A "Tall" Story

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"When I make a word do a lot
of work like that," said Humpty Dumpty,
"I always pay it extra".
Lewis Carrol *"Alice Through the Looking Glass"*

1. A Stochastic Model for the Use of the Predicate "Tall"

Grammatical investigations ([1]) have led to the idea that vague predicates such as "heavy", "short", "bald" are used on the basis of a comparison with an implicit standard. In the following we consider the predicate "tall", as applied to the set of men of a restricted age range living in a small community. (We do not consider problems arising from the differing use of "tall" across large communities such as the Japanese and the North American). Also, "tall" is taken to be a function of an individual's height, discarding other features that might also be important, for instance that a certain choice of clothes might make one appear of larger height.

Essential to declaring "tall" to be a vague predicate is that there are cases in which the same height is declared at one time, by the same individual, to be tall and at another time not tall. This conception of vagueness is called "borderline" and has been presented in [2].

The different decisions an individual makes for borderline cases can be formalized as a stochastic process, which is formally identical to the process that describes measurement errors.

The following equation gives the probability that a certain height h is declared tall:

$$p(h) = \begin{cases} 1, if\ h > hs + \varepsilon \\ 0, if\ h < hs - \varepsilon \\ r(|h - hs|), if\ |h - hs| \le \varepsilon \end{cases}$$

where

hs is the standard value taken for comparison, and

ε is the threshold value for perception (the value below which the human eye can no longer distinguish two heights), and

*r : **R** -> [0, 1]* is a radial function (symmetric and reaching a maximum in zero) such as a Gaussian.

2. The Sorites Paradox

Let us describe the sorites paradox as applied to "tall":

Consider a man of height 2m. He is undoubtedly tall. But there is a certain threshold for visual perception, say 1mm, under which you can no longer distinguish the difference in height. Therefore, a man of height 1.999 will also be declared tall. By continuing the reasoning, we reach the conclusion that any man, of any height is to be declared tall. And therefore, there are no short men, which contradicts common sense (and the common usage of the words tall and short).

But then, let us imagine how the presentation of concrete men is to be made to the decision maker. For if the men are presented as a row, the decision maker will move along the row of men and select a certain individual, saying "from here on they are all tall", thus cutting the row; but a future cut will most probably not be made in the same place. And if the individuals are to be presented one by one, over a period of time, than those having borderline heights will sometimes be declared as tall, and sometimes not.

The model described in Section 1 fits in both cases, and it gives the probability for a certain height to be declared as "tall".

In view of the above, the sorites paradox is flawed in a linguistic way: where it says "therefore will also be declared tall" it should read "therefore it will be treated in the same way from the point of view of "tallness"". And that is correct, for a small difference in height would give a small difference in probability, which cannot be determined by experiment. (Imagine an experiment that would show in a statistically significant manner that a man treats the height of 1.80m in a different manner from the height 1.799m.)

3. Fuzzy Sets and Vague Predicates

Fuzzy sets assign membership degrees to the set of heights, interpreting these values as "truth values". As explained in [3], this is not a satisfactory solution to the problem raised by the sorites paradox.

However, the fuzzy model and the "hidden standard" model are very similar. Both use numerical values to give a certain measure of the reliability of the judgement "this height is tall". The difference stems from the interpretation of the numerical values assigned in the two models. In [4] we have explained in detail how this difference of interpretation can affect the way fuzzy logic is applied in problems of control. In brief, when applying fuzzy logic to industrial control, it is necessary to find the degree of truth of a composite sentence as a function of the truth values of the composing sentences. This is usually done by taking the minimal truth value in conjunctions and the maximal truth value in disjunctions. If one interprets "truth values" as giving the probability that a certain statement is true, then the computation described above should be performed according to Bayes' rule instead. Therefore analyzing the terms of the fuzzy rules used in a controller in terms of hidden standards model might have practical consequences.

4. Conclusion

We have presented a model for the use of the predicate "tall" by formalizing the grammatical conclusion that some vague predicates contain a comparison with an implicit standard. We have then tried to use this model to understand the sorites paradox. Finally, the relation between such a "hidden standard" model and fuzzy logic was briefly discussed.

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