Metric Constrained Interpolation, Commutant Lifting and Systems

by C. Foias, A.E. Frazho, I. Gohberg and M. A. Kaashoek

"Operator Theory. Advances and Applications, Vol. 100"

Birkhäuser Verlag, Basel, 1998, 587+xii p.

ISBN 3-7643-5889-0

This well-written book is a self-contained and unified presentation of the solution of stationary and nonstationary interpolation problems with norm constraints, in finite or infinite dimensions. The basic techniques are the commutant lifting theorem from operator theory and the state space method from mathematical system theory, which are combined to solve these problems. System theory is used not only for stating the problems, but also for delivering state space formulas which describe the solutions. The interaction between interpolation and system theory proved very useful for recent advances in both fields.

The book consists of **Introduction, 14 Chapters** (grouped in two Parts), and **one Appendix**.

Part A, *Interpolation and Time Invariant Systems*, contains the first seven Chapters, and develops a general theory for H^∞ interpolation problems for operator-valued functions with operator argument. The main results include central state space solutions, state space formulas for describing all solutions, and applications to H^∞ and mixed H^∞ and L^2 control problems.

Chapter I, *Interpolation Problems for Operator-Valued Functions*, gives necessary and sufficient conditions for the existence of solutions to H^∞ interpolation problems for operator-valued functions with operator arguments. After introducing notation and terminology, the classical and "tangential" Nevanlinna-Pick, Hermite-Fejér, Nehari, Sarason, and Nudelman interpolation problems, with both one-sided and two-sided versions, are discussed.

Chapter II, *Proofs Using the Commutant Lifting Theorem*, contains proofs for the operator-valued theorems presented at Chapter I.

Chapter III, *Time Invariant Systems*, is an introduction to the state space theory for discrete time invariant systems, including structural properties (controllability, observability, stability), realization theory, point evaluation, computing the norm for (block) Hankel operators, etc. Explicit state space formulas for some interpolation problems are also given, allowing conversion between specific problems.

Chapter IV, *Central Commutant Lifting*, introduces the "central intertwining lifting" concept, and uses it to prove the commutant lifting theorem. Explicit formulas for this central solution are given for some interpolation problems, including the operator-valued Schur problem. It is shown that central intertwining lifting maximizes a certain entropy function, and has some bounds useful for solving certain mixed H^∞ and L^2 interpolation problems.

Chapter V, *Central State Space Solutions*, derives explicit state space formulas for the central solutions of the Nevanlinna-Pick, Nehari, Sarason, Nudelman, and two block interpolation problems, and part of their H^∞ and L^2 bounds. Some of the formulas are new indeed.

Chapter VI, *Parameterization of Intertwining Liftings and its Applications*, shows that the set of all intertwining liftings is parameterized by a natural set of contractive analytic operator-valued functions, and gives a Redheffer scattering interpolation of this parameterization. State space formulas for the Redheffer scattering matrix, which parameterizes the set of all solutions for some standard interpolation problems, are presented.

Chapter VII, *Applications to Control Systems*, uses norm constrained interpolation for solving problems in systems and control, mainly for single-input single-output systems. The Youla parameterization of all stabilizing controllers of a basic feedback control problem is derived. Interpolation results are used to solve H^{∞} and H^2 error tracking problems, and some numerical examples are given. The multivariable case and the model matching problem are also briefly commented.

Part B, *Nonstationary Interpolation and Time Varying Systems*, contains the last seven Chapters, and deals with norm constrained interpolation problems for doubly infinite lower triangular matrices with operator entries, the nonstationary analogues of the H^{∞} interpolation problems, which were studied in **Part A**. Here either the reduction to infinite dimensional-invariant problems, or the three chains completion theorem, is used. Time-variant versions of central solutions, the description of all solutions for the three chains problem, and their application to time-dependent H^{∞} control problems, are shown.

Chapter VIII, *Nonstationary Interpolation Theorems*, gives necessary and sufficient conditions for the existence of solutions for the nonstationary Nevanlinna-Pick, Hermite-Fejér, Nehari, Sarason, and Nudelman interpolation problems for doubly infinite lower triangular operator matrices.

Chapter IX, *Nonstationary Systems and Point Evaluation*, presents the nonstationary version of point evaluation, and shows how a time-varying system may be converted to an infinite dimensional time-invariant system, suggesting reduction techniques to be further used.

Chapter X, *Reduction Techniques: From Nonstationary to Stationary and Vice Versa*, defines techniques for converting nonstationary interpolation problems to stationary ones, bringing forth a left invertible transformation, which maps a doubly infinite operator matrix onto a doubly infinite block Laurent matrix. A particular left inverse is also defined.

Chapter XI, *Proofs of the Nonstationary Interpolation Theorems by Reduction to the Stationary Case*, shows that each nonstationary interpolation problem considered is equivalent to a stationary one, and gives some state space formulas for the solutions.

Chapter XII, *A General Completion Theorem*, develops a time-varying version of the commutant lifting theorem, and provides three different proofs. A nonstationary maximum entropy principle is stated.

Chapter XIII, *Applications of the Three Chains Completion Theorem to Interpolation*, contains a direct proof of the nonstationary interpolation theorems, which were stated at Chapter VIII, and shows that the three chains completion problem is equivalent to a nonstationary four-block problem.

Chapter XIV, *Parameterization of all Solutions of the Three Chains Completion Problem*, proves that the set of all solutions for the three chains problem with a given tolerance is parameterized by a natural set of contractive upper triangular operators, and obtains the set of all solutions to a nonstationary Nehari problem with data in state space form.

The **Appendix** deals with positive (semi)definite matrix -valued functions on the unit circle, and refers constructive methods for spectral factorization of matrix-valued functions, based on finite sections of Toeplitz matrices and associated Riccati difference equations. Explicit state space formulas for the rational case are included.

Written in the standard mathematical theorem-proof style, the book makes a valuable theoretical contribution. (Of course, not all theorems are proved).

In order to avoid cross references in excess, definitions and results are often recalled, and parts of the proofs are resumed whenever necessary. There are few numerical examples. The statement of theorems with important potential applications is as self contained as possible. A Notes section ends each Chapter with references to the literature. Short, but interesting historical outlooks are also had in these sections, as well as in the Introduction. There is a Bibliography section including over 170 references. A list of symbols and a concise subject index come at the end.

By its comprehensive coverage, good organization, the readability of the exposition, the theoretical results included, and the highlighted potential applications, the reviewed book recommends itself as an advanced graduate course in mathematical system theory. The intended audience is that of mathematicians, control engineers and researchers, as well as students in the corresponding disciplines. The assumed background for the book includes linear algebra, complex analysis, and Hilbert space operator theory.

Vasile Sima