Differential and Integral Operators

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This and the next volume of the Operator Theory series contains the Proceedings of *International Workshop on Operator Theory and Applications, IWOTA '95,* which was held at the University of Regensburg, Germany, from July 31 to August 4, 1995. It was the eighth edition of such an workshop.

IWOTA '95 offered a rich programme on a wide range of recent developments in operator theory and its applications. The programme consisted of 6 invited plenary lectures, 54 invited special topic lectures and more than 100 invited session talks.

About 180 participants from 25 countries attended the workshop. More than one third came from Eastern Europe.

This volume contains 22 papers, starting with problems for abstract operators to spectral theory of ordinary and partial differential operators, pseudodifferential operators and integral operators.

The applications addressed problems in mathematical physics, hydrodynamics, magnetohydrodynamics, quantum mechanics, astrophysics as well as theory of networks and systems.

A brief description of the papers follows.

1. *Limit behaviour in a singular perturbation problem, regularized convolution operators and the three - body quantum problem,* by S. Albeverio and K. A. Makarov, studies a model of a quantum mechanical system related to the three - body problem. The model is defined in terms of a symmetric pseudodifferential operator with unbounded symbol.
2. *Banach algebras of functions on nonsmooth domains*, by F. Ali Mehmeti and S. Nicaise, investigates the spaces of functions on domains with a conical singularity, which arise naturally in the study of hyperbolic and parabolic evolution equations on such domains. In this paper are stated the conditions under which these spaces are Banach algebras, a result which is of independent interest.
3. *A nonlinear approach to generalized factorization of matrix functions,* by M. Câmara and A. F. Dos Santos, determines the generalized factorization of some classes of 2x2 matrix symbols by reduction to the study of certain scalar non - linear Riemann - Hilbert problems.
4. *Completeness of scattering systems with obstacles of finite capacity,* by J. Van Casteren and M. Demuth, proves that the scattering system established by some couple of operators is complete if the singularity region Γ has finite capacity.
5. *Examples of positive operators in a Krein space with 0 a regular critical point of infinite rank,* by B. Curgus and B. Najman, proves that the operators associated with the perturbed wave equation in $\mathbf{R}^n$ are similar to a self - adjoint operator in a Hilbert space. These operators have the whole $\mathbf{R}$ as the spectrum.
6. *On Hilbert - Schmidt operators and determinants corresponding to periodic ODE systems,* by R. Denk, investigates the structure of infinite determinants corresponding to linear periodic ODE systems.
7. *On estimates of the first eigenvalue in some elliptic problems,* by Yu. V. Egorov and V.A. Kondratiev, studies the boundary value problems of the form $Lu = \lambda V(x)u$ where L is an elliptic operator in a domain of $\mathbf{R}^n$ and gives estimates of the first eigenvalue. Note that the first eigenvalue for the Dirichlet problem is often a very important characterization of a system in Physics, Elasticity, Biology etc. In the theory of boundary value problems it serves for estimation of the domain of existence or uniqueness of solutions.
8. *Nonsingularity of critical points of some differential and difference operators,* by A. Fleige and B. Najman, proves regularity of the critical points 0 and ∞ for two different examples of positive definitizable operators in Krein spaces.

9. *A nonlinear spectral problem with periodic coefficients occurring in magnetohydrodynamics,* by A. Lifschitz, describes a method for studying instabilities of plasma equilibria and waves.

10. *An evolutionary problem of a flow of a nonlinear viscous fluid in a deformable visoelastic tube,* by W. -G. Litvinov, studies the problem of a nonsteady flow of a nonlinear viscous fluid in an oscillating tube. The existence of a solution is proven.

11. *Quantum compound Poisson processes and white noise analysis,* by E. W. Lytvynov, constructs the Fourier transform in generalized joint eigenvalues. This construction gives the possibility of introducing spaces of test and generalized functions.

12. *Invariant and hyperinvariant subspaces of direct sums of simple Volterra operators,* by M. M. Malamud, describes in geometrical terms the lattices of invariant and hyperinvariant subspaces of some Volterra operator defined on $L_p[0,1]\otimes \mathbf{C}^n$.

13. *Some interior and exterior boundary - value problems for the Helmholtz equation in a quadrant,* by E. Meister, F. Penzel, F. - O. Speck and F. S. Teixeira, studies boundary transmission problems for two – dimensional Helmholtz equations in a Sobolev space setting. The first problem of a quadrant with Dirichlet condition on the one side, and transmission condition on the other, is solved in closed form for the case where all quadrants are occupied by the same medium.

14. *Interpolation of some function spaces and indefinite Sturm - Liouville problems,* by S. G. Pyatkov, considers self – adjoint Sturm – Liouville problems of the form $Lu = \lambda g(x)u$ where L is an ordinary differential operator of order 2m, defined on the interval $(0,1)$. The author proves, under some assumptions, that the eigenvectors and the associated vectors constitute a Riesz basis in the space L_2 with the weight $|g|$.

15. *Mellin pseudodifferential operator techniques in the theory of singular integral operators on some Carleson curves,* by V. S. Rabinovich, proves some results of the Mellin pseudodifferential operators theory.

16. *Wiener - Hopf factorization of singular matrix functions,* by M. Rakowski, discusses properties of Wiener – Hopf factorization of singular matrix functions relative to a rectifiable contour.

17. *Elliptic boundary value problems for general elliptic systems in complete scales of Banach spaces,* by I. Roitberg, studies elliptic BVPs for general elliptic systems in the case where the boundary conditions contain both the function from the systems and the additional functions defined at the boundary of the domain.

18. *Classic spectral problems,* by L. A. Sakhnovich, shows that the string matrix equation and the Sturm – Liouville equation belong to a class of canonical systems.

19. *Mellin operators in a pseudodifferential calculus for boundary value problems on manifolds with edges,* by E. Schrohe and B.- W. Schulze, shows how to associate an operator valued Mellin symbol with an arbitrary edge – degenerate pseudodifferential BVP, the so-called "Mellin quantization" procedure.

20. *On some global aspects of the theory of partial differential equations on manifolds with singularities,* by B. -W. Schulze, B. Sternin and V. Shatalov, investigates the connection between asymptotic expansions of solutions to elliptic equations near different points of singularities of the underlying manifolds.

21. *Green's formula for elliptic operators with a shift and its applications,* by Z. G. Sheftel, introduces the notion of normal boundary conditions with a shift, and deduces the Green's formula for such boundary conditions and partial differential equations of even order.

22. *On second order linear differential equations with inverse square singularities,* by R. Weikard, studies the differential equation:

$$y'' + \left(\frac{a}{x^2} + q(x)\right)y = Ey$$

where $a, E \in \mathbf{C}$ and q is a complex – valued function which is locally integrable on $\mathbf{R} - \{0\}$ and analytic in a neighbourhood of $x = 0$.

The contributions to different aspects of operator theory and its applications contained in this volume are of interest for the research workers in the domain.

Marius Rădulescu