BOOK REVIEWS

Elements of Multivariate Time Series Analysis

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This book is justified by the use of methods of time series analysis in the study of multivariate time series, that has become of increased interest in recent years. In spite of the fact that the methods are rather well- developed and understood for univariate time series analysis, the situation is not so good for the multivariate case. The present book is dedicated to introducing the basic concepts and methods that are useful in the analysis and modelling of multivariate time series. The book includes both traditional topics such as auto-covariance and auto-correlation matrices of stationary processes, properties of vector ARMA models, forecasting of the ARMA processes, least- squares and maximum likelihood estimation techniques for AR and ARMA models, and model checking diagnostics for residuals, as well as topics of more recent interest for vector ARMA models such as reduced-rank structure, structural indices, scalar component models, canonical correlation analyses for vector time series, multivariate unit-root models and cointegration structure, state-space models and Kalman filtering techniques and applications. The book concentrates on the time-domain analysis of multivariate time series, the spectral analysis being not considered here.

This is a revised edition of the book, with some additional topics taken on as to the original version, and certain existing materials expanded, in an attempt to provide full coverage of the topics in time-domain multivariate time series modeling and analysis. Notable new additions are: an entirely new chapter on various topics that arise when exogenous variables are involved in the model structures, inclusion of new sections on state-space forms of the vector ARMA model, on the background and motivation for the information criterion (AIC), on canonical analysis for vector AR processes, inclusion of new materials in some sections on canonical correlation analysis, some details on reduced-rank multivariate linear regression model results, on the concepts of controllability, observability, and minimality for time-invariant state-space models, on estimation of missing values in vector ARMA models, and on the initialization for Kalman filtering, smoothing and likelihood evaluation in non-stationary models. Also, some substantial revisions have been made on sections concerned with estimation and testing of non-stationary (unit-root) processes to give evidence of further developments on the topic.

The book contains eight chapters, an Appendix with time series data sets used in the text, references, subject and author indices. Also, a set of exercise problems are included at the end of the book.

Chapter 1 *"Vector Time Series and Model Representations"* introduces the concept of stationarity of a vector time series process, and some basic covariance and correlation matrix properties and spectral properties of such a process are presented. Some features of linear filtering of a stationary time series are considered. Vector autoregressive moving average (ARMA) models are then introduced, and some of their different model representations are discussed. Appendix to this Chapter provides a review of multivariate normal distribution and related topics.

In **Chapter 2**, titled *"Vector ARMA Time Series Models and Forecasting"*, the ARMA models introduced at Chapter 1 are examined, and the stationarity and invertibility aspects of vector ARMA processes are considered. The covariance matrix structure of vector ARMA processes is analysed in general as well as for special cases such as first-order MA, AR, and ARMA models. In addition, parameter identifiability of mixed ARMA model representations is remarked. Non-stationary ARMA processes are also considered, and the concept of cointegration among the component series of a non-stationary process is defined. Forecasting of vector ARMA models, including computation of forecasts and mean squared error matrix of the forecast errors, is presented. A distinct space is reserved to state-space form of the vector ARMA model. This Chapter also contains an Appendix with two methods for obtaining autoregressive and moving average parameters from co-variance matrices.

In **Chapter 3**, *"Canonical Structure of Vector ARMA Models"*, this problem is briefly discussed through the introduction of such concepts as the Kronecker indices and the McMillan degree of a vector process, and the echelon canonical form of the vector ARMA model is expressly presented. The canonical correlation structure for stationary vector ARMA processes is examined, and the relation between canonical correlation structure and the associated notion of scalar component models, is approached. Partial correlation matrices and their canonical correlations in a stationary vector process, and their special features for pure AR models, are also considered.

In **Chapter 4**, *"Initial Model Building and Least Squares Estimation for Vector AR Models"*, preliminary model specification techniques based on sample statistics such as correlations and partial correlations are stated and discussed. Least- squares estimation for vector AR models and associated tests on hypotheses for the order of the AR model are made. The properties of least squares estimates for vector AR models are discussed, and additional methods for initial specification and selection of an appropriate ARMA model, including the use of canonical correlation methods and information-theoretic model selection criteria such as AIC and BIC, are explored. An Appendix to this Chapter summarizes and reviews some basic results on general multivariate linear regression model.

Chapter 5 of the book, titled *"Maximum Likelihood Estimation and Model Checking for Vector ARMA Models"*, presents some conditional and exact maximum likelihood (ML) estimation procedures for vector ARMA time series models, and examines their properties. For conditional maximum likelihood, an explicit iterative computation of the ML estimator in the form of a generalized least squares estimation is yielded, while for the exact likelihood method, two different approaches to computation of the exact likelihood function are made. An ML estimation of vector ARMA models under linear constraints on parameters, and an associated likelihood ration (LR) testing of the hypothesis of linear constraints are also undertaken. Model checking techniques for an estimated model, based on correlation matrix properties of model residuals, are explored. The effect of parameter estimation errors on mean square error for prediction in an estimated model and the motivation for AIC as a model selection criterion are shown. Two numerical examples for fitting and checking vector ARMA models are also given.

Chapter 6, *"Reduced-Rank and NonstationaryCointegrated Models"*, presents some new topics on the modeling of vector time series. These include the examination of models that incorporate special structures in their parameterization, in particular the nested reduced-rank models, which attempt to cope with the problem of high dimensionality of the parameters in vector models. Model specification methods, based on partial canonical correlation analysis, and parameter estimation will be presented for the nested reduced-rank AR models. Due consideration is given to the estimation and testing issues related with multivariate nonstationary models that contain unit roots in their AR operator, and to the associated concept of cointegration among the components of a non-stationary vector process. An alternate canonical correlation analysis procedure and multiplicative seasonal vector ARMA models are discussed as additional special topics in this Chapter.

A state-variable (state-space) model and its basic properties are described in **Chapter 7**, *" State-Space Model, Kalman Filtering, and Related Topics"*. The Kalman filtering and smoothing procedures associated with state-space models are derived. The relation of time-invariant state-variable models with vector ARMA models, including the state-space representations for vector ARMA models, and associated topics of minimal dimensionality of the state vector and the relation with the Kronecker indices and the McMillan degree of a process, is also discussed. The use of a state-space formulation for construction of the exact likelihood function for the vector ARMA model, as well as comments on the results of a classical approach to smoothing and filtering of time series are there.

Chapter 8, *"Linear Models with Exogenous Variables"*, makes a thorough analysis of this type of model, where the output variables of interest may be influenced by other observable input variables, which are determined outside the system concerned. The main topics discussed are: representation of linear models with exogenous variables (ARMAX), forecasting on ARMAX models, optimal feedback control in ARMAX models, model specification, ML estimation and model checking for ARMAX models. The last section of this Chapter presents a numerical example, to illustrate the model building procedures for the type of model considered.

Appendix to the book includes nine data sets of uni- and multivariate time series. The set of exercise problems makes the book more valuable for textbook use. The book contains ample and informative references to the literature.

Readers who have some background in univariate time series methods will find it useful in getting familiar with topics of multivariate time series. Some knowledge of matrix algebra techniques and results will be needed to fully grasp all the developments of the topics in the book.

The book aims to provide the basic concepts for an adequate understanding of the material, so that elaborate and detailed mathematical developments and arguments are adjacent concerns,although substantial references are provided for further mathematical reading. The book is accessible to a wider audience who has a working background in univariate time series analysis and some knowledge of matrix algebra methods, and who wishes to use the multivariate time series modelling techniques in applications.

The book could serve as a graduate-level textbook on multivariate time series for a second course in time series and be considered a reference book for the researchers and practitioners in the area of engineering, physical, biological, economic, natural and social sciences.

From a pedagogical point of view, there is much merit in the book. It contains many remarks, comments and examples, developed in sufficient detail, to actually help reader understand.

The material is well-selected and organized. Its style, the illustrations and discussions are clear and logically correct.

In my opinion the book is an exceptionally well-written research level monograph. In spite of the rapidly developing field of multivariate time series and of the limitations imposed by space and time, I found the book to be very interesting and comprehensive, and a valuable acquisition to the existing good literature on time series analysis. I recommend it without any reservation to an extensive readership. Buy it. Read it !

The reviewer is most grateful to Springer-Verlag for having been offered a complimentary copy of the book and so given the opportunity of focussing the best in one of his own domains of interest.

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