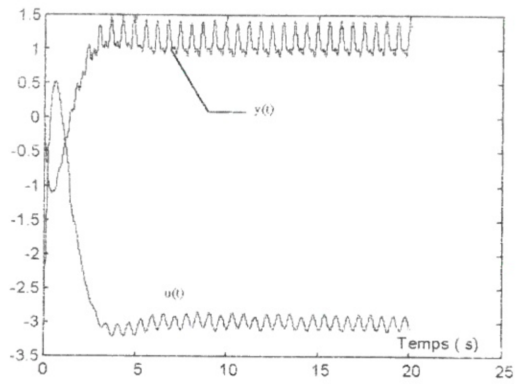
$$\underline{b_1} = -1.039; \quad \underline{b_2} = 0.7;$$

Besides, we propose that the average model is included in the model base. It is expressed as an average of the four extreme models. We then have:

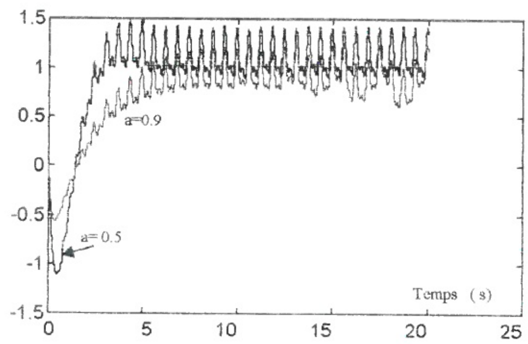
$$A_m = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -1 & 1 \end{pmatrix}; \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; \quad C = (-1 \;\; 1 \;\; 2)$$

At this level it is interesting to have an idea of the dynamical behaviour of the extreme models. The poles determination shows that the four models are unstable and that their instability is in an oscillatory form. Besides, the four transfer functions have a non-minimal phase angle.

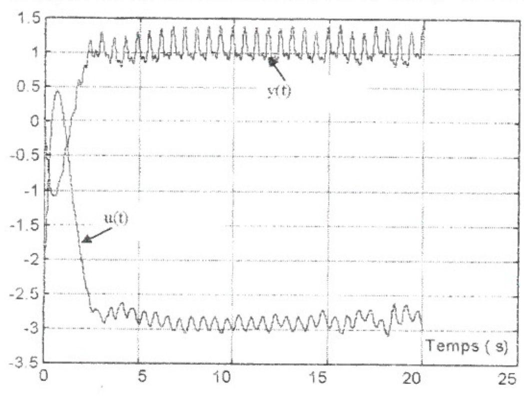
The simulation results give accurate information about the robustness of the proposed approach.



As we can notice in the previous Figure, which represents the evolution of both the output y(t) and the control variable u(t) for a parameter in the membership function fixed at a=0.5, despite some important variations in terms characterising the output vector via the matrix C, the output is practically stabilised in almost 5 seconds.



It is interesting, at this stage, to study the influence of the parameter "a" appearing in the membership functions.



We can remark that when a=0.9, the process presents a slower dynamics, however the non-minimal phase angle is attenuated. It would be interesting, in further development of this approach, to search for the optimal membership functions and to study the influence of the different techniques of fuzzification and defuzzification.

Besides, the previous simulation results do not take into account the average model in the base. In fact, this model can serve as a reference model to the other four models in the base which enter comparison.

Hence, we can notice that with the introduction of the average model, the dynamical behaviour of the process gets slightly better.

7. Pole Assignment Strategy

In this Section we propose, for the multimodel control, the application of a pole assignment strategy based on the use of the same fuzzy fusion with the five models in the base. The interest of this strategy resides in the form of the four models in the base which can be derived directly in a controllable form. Besides, a new theorem on the stability of multimodel control has been proposed [8] and is based on this control strategy.

In order to carry out this control strategy, let us propose a five -order process represented by the following state space model:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 & -a_4 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

The parameters a_i vary within the limits presented in the Table.

	$a_{average}$	Δa	min	max
$-a_0$	-1	1	-2	0
$-a_1$	1.2	0.5	0.7	1.7
$-a_2$	0.5	1	-0.5	1.5
$-a_3$	-0.3	0.2	-0.5	-0.1
$-a_4$	1	0.1	0.9	1.1

As concerns the output vector C, first we choose C=[1 0 0 0 0] then C=[1 1 1 1 1]. The extreme models are then derived and can be expressed by:

$$A_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -\underline{a_0} & -\overline{a_1} & -\overline{a_2} & -\underline{a_3} & -\underline{a_4} \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -\underline{a_0} & -\underline{a_1} & -\overline{a_2} & -\overline{a_3} & -\underline{a_4} \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -\overline{a_0} & -\overline{a_1} & -\underline{a_2} & -\underline{a_3} & -\overline{a_4} \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -\overline{a_0} & -\underline{a_1} & -\underline{a_2} & -\overline{a_3} & -\overline{a_4} \end{pmatrix}$$

The state matrix of the average model is determined as the average of the coefficients of the previous four models and is expressed by:

$$A_m = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & 1.2 & 0.5 & -0.3 & 1 \end{pmatrix}$$

A pole analysis shows that the four models in the base plus the average model are unstable. Moreover by choosing the output matrix C=[1 1 1 1 1] we introduce unstable zeros with a strong imaginary part.

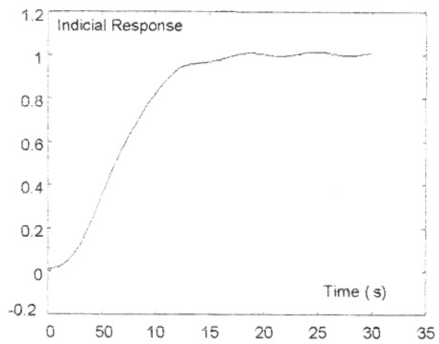
Even though this illustrative example may not have a physical reality, it was deliberately introduced in order to test the efficiency of the proposed approach under extremely severe conditions.

The principle underlying the pole assignment control law is the following: determine for each partial model in the model base a feedback gain such that the poles of each model are assigned identically to:

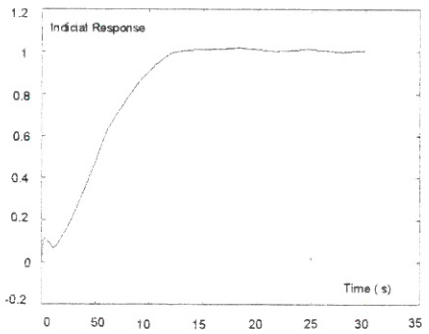
$$\text{Poles} = [-5, -0.7, -0.6, -0.55, -0.5]$$

These new poles are determined by looking out to the average natural dynamics of the process [8].

The simulation results in both cases for the output vector are as follows:



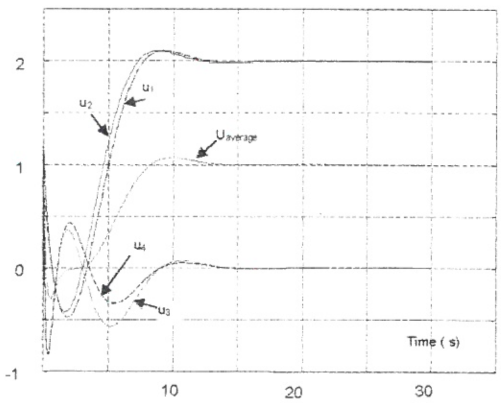
Pole assignment for
C= [1 0 0 0 0]; $\Delta a_0 = \Delta a_4 = 0.1$



Pole assignment for
C= [1 1 1 1 1]; $\Delta a_0 = \Delta a_4 = 0.1$

We can remark that a good stabilisation of the system can be obtained. However, a bad choice of the new dynamics of the closed loop system can drive an unstable behaviour.

It may appear interesting now to get an idea about the contribution of each model to the global control law. Indeed, the following simulation result shows the evolution of the control variable for each partial model including the average model. We have:



8. Conclusion

This paper presented different new results: the introduction of a systematic method for validity determination when dealing with a multimodel approach. Based on the fuzzy logic and using the fusion technique, this method consists in an extension of the classical definition of residue and then of the validity especially adapted to the multimodel approach. Besides, a systematic method was proposed for model base generation whatever the order and the complexity of the process of which main advantage was the limitation to four, five or six models. Moreover, drawing up of a control law using the LQ technique was proposed in this context using the fusion technique weighted by the validity indexes. The pole assignment control was also considered under extremely severe conditions for the system and for deriving good results. The model base was enriched by an average model thought to be a reference model which the other four models were compared with. It is interesting to note that this strategy grants higher robustness to the results even in largely complex and ill-defined systems.

REFERENCES

1. MURRAY-SMITH, R. and JOHANSEN, T. A., **Multiple Model Approaches to Modeling and Control**, TAYLOR & FRANCIS, 1997.

2. KSOURI-LAHMARI, M., EL KAMEL, A., BENREJEB, M. and BORNE, P., **Multimodel, Multicontrol Decision Making in System Automation,** IEEE-SMC'97, Orlando, Florida, USA, October 1997.

3. ISERMANN, R., **Process Fault Detection Based On Modelling and Estimation Methods: A Survey**, AUTOMATICA, Vol. 20, No. 4, 1984.

4. STAROSWIECKI, M., COCQUEMPOT, V. and CASSAR, J. P., **Observer Based and Parity Approaches for Failure Detection and Identification,** IMACS Symposium, MCTS, Lille, France, 1991.

5. V.L. KHARITONOV, **Asymptotic Stability of An Equilibrium Position of A Family of Systems of Linear Differential Equations**, DIFFERENTIAL, Uravnen, Vol. 14, 1978.

6. CHAPELLAT, H. and BHATTACHARYYA, S. P., **An Alternative Proof of Kharitonov's Theorem** , IEEE TRANS. ON AUTOMATIC CONTROL, Vol. 34, No. 4, 1989.

7. EL KAMEL, A., BORNE, P., KSOURI-LAHMARI, M. and BENREJEB, M., **On the Stability of Non-linear Multimodel Systems** , proposed for publication to INT. J. STABILITY AND CONTROL: THEORY & APPLICATIONS.

8. BORNE, P., DAUPHIN-TANGUY, G., RICHARD, J. P., ROTELLA, F. and ZAMBETTAKIS, I., **Analyse et régulation des processus industriels**, TECHNIP, France, 1993.