Modeling With Fuzzy Sets

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One of the first reasons for introducing fuzzy sets was that of the difficulties encountered by the classical set theory in dealing with vagueness. Once vague concepts are asserted, it seems hard to describe their extensions as "crisp" sets. In this paper we try to analyse the situation and see if fuzzy sets theory can do better. To make this analysis more specific we shall consider the concept "tall" which is almost unanimously declared to be vague.

Our task will be easier if we restrict the context: "tall" will be applied only to men living within a limited area, etc. It seems that the vagueness of "tall" is already present in this simplified universe. We do not consider here the problem of vagueness as a result of the "subjectivity" of different people. The problem is if a reasonable set of tall men can be defined; so we look for "rules" for deciding about membership or non-membership to this set. Sorensen (2) thinks this to be impossible: for example, "tall" is vague because a man of 1.80 meters in height is neither tall nor clearly not tall. No amount of conceptual analysis or empirical investigations can settle whether a man whose height is 1.80 meters is tall. Borderline cases are inquiry "resistant".

We think the relation between "tall" and "height" is the first to be understood. Let us start our investigations by recallling some definitions (see Oxford Advanced Learner's Dictionary 1989):

1. height = measurement from head to foot of a standing person

2. tall = of more than average height

and a note of usage:

3. the adjective "tall" relates (height) to sense

We shall take these definitions as a starting point for our discussion. It is important to say that we do not study the relation "x is taller than y" which, from our point of view, is less vague than "x is tall". Of course the connections between "tall" and "taller than" are quite interesting (not only from a grammatical point of view), but this subject makes a different discussion. It is worth remarking that in using "tall" we make a comparison, but with an average height. It is this hidden threshold which makes things "fuzzy".

On the other hand, the previous definitions lead to an algorithm for defining a set of "tall men": given a set of men M, compute (estimate) the average height and then consider the subset T of M consisting of those men of height more than the average. The problem is not so simple because of one's dispositional belief in the possibility of deciding about a man being tall without any reference to a concrete set M. Let us find some "rational axioms" for the definition of "tall" in terms of height and senses. First, making some reductions, we assume that the height is the only "objective" property involved. Imagine an interval (0, L) of possible heights being given and try to define a subset T of (0, L) as the subset of "tall" heights. The second axiom is about perception:

(*) if $a \in T$ then there is a neighbourhood V of a such that $V \subseteq T$ and the same condition for CT (in (0, L)).

This is a condition of "openness" for empiric predicates (see also Goguen [1]). The third axiom is that T and CT are nonempty (there are tall heights but not all heights are tall).

Unfortunately there is no such T in (0, L); in fact T should be open, nonempty and closed in (0, L) which is connected so necessarily

T= (0, L) — a contradiction.

So, we can say that classical set theory is not very useful in modeling the empiric predicates. Now, let us consider fuzzy sets. The point is to define the extension of "tall" as a function $f : (0, L) \rightarrow [0, 1]$; for any $a \in (0, L)$, f(a) is the grade of membership of a to the fuzzy set f. We are ready to accept that membership is gradual and also that it can be measured by reals.

Of course, f(a)=1 has the meaning: a is surely tall. It is natural to assume the continuity of f as we imagine small changes in heights.

Now, we consider the set T of the points $a \in (0, L)$ such that f(a)=1. It is a nonempty set (we have tall heights), having nonempty complement (we surely have heights which are non tall) and which is closed by the continuity of f and open by the axiom (*). So, again T= (0, L) which is a contradiction. It seems that modeling with "fuzzy sets" though giving some refinement by using grades of membership does not solve the basic "contradiction" between numbers and perceptions. In a forthcoming paper we shall analyse this situation from the point of view of probability theory. Anyway, the idea of "grade of membership" is a very interesting one and modeling with fuzzy sets is in general a good approximation.

REFERENCES

1. GOGUEN, J.A., **The Logic of Inexact Concepts**, SYNTHÈSE 19, 1968-69, pp. 325-373.

2. SORENSEN, R., **Vagueness**, Stanford Encyclopedia of Philosophy, 1997.