Approximate Reasoning

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Consider the beginning of the first Chapter in Lee and Zhu [1] : " Human reasoning usually is approximate in nature and involves various uncertainties. For example, given that *John is tall and Tom is a little bit shorter than John* we can infer that *Tom is tall also*." Let us try to understand why the inference above is *approximate* and to detect *uncertainties*, if any. Replace, first, the inference by the following: if *x is tall and y is a little bit shorter than x then y is tall also*. This seems more general; a possible source of vagueness is in passing from "x is tall" to "John is tall " due to the fuzzy predicate "tall" (see [2]). But this decision problem is not crucial for the inference we are studying. The same keeps valid for the fuzzy relation " a little bit shorter than ". Another problem is if the inference (in the general form) could be accepted as valid from, say, an empirical or elementary point of view: that is, if someone is tempted to accept it as clear or evident. Of course, in asking such a question, we introduce a vagueness of the second order (to speak in vague terms of vague things) but our feeling is that such questions bear relevance (and, as a matter of fact, it is very difficult to avoid second order vagueness). It seems quite possible to reject the inference because of lacking information about the relation "a little bit shorter than ". But the meaning of this relation can be explained, for example, as follows: "x is a little bit shorter than y" means that, after a careful examination (not necessarily after measuring their heights [2]), "x is found to be a little bit shorter than y"; we prefer not to introduce heights. Of course such an explanation is in fact fuzzy but in our opinion it helps make things more precise. Now, it seems quite possible to intuitively accept our inference as being valid. So if we do not bother ourselves with the problem of deciding if John is tall (or if Tom is a little bit shorter than John) we may consider the inference as empirically valid (and, by way of consequence, no uncertainties will result).

Another aspect should nevertheless be revealed; something subtler and almost imperceptible, the slippery slope phenomenon. Suppose we know (this time empirically) that John is tall (if you want "John is tall" is a fact) and that Tom is a little bit shorter than John. Using our basic inference, we find that Tom is also tall. This is a consequence and if we take this consequence as a matter of fact (so if we take it from the context) we can use it in an inference like *Tom is tall and Peter is a little bit shorter than Tom so Peter is also tall*. By doing this "many times" we may come to declare as tall someone who is definitely not tall. Is this strange phenomenon an enough reason why to call the starting inference "approximate" or "uncertain"? The point is that the inference may not be repeated freely any time one likes (in a different context the situation is the same as in quantum logic). It is worth- mentioning that the consequence appears in the form "Tom is tall also"; this *also* looks like a mark for the fact that the conclusion was reached from an inference and not as an observed fact (we do not see Tom without John to decide about his tallness). Here some kind of a modality is present: an entropic like aspect of the validity of some inferences. There are voices proclaiming that fuzzy sets can solve the slippery slope paradox; the problem complexity prevents us from entering into more detail (see [3]).

The following of the paper analyses some elementary aspects of the "generalized modus ponens" (see [4]). To be more specific, let consider the following simple case: suppose X, Y are variables having the linguistic values of small (s), medium (m) and large (l) and consider the implication:

(*) If X is small then Y is large

From all (not few) interpretations of (*) we have chosen the dynamic one: X is the set of all possible inputs and Y the set of the outputs of an input-output system. Obviously (*) can be a rule in an expert system, etc.

So (*) shows that the pair (s, l) belongs to the input-output relation of the system. If no other information is available then all the pairs (m,y), (l,y) $y \in Y$ are equally possible members of the input-output relation. Therefore if we want to describe our knowledge about the system we have to consider non-deterministic systems: we associate the inputs m and l with the whole Y. This discussion was meant to enable a better understanding of the problem of the "generalized modus ponens":

If X is small then Y is large

X is medium

?

It seems that a rational answer could be "everything" in the sense that from the information (*) and the fact that X is medium there derives the conclusion Y is Y. Note that the values of X and Y are fuzzy predicates but, as linguistic variables, they are crisp. Sometimes the input and output sets of a system include a notion of closeness (for example a topology or a tolerance) and from " If X is A then Y is B" one derives something like "if X is close to A then Y is close to B". This conclusion can be dangerous if A is not "stable" (strong dependence on initial conditions).

The use of the semantics of fuzzy sets helps smoothing the crisp character (small and medium intersect with each other), and formulas are found to compute the conclusion in the generalized modus ponens. In general a fuzzy set resulted from the computation does not correspond to a linguistic value; this is to be expected if minding the lack of information at the level of crisp sets.

REFERENCES

1. LEE, E. S. and ZHU, Q., **Fuzzy and Evidence Reasoning**, PHYSICA-VERLAG, 1995.
2. FLONDOR, P., **Modeling with Fuzzy Sets**, STUDIES IN INFORMATICS AND CONTROL, Vol. 7, No. 3, 1998, pp. 207-208.
3. GOGUEN, J. A., **The Logic of Inexact Concepts**, SYNTHESE 19, 1968-1969, pp. 325-373.
4. ZADEH, L. A., **The Concept of A Linguistic Variable and Its Application to Approximate Reasoning**, AMERICAN ELSEVIER PUBLISHING COMPANY, New York, 1973.