Outlines Of A Non-classical Logic for the Approach Of Fuzziness

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1. The concept of fuzziness may be approached more efficiently if it is considered as a fact of **indeterminacy**, that also covers many other concepts known under several labels like ambiguity, equivocacy, undecidability, uncertainty, vagueness, or crisis, chaos, disorder, entropy, etc. In this paper, for clarity's sake, semantical aspects will be, as much as possible, separated from the logical ones on the reason that the lack of verbal exactness may be remedied by avoiding, or replacing vague or ambiguous terms and expressions or/and stressing the intended meaning by redundancy. Semantic indeterminacy is usually provisional and it is to be eliminated, by convenient operations, even in mathematics.

2. The problem of indeterminacy is for logicians as old as that of determination, but the latter got priority since the former was considered as lying without the scope of science. However Aristotle dealt with it in "*De Interpretatione*", Chapters 8 and 9, in the case of undecided and undecidable statements. The non-provisional indeterminacy, i.e. that cannot be eliminated, pervades all fields of human knowledge from philosophy to technology and from mathematics to empirical sciences. As logic was to provide both criteria and rules for expressing determinations, the setting up of a logical status of indeterminacy was postponed until recent developments in quantum physics and informatics compelled scientists to cope with the problem of indeterminacy. Since old times logicians' endeavours resulted in forging and refining tools for the construction of a consistent "discourse", i.e. a chain of statements, on a "universe", i.e. a set of objects like things, events, states, mental products, etc. For our purpose the "universe of discourse" consists of two kinds of objects: determinate and indeterminate. Some examples from several fields may give a hint at the class of indeterminate objects: undecidable sentences in logic, fractals in geometry, uncertain relations in quantum mechanics, crises in biology or sociology, fuzziness in computer science, etc. An adequate metaphor for indeterminacy may be thought as a field that is in itself neither infinite nor finite, but which gets determinate as soon as a path is imagined to divide the whole field into parts interrelated with respect to the path itself. In the pair *field-path* the first member exists alone even without a path, while the second member may be thought as a mere possibility and cannot be imagined in the absence of the field. From this example one may conclude that indeterminacy possesses a structure of possible components which can eventually change even the character of indeterminacy, but they conserve it as long as these components remain mere possibilities with equal chances to turn true. The metaphor of the pair field-path may also serve to reveal the ontological roots of **binarity** that supplies a general frame of the logical thinking. As any path divides a field into two sides, and taking into account the symmetry of the human anatomy as well as the fact that the human speech consists of sequences of at least two sounds, it becomes obvious that the ontological origin of the binary thought is fundamental. Consequently, the thinking that deals with determinate objects is to be expressed through affirmative or negative statements. In this case the opposition true-false is rooted in the very act of determination, but in the case of indetermination such an opposition does not work.

3. For the purpose of explaining the difference between the determinate and the indeterminate parts of the universe of discourse, it seems necessary to examine

whether indeterminacy is compatible with the concept of **order**. Since thermodynamical processes were associated with disorder, which was assimilated to indeterminacy, one may conclude that the latter cannot be compatible with any kind of order. However, the long tradition of the European thinking does not dissociate the being and the order. From the Holy Writ up to the contemporary physicist David Bohm, order is considered intrinsic to every existing entity. If according to Platonic idealism matter was "me on", and, according to Aristotle, the same matter was a mere shapeless possibility, for the Christian mediaeval thinkers, who learnt from the Bible (the Book of Wisdom) that God said "Omnia in mensura et numero et pondere disposuisti" ordo was identified with being. Albertus Magnus summarized this conception as follows: "ipsa materia per modum et mensuram sui esse et aptitudinem ad formam et ordinem quo ut subjectum ordinatur motus et mutationis, non sine modo et specie et ordine est " (S. Th. II, tract I, qu.3). Thus matter cannot be without structure (sine modo et specie et ordine), as quantity (per modum et mensura sui), as capability to get form and order (per aptitudinem ad formam et ordinem), and as a subject to motion and change (subjectum motus et mutationis) [1]. David Bohm, a contemporary physicist, formulated a doctrine of the wholeness opposed to fragmentation and he argued [2] that matter possesses an implicate order which is manifest in any sequence or succession. Within the whole set of implicate orders there is a totality of forms that have an approximate kind of recurrence, stability and separability. He explains the implicate order as a process of enfoldment and unfoldment. A significant development took place in mathematics when Bourbaki [3] tried to get to its foundations using the concept of set: he could not work with mere sets and, consequently, he had to choose as a fundamental structure the "magma", i.e. a set provided with an operator. Otherwise "set" is a word of no use in mathematics.

4. Both philosophers and scientists have come to agree that indeterminacy is compatible with order if the latter is intrinsic and not inserted from outside. We are now in the position to proceed on a logical approach of indeterminacy that also includes fuzziness. Since the classical logic is not apt to deal with indeterminacy, a new approach of the latter will be a non-classical one that may be called an aoristic logic (aoristic means indefinite, as being formed from "a"- "not" and "horizo"-"limit"). In the definition of an **aoristic logic** the first task is to select and verify concepts and rules from the classical logic according to the specificity of indeterminacy. The structure of any indeterminacy includes one or more components which may be called alternatives, as to suggest that they are in a finite number and present full equiprobability. Such a structure is ipso facto ordered and, owing to the equiprobability of the alternatives, it has two essential properties: stability and homogeneity. Any form of reasoning on the structure of indeterminacy as a whole implies the following principles or laws: the dual pair of identity and alterity, and the principle of sufficient reason. The first two principles serve to obtain and structure information about a particular area of indeterminacy, and to distinguish its component alternatives with a view at making correct statements about the state of indeterminacy and, at the same time, discerning the state of equiprobability of each alternative. The principle of sufficient reason guides particularly the chain of statements about the ordered structure of indeterminate entities for the purpose not to mistake inherent order for eventual external determinations operating on alternatives. Besides, the principle of sufficient reason emphasizes the reasonableness of any approach to the structure of indeterminate entities.

Valid statements on indeterminacy are generated with the help of two unary operators: "idem" (symbol:=), and "aliter" (symbol: $\neq$), and of a dyadic operator: "neither-nor" (symbol:/) discovered by Peirce in 1880, re-discovered by Sheffer in 1913, reformulated by Nicod in 1917, and adopted by Russell in 1925 [4]. "idem" serves to identify an indeterminate entity; the truth value of it results from the expression $id(a)=a$ or $id(a)\neq b$. "aliter" is used to state a difference between indeterminate entities or between component equiprobable alternatives inside the same indeterminate entity. Its truth table has the form: al(a)=b or al(a)≠a. The truth table of Sheffer's operator has two entries:

A\B	t	f
t	f	f
f	t	t

Summarily:

(i) identity- the function takes the same value as the argument

(ii) alterity-the function takes a different value than the argument

(iii) rejection –the function is always true except for the case when both arguments are true

Some many-valued logical systems (e.g. Kleene's system) introduced a third truth value: "undetermined" or "undecided" or "indifferent" or "neutral" to be defined together with "true" and "false" [5] but this third value- except the deontic logic- is not used extensively in particular problems. With the operators "idem", "aliter" and "neither-nor", a logician may succeed to construct valid statements describing indeterminate entities and their ordered equiprobable alternatives. The component alternatives have the value 1/n in the interval (0,1), where n is the number of alternatives, 0 means non-existence, and 1 means certainty or complete determination. If an alternative has another value than 1/n, then the stability inside the indeterminate entity gets lost and the whole entity changes its character. The term coined for the stable state of an indeterminate object is compossibility, which means that the respective indeterminacy enjoys a sui-generis consistency.

If an alternative of an indeterminate entity gets divided into equiprobable subalternatives, while other alternatives remain unchanged, then the primitive entity loses indeterminacy and, consequently, two new indeterminate entities are resulting: one as a restriction of the primitive entity and the second one consisting of the equiprobable subalternatives of the previously divided alternative in the primitive indeterminate entity. The process of generating indeterminacy may continue so that its milestones are to be called levels of compossibility and these may serve as a measure of uncertainty: the greater the number of levels, the nearer the certainty, because each new level of compossibility reduces the area of the primitive indeterminate entity. Some similarity to levels of compossibility can be found in the introduction of grades in the relation of membership by L. Zadeh [6], when he formulated the theory of fuzzy sets and systems. Fuzziness is, indeed, indeterminacy applied mainly to the membership relation with regard to sets and systems.

5. The aoristic logic deals, thus, with indeterminacy that is usually associated with a lack of precision caused by insufficient knowledge or by the failure to obtain complete information. However, some scientists have recently found examples of indeterminacy caused by deeper knowledge and more information. Such an indeterminacy may be called **overdetermination,** suggested by the German term "Mehrwissen" introduced by C.-F. Weizsäcker [7]. "Unscharfrelationen" discovered by W. Heisenberg [8] in quantum physics belong to the class of indeterminate entities. Quite recently Gregory Chaitin [9] described randomness in arithmetic following Gödel's and Turing's results, as he applied the method of Diophantine equations to arithmetise a program for computer. He concluded that it is impossible to prove whether each member of a family of algebraic equations has a finite or infinite number of solutions. Another example of randomness is present in the concept of chaos [10] that can be generated by simple deterministic systems with few elements. H. Poincaré's definition of fortuitous phenomena of 1903 may be interpreted as a rejection of the Laplacean determinism: "arbitrarily small uncertainties in the state of a system may be amplified in time and so prediction of the distant future cannot be made". The conclusion is that more information does not make randomness go away. Indeterminacy is thus not only inherent to entities that are not-yet or never-to-be determinate; it can also be the outcome of overdetermination. Determinate entities

are like archipelagoes or continents surrounded by an ocean of indeterminacy, the waves of which are permanently touching their shores and sometimes overflowing them unexpectedly.

6. The aoristic logic may appear rather as a reduced form of the classical logic with fewer principles and connectives. On the other side the central role of equiprobable alternatives reminds of a many-valued logic or of the logic of modalities. The aoristic logic is unquestionably related to both classical and non-classical logics, but, nevertheless, it claims its own status on the reason that it deals with a subject-matter – the indeterminacy-which is quite distinct from the world of determinations. More than that, most scientists relegated indeterminacy to the minor province of the qualitative with which they were rather reluctant to deal, since qualitative was blamed-according to Rutherford-as poor quantitative. In spite of the pioneering work of H. Reichenbach, G. Birckoff and J. von Neumann in the thirties by setting up ingenious calculi for a specimen of indeterminacy, a conceptual frame for the whole realm of indeterminacy was scarcely available. The aim of the present paper has been to give some hints at a task that is entirely within reach of logicians.

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