Aed Theory and Hierarchical Knowledge Networks

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Abstract: Aed theory and hierarchical networks of aed processors are considered in the paper. Unlike the means of mathematics and cybernetics based on set theory the new knowledge means are coherent with design&control tasks requirements. They allow to realize the main law of hierarchical level space: all units arise in lower levels multiplying and uniting, create more high levels units and are changed by higher levels. It is the law of new mathematics with units and actions which is changed when higher units and actions arise in lower actions carrying out. Aed processors networks developed in conformity with this law (as well as the units and processes of all known levels: physical, chemical, biological, demographical, engineering and knowledge) are able to connect various levels and to design their new states with taking into account their own changing constructions and activity and their interactions.

Keywords: hierarchical knowledge networks, design, control

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1. The Reasons for Aed Theory Set Up

Aed theory set up reasons are the design&control process requirements. The knowledge means in this process must carry out the main design&control task for units at any level under any knowledge uncertainty:

- to create and change the unit construction & technology by means of lower levels units selection and their interacting , to make the state and activity at higher levels as close as possible to their aims (uniting & selecting stratum);
- to change the ways (strategies) of design&control when the designed unit is multiplied and knowledge uncertainty is removed (multiplying & learning stratum);
- to change all strata as new (more high levels) knowledge is created (uniting & multiplying strata of the knowledge base).

According to these requirements, the knowledge base of the design process must be the unit with a hierarchical construction which connects any level unit, its lower and higher levels. Uncertain higher levels must be described in it by setting out a strategy, and higher levels arising must change all lower levels.

Mathematics and cybernetics theories based on set theory are incoherent with design requirements since set theory describes an one-level world outlook. In line with it the connections in systems constructions are stronger than systems connections with their holding constructions, the laws of levels weaken with level increasing, and only one (initial) level is always leading in the level space.

The general image of all one-level theories of mathematics and cybernetics in Figure 1 allows to see the boundaries of a widespread understanding of an abstract system and the boundaries of knowledge networks based on this understanding. The systems are defined in Figure 1 by rectangles, their connections -by a certain number of arrows; the measures of the system internal connections outnumber the measures of connections in their holding systems as well as in a widespread understanding. As a result the details look like indivisible atoms in holding systems and the last holding system is the set - a chaotic construction with untied acts of its atoms, which does not control the atoms activity, not to mention their construction changing.

Using such a system understanding in knowledge networks set up restricts their abilities. The existing knowledge networks are limited to carrying out design tasks when lower level units and acts must be changed by higher levels (resulted from their interactions).

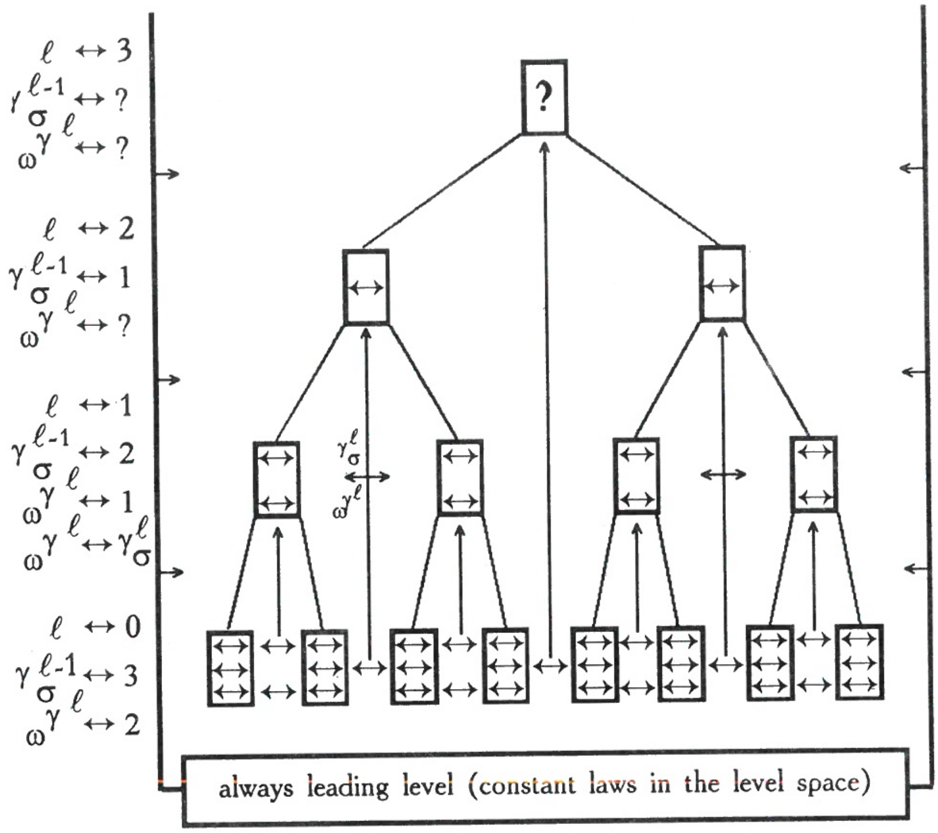

Figure 1. Exact (graphic-number) image of one-level world outlook kinds. They are based on the understanding of system as the construction which connects its own details stronger than its connections with other systems in its environment. ω^{ℓ} - system of level ℓ , $\sigma^{\ell-1}$ - its construction, ${}_{\omega}\gamma^{\ell}$ - its connections in its holding system, which has the construction σ^{ℓ} with the connections γ^{ℓ}_{σ} ; ω^{ℓ} are defined by the rectangles, γ^{ℓ} -by the arrows and arrows number is the measure of connection. This measure decreases with level increasing and higher systems cannot control the constructions and activity of lower systems. The construction of the last holding system cannot be described by system axioms, it means that the constant laws (axioms) of system are incomplete or contradictory.

Best of all for the design aim in current cybernetics is a two-level system (Mesarovic et al, 1970). However it is defined as a symbol realization of an one-level world outlook: its coordinator cannot see the internal constructions and the activity of lower level systems, their laws are more powerful than the laws of their interactions at higher levels. That is why a two-level system was extended to the whole level space and its new state got its own name - aed (Aed is a Hellenic word meaning the changing symbol unit with unlimited outlook in the level space). At first aed realization was chimerical, and described by diverse knowledge means: dynamical systems (Mesarovic and Takahara, 1975), geometry and number code. Then all the named and other knowledge means were described in an initial realization as aed connected states (Novikava et al, 1990). Now aed theory has its own language with new graphics and finer settlement of the knowledge space.

2. Aed Statute in Mathematics Symbols

Aed theory describes a new world outlook coherent with the main law of level space dynamics: all levels arise on lower levels multiplying and uniting, set up more high levels and are settled by higher levels realisations. In conformity with the level upgrading law, the guiding laws are the laws of the current highest levels and they change all levels set up before. The main law may be changed by new (arising) levels. This theory is beyond the set theory boundaries, but it belongs to mathematics as an exact science.

Now the level space contains physics, chemistry, biology, demography, engineering and knowledge levels (Novikava et al, 1991; Novikava et al, 1993). It was created by the uniting and multiplying strategy and the current highest level is knowledge. It contains language, art, learning, design&science. Its activity changes the engineering level (industry, service, trade, currency), gives new stratification to demographical units (a creative work shared by the most considerable strata), reconstructs all known levels, and maintains new levels arising.

The statute considered below unites the codes of a two-level system and the general systems theory by (Mesarovic et al, 1970; Mesarovic and Takahara, 1975), the number code L^S (Lebeg, 1938), geometry (G) and cybernetics technologies (CT) methods; dynamical systems $(\overline{\rho},\overline{\varphi})$ are the main means for the description of the named codes. Then (in a multiplying act) they are described by aed means. The details of aed statute S^ℓ are the connected laws of aed strata: level (time), unit, construction, act (process), statute (laws, connections), power (coordinator) and outlook in the level space (design field). They are united by the main law of the hierarchical space (the constructions of lower levels create the units of higher times and these units multiply by new level constructions arising).

S^ℓ is described by the following symbol construction:

$$S^\ell \leftrightarrow \{\omega, S_0, \sigma\}^\ell$$

ω^ℓ - an aggregated dynamical realization of the units and acts, σ^ℓ - construction, S_0^ℓ - coordinator, ℓ - index of level, $\ell \quad L^s$.

$$\omega^\ell \leftrightarrow \{\widetilde{\omega}, S_0\}^\ell, \sigma^\ell \leftrightarrow \{S_0, \widetilde{\sigma}\}^\ell,$$

$\widetilde{\omega}^\ell$ and $\widetilde{\sigma}^\ell$ are connected by S_0^ℓ and contain the dynamical realizations and constructions of unit (object) ${}_oS^\ell$, its environment ${}_\varepsilon S^\ell$ (other units of its level), acts (processes) ${}_{o\pi}S^\ell$ of ${}_oS^\ell$ in ${}_\varepsilon S^\ell$ and acts ${}_{\pi\varepsilon}S^\ell$ of ${}_\varepsilon S^\ell$ with ${}_oS^\ell$:

$$\{{}_oS, {}_{o\pi}S\}^\ell \leftrightarrow S^{\ell\pm0}, \{{}_{\pi\varepsilon}S, {}_\varepsilon S\}^\ell \leftrightarrow S^{\ell\pm\tau};$$

that is level ℓ is discovered in the interlevel connections:

$$\widetilde{\omega}^\ell \leftrightarrow \{\{{}_o\omega, {}_{o\pi}\omega\}, {}_\omega\gamma, \{{}_{\pi\varepsilon}\omega, {}_\varepsilon\omega\}\}^\ell \leftrightarrow$$

$$\leftrightarrow \{\{\omega^{\ell\pm0}, {}_\omega\gamma^\ell, \{\omega^{\ell\pm\tau} : \tau \in L^s, \tau \neq 0\}\} \leftrightarrow$$

$$\leftrightarrow \{\{\omega^{\ell\pm\tau} : \tau \in L^s\}, {}_\omega\gamma\}^\ell$$

$$\widetilde{\sigma}^\ell \leftrightarrow \{\{\{\omega_i : i \in I\}^{\ell\pm\tau} : \tau \in L^s\}, {}_\sigma\gamma\}^\ell \leftrightarrow$$

$$\leftrightarrow \{\{\sigma^{\ell\pm\tau} : \tau \in L^s\}, \omega^{\ell\pm0}\}^\ell.$$

$\widetilde{\omega}^\ell$ contains the dynamical systems

$${}_k\omega^\ell \leftrightarrow {}_k(\overline{\rho},\overline{\varphi})^\ell, k \in_k L \leftrightarrow \{o, o\pi, \pi\varepsilon, \varepsilon\},$$

${}_\omega\gamma^\ell$ - connections of ω^ℓ with other units and acts, and the construction of ${}_\omega\gamma^\ell$ connects the details of ${}_k\omega^\ell$ (their states ${}_kX^\ell$, inputs ${}_kX^\ell$ and outputs ${}_k\psi^\ell$):

$${}_\omega\gamma^\ell \leftrightarrow \{{}_k\{X, C, Y\} : k \in_k L\}^\ell.$$

$${}_k(\overline{\rho},\overline{\varphi})^\ell :$$

$${}_k\overline{\rho}^\ell = {}_k\{\rho_t : C_t \times X_t \to Y_t \,\&\, t \in T\}^\ell$$

$${}_k\overline{\varphi}^\ell = {}_k\{\varphi_{tt'} : C_t \times X_{tt'} \to C_{t'} \,\&\, t, t' \in T \,\&\, t' > t\}^\ell$$

The network of connections of ${}_kX^\ell$, ${}_kX^\ell$, ${}_kY^\ell$ is described in Table ${}_\omega\gamma^\ell$.

$S^{\ell\pm\tau}\,(\tau \neq 0)$ has some details, which show increasing uncertainty in the signs of $S^{\ell\pm0}; S^{\ell\pm0}$ sets ${}^{\ell\to(\ell\pm\tau)}$ and gets $X^{\ell\leftarrow(\ell\pm\tau)}$ - outputs of level ℓ to the lower ${}^{\ell\to(\ell-\tau)}$ and higher ${}^{\ell\to(\ell+\tau)}$ levels and inputs from levels $\ell-\tau$ and $\ell+\tau$ $(X^{\ell\leftarrow(\ell-\tau)}, X^{\ell\leftarrow(\ell+\tau)})$; states ${}_oX^\ell$ are the own inputs and outputs of level ℓ :

$${}_oX^\ell \leftrightarrow \{X^{\ell\to\ell}, Y^{\ell\to\ell}\}$$

Thanks to the connections in ${}_\omega\gamma^\ell$ any detail of ω^ℓ is restored by its other details with becoming uncertainty.

Table $_{\omega}\gamma^{\ell}$

$_{\omega}\gamma^{\ell}$	States	Inputs	Outputs
$_{o}S^{\ell}$	$_{o}X^{\ell}$	$_{o}X^{\ell} \leftrightarrow X^{\ell\leftarrow(\ell\pm\tau)}$	$_{o}Y^{\ell} \leftrightarrow {}_{o}C^{\ell}$
$_{o\pi}S^{\ell}$	$_{o\pi}X^{\ell} \leftrightarrow {}_{o}X^{\ell}$	$_{o\pi}X^{\ell} \leftrightarrow {}_{o}C^{\ell}$	$_{o\pi}Y^{\ell} \leftrightarrow Y^{\ell\rightarrow(\ell\pm\tau)}$
$_{\pi\varepsilon}S^{\ell}$	$_{\pi\varepsilon}C^{\ell} \leftrightarrow {}_{\varepsilon}X^{\ell}$	$_{\pi\varepsilon}X^{\ell} \leftrightarrow {}_{\varepsilon}C^{\ell}$	$_{\pi\varepsilon}Y^{\ell} \leftrightarrow \{Y^{(\ell\pm\tau)}_{(\ell\pm\tau)\rightarrow(\ell\pm\tau)}\}$
$_{\varepsilon}S^{\ell}$	$_{\varepsilon}X^{\ell}$	$_{\varepsilon}X^{\ell} \leftrightarrow \{X^{(\ell\pm\tau)\leftarrow\ell}, X^{(\ell\pm\tau)\leftarrow(\ell\pm\tau)}\}$	$_{\varepsilon}Y^{\ell} \leftrightarrow {}_{\varepsilon}C^{\ell}$

The coordinator is described as follows:

$$S_0^{\ell} \leftrightarrow \{\omega, S_0, \sigma\}_0^{\ell},$$

that is S_0^{ℓ} has its own aggregated dynamical realization ω_0^{ℓ} and the construction σ_0^{ℓ}; the availability of S_{00}^{ℓ} (the connection with higher levels) allows to account for and to change S_0^{ℓ} by its own activity.

Let

$$\lambda \leftrightarrow \ell \pm \tau_{\lambda}, \varphi \leftrightarrow \lambda \pm \tau_{\varphi}, \chi \leftrightarrow \varphi \pm \tau_{\chi} \quad \text{etc.,}$$
$$\psi \leftrightarrow \chi \pm \tau_{\psi}, ? \leftrightarrow \psi \pm \tau_{?},$$

$^{\beta}L \leftrightarrow \{\lambda, \varphi, \chi, \psi, ?, \ldots\}$. Then $S_{00}^{\ell} \leftrightarrow {}^{\beta}S_0^{\ell} \,\&\, \beta \in L^{\beta}$; $^{\beta}S_0^{\ell}$ is the contraction of field S^{β} on S^{ℓ}: $^{\beta}S_0^{\ell} \leftrightarrow S^{\beta} / S^{\ell}$ and

$$^{\lambda}S_0^{\ell} \leftrightarrow \{{}^{\lambda}\omega, {}^{\varphi}S_0, {}^{\lambda}\sigma\}_0^{\ell}$$
$$^{\varphi}S_0^{\ell} \leftrightarrow \{{}^{\varphi}\omega, {}^{\chi}S_0, {}^{\varphi}\sigma\}_0^{\ell}$$
$$^{\chi}S_0^{\ell} \leftrightarrow \{{}^{\chi}\omega, {}^{\psi}S_0, {}^{\chi}\sigma\}_0^{\ell}$$

. . . .

The fields $^{\beta}S_0^{\ell}$ are strata of S_0^{ℓ} and β is an outlook in the level space. The knowledge uncertainty of S_0^{ℓ} is increased with the distance from ℓ. Every level $_{\beta}\tau$ of uncertainty on each stratum $^{\beta}S_0^{\ell}$ has its own coordinating strategy. The strategies of $^{\lambda}S_0^{\ell}$ (processes $_{o\pi}^{\lambda}S_0^{\ell}$) connect the changes of constructions $\sigma^{\ell-\tau}$ and $\sigma^{\ell+\tau}$ by using ω^{ℓ}. The act of key unit ω^{ℓ} creation in $\sigma^{\ell-\tau}$ is the uniting process, the act of $\sigma^{\ell+\tau}$ creation when ℓ is the highest level is the multiplying process with the initial unit ω^{ℓ}.

The changing of strategies $_{o\pi}^{\lambda}S_0^{\ell}$ as far as one can be executed by stratum $^{\varphi}S_0^{\ell}$ and be controlled by following strata. At the same time the outlook in the level space extends from $\lambda \leftrightarrow \ell \pm \tau_{\lambda}$ to $\varphi \leftrightarrow \lambda \pm \tau_{\varphi}$ and so on.

Uncertainty removal in the S_0^{ℓ} outlook is equivalent to system organization improvement (an increase in the interactions level), when $^{\lambda}S_0^{\ell}$ realizations are united and multiplied by $^{\beta}S^{\ell}(\beta > \lambda)$, thus realizing the level increasing process in a hierarchical space S^{ℓ}.

In accordance with the run of events in time increasing process, the uniting stage (aed chimeric statute creation in the known mathematics and cybernetics means) becomes the multiplying act which realizes the definition of the known means $(L^{s}, (\overline{\rho}, \overline{\varphi}), \mathrm{G}, \mathrm{CT}$ and others) by aed technology.

3. The Main Knowledge Constructions in Aed Theory

Dynamical systems are described in aed terms in (Novikava and Gancharova, 1990). The result is almost obvious. Since $(\overline{\rho}, \overline{\varphi})$ is a generalization of all existing set theory means (Mesarovic and Takahara, 1975), the mathematics and artificial intelligence means become coordinated details of the general knowledge construction. Besides, they acquire larger abilities for carrying out habitual and new tasks .

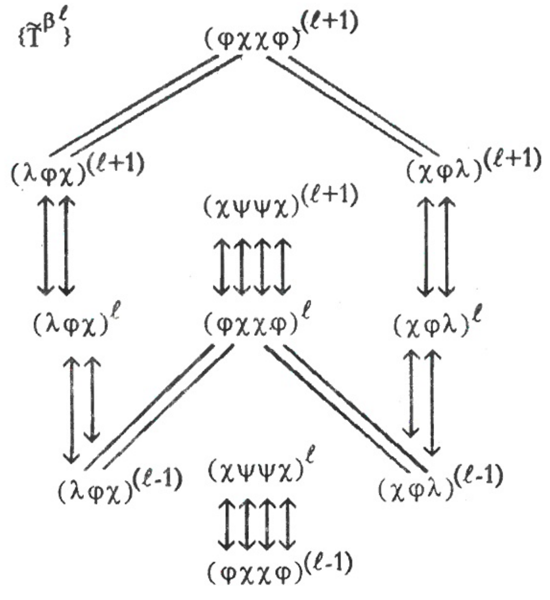

Figure 2. Interlevel connections in $L^{S\ell}$ (fragment)

The description of L^S is made as follows. The existing number code of L^S has not all signs of S^ℓ : the lower units construction and activity are not changed by higher levels. In aed theory it acquires these signs. The most considerable states of the number code (from integer to hypercomplex numbers) are described by the standard unit $L^{S\ell}$, which is founded on the next basic construction $\tilde{}^{\ell}$:

$$\tilde{T}^\ell \leftrightarrow \left\{ {}_\beta\tilde{\tau}^\ell \leftrightarrow \tilde{\beta}^\ell : \beta \in L^\beta \right\}$$

$$\tilde{\lambda}^\ell \leftrightarrow {}^-\tilde{\lambda}, {}^0\tilde{\lambda}, {}^+\tilde{\lambda}\} \leftrightarrow$$

$$\leftrightarrow \left\{ \left\{ \tilde{\lambda}, \tilde{\varphi}, \tilde{\chi}, \tilde{\psi}, \tilde{?} \right\}^\ell, \tilde{\varphi}^\ell, \left\{ \tilde{?}, \tilde{\psi}, \tilde{\chi}, \tilde{\varphi}, \tilde{\lambda} \right\}^\ell \right\}$$

$$\tilde{\varphi}^\ell \leftrightarrow {}^0\tilde{\lambda}^\ell \leftrightarrow {}^-\tilde{\varphi}, {}^0\tilde{\varphi}, {}^+\tilde{\varphi} \leftrightarrow$$

$$\leftrightarrow \left\{ \left\{ \tilde{\varphi}, \tilde{\chi}, \tilde{\psi}, \tilde{?} \right\}^\ell, \tilde{\chi}^\ell, \left\{ \tilde{?}, \tilde{\psi}, \tilde{\chi}, \tilde{\varphi} \right\}^\ell \right\}$$

$$\tilde{\chi}^\ell \leftrightarrow {}^0\tilde{\varphi}^\ell \leftrightarrow {}^-\tilde{\chi}, {}^0\tilde{\chi}, {}^+\tilde{\chi}\} \leftrightarrow$$

$$\leftrightarrow \left\{ \left\{ \tilde{\chi}, \tilde{\psi}, \tilde{?} \right\}^\ell, \tilde{\psi}^\ell, \left\{ \tilde{?}, \tilde{\psi}, \tilde{\chi} \right\}^\ell \right\}$$

the other details of $\tilde{T}^\ell$ are defined in line with this:

$$\tilde{\lambda}, \tilde{\varphi}, \tilde{\chi}, \tilde{\psi}, \tilde{?}\}^\ell \leftrightarrow \left\{ {}^-\tilde{\lambda}, {}^-\tilde{\varphi}, {}^-\tilde{\chi}, {}^-\tilde{\psi}, {}^-\tilde{?} \right\}^\ell,$$

$$\tilde{?}, \tilde{\psi}, \tilde{\chi}, \tilde{\varphi}, \tilde{\lambda}\}^\ell \leftrightarrow \left\{ {}^+\tilde{?}, {}^+\tilde{\psi}, {}^+\tilde{\chi}, {}^+\tilde{\varphi}, {}^+\tilde{\lambda} \right\}^\ell.$$

The diagram $\tilde{T}^{\beta\ell}$ (Figure 2) describes the connections of fields $\tilde{T}^{\beta\ell}$.

Figure 3 gives an idea of the construction of unit $L^{S\ell}$ and of its prolongation ${}_\omega\gamma^{\ell\leftrightarrow\ell} \leftrightarrow \tilde{L}^{s\ell}$, which connects, without breaks, $L^{S\ell}$ with the other discrete units of level 1. The prolongation occurs in the unit outlook of the level space. Figure 4 describes the activity of L^S , when discrete levels are connected without breaks too. The outlooks in the level space become the units of higher levels in this activity.

According to { $\tilde{T}^{\beta\ell}$ }, the connections of number characteristics in L^S are described as below:

$$\lambda^l \qquad \varphi^{l+1} \qquad \chi^{l+2} \qquad \psi^{l+3}$$

$$1^{\updownarrow l}_{\downarrow} \leftrightarrow 0.1^{\updownarrow l+1}_{\downarrow} \leftrightarrow 0.01^{\updownarrow l+2} \leftrightarrow 0.001^{\updownarrow l+3}$$

$$\sigma^l\left(\overline{L}^{s(l-1)}\right) \leftrightarrow 10^{l-1} \quad \omega^l\left(L^{sl}\right)$$

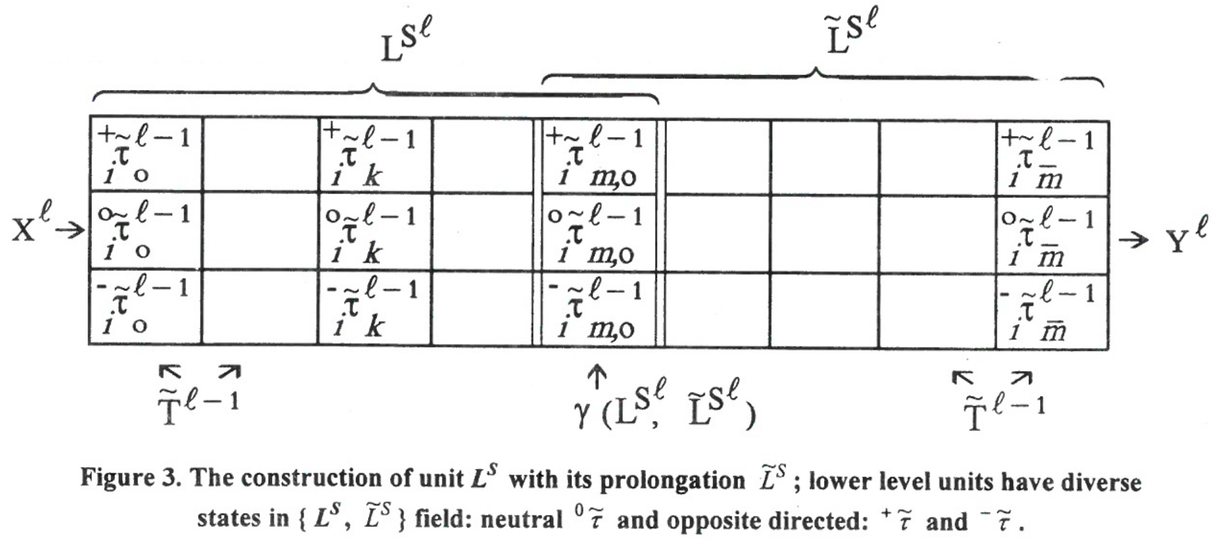

Figure 3. The construction of unit L^S with its prolongation $\tilde{L}^S$; lower level units have diverse states in { L^S, $\tilde{L}^S$ } field: neutral ${}^0\tilde{\tau}$ and opposite directed: ${}^+\tilde{\tau}$ and ${}^-\tilde{\tau}$.

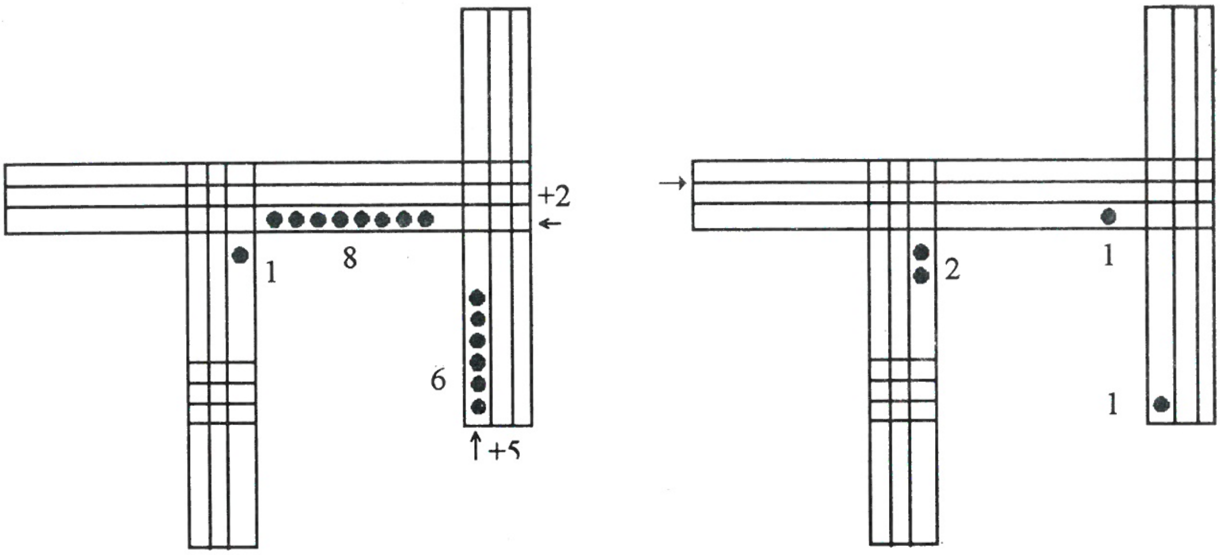

Figure 4. The uniting action in L^S : 186+25=211

Table $R^{(*)}$

C^l \ X^l	$^-\lambda$	φ	$^+\lambda$
$^-\lambda$	$^+\lambda$	φ	$^-\lambda$
φ	φ	φ	φ
$^+\lambda$	$^-\lambda$	φ	$^+\lambda$

where 1^ℓ - the unit of level $\ell; m^{\ell-1}, m^\ell, m^{\ell+1}, m^{\ell+2}, m^{\ell+3}$ are (not necessarily equal) bases (the base is a radix in the given case).

The unit L^S works in the following way. At first $\left\{L^{s\ell}, \widetilde{L}^{s\ell}\right\}_i$ contains the neutral lower units and gets on the input X^ℓ the lower units of diverse directions (signs); uniting opposite oriented units gives the neutral unit again. If an amount of identical directed units comes to m^ℓ then $L^{S\ell}$ sends to the level $(\ell+1)$ the unit of level $(\ell+1)$ and changes its own state from $\widetilde{\varphi}^\ell$ to $\widetilde{\lambda}^\ell$. In this state every unit $L_i^{s\ell}$ yields a prolongation $\widetilde{L}_i^{s\ell}$ to the unit $L_{i+1}^{s\ell}$ and then inputs of level ℓ are taken over by $L_{i+1}^{s\ell}$. In this way the uniting action leads to the unit multiplying, when a new unit of its level arises in the level space.

State changing in the space $^\beta L$ is the task of the coordinator S_0^ℓ / L^s of unit L^S; this task is carried out by uniting and multiplying actions. The next Tables on the contractions of $\widetilde{T}^\ell$ connect L^S with algebra systems.

Table $C^{(*)}$

C^l \ X^l	$^-\lambda$	$^-\varphi$	χ	$^+\varphi$	$^+\lambda$
$^-\lambda$	$^+\lambda$	$^+\varphi$	χ	$^-\varphi$	$^-\lambda$
$^-\varphi$	$^+\varphi$	$^-\lambda$	χ	$^+\lambda$	$^-\varphi$
χ	χ	χ	χ	χ	χ
$^+\varphi$	$^-\varphi$	$^+\lambda$	χ	$^-\lambda$	$^+\varphi$
$^+\lambda$	$^-\lambda$	$^-\varphi$	χ	$^+\varphi$	$^+\lambda$

Table $Z^{(*)}$

C^l \ X^l	?	$^+\psi$	$^+\chi$	$^+\varphi$	$^+\lambda$
?	?	?	?	?	?
$^+\psi$	?	$^-\lambda$	$^-\varphi$	$^+\chi$	$^+\psi$
$^+\chi$	?	$^+\varphi$	$^-\lambda$	$^-\psi$	$^+\chi$
$^+\varphi$	?	$^-\chi$	$^+\psi$	$^-\lambda$	$^+\varphi$
$^+\lambda$	?	$^+\psi$	$^+\chi$	$^+\varphi$	$^+\lambda$

The multiplication of real numbers is described in Tables on the state changing function $R^{(*)}$ of S_0^ℓ on the ${}_{\mathrm{R}}\widetilde{T}^{\ell}$:

$$ {}_{\mathrm{R}}\widetilde{T}^{\ell} \leftrightarrow \left\{ {}^{-}\widetilde{\lambda}, \widetilde{\varphi}, {}^{+}\widetilde{\lambda} \right\}^{\ell} \& \widetilde{\varphi}^{\ell} \leftrightarrow 0^{\ell} \& \pm\widetilde{\lambda}^{\ell} \leftrightarrow 1^{\ell} $$

The multiplication of complex numbers is shown in Table $C^{(*)}$ on

$$ {}_{\mathrm{C}}\widetilde{T}^{\ell} \leftrightarrow \left\{ {}^{-}\widetilde{\lambda}, {}^{-}\widetilde{\varphi}, \widetilde{\chi}, {}^{+}\widetilde{\varphi}, {}^{+}\widetilde{\lambda} \right\}^{\ell} \& \pm\mathrm{Re}\left(\omega^{\ell}\right) = \pm\widetilde{\lambda}^{\ell} $$
&
$$ \pm\mathrm{Im}\left(\omega^{\ell}\right) = \pm\widetilde{\varphi}^{\ell} \& \widetilde{\chi}^{\ell} \leftrightarrow 0^{\ell} \& \omega^{\ell} \in \mathrm{C}^{\ell}. $$

Table $Z^{(*)}$ of quaternion multiplication and the diagram of this Table (Figure 5) are defined on ${}_{\mathrm{Z}}\widetilde{T}^{\ell} \leftrightarrow \left\{ {}^{-}\widetilde{\lambda}, {}^{-}\widetilde{\varphi}, {}^{-}\widetilde{\chi}, {}^{-}\widetilde{\psi}, \widetilde{?}, {}^{+}\widetilde{\psi}, {}^{+}\widetilde{\varphi}, {}^{+}\widetilde{\lambda} \right\}^{\ell}$.

$\widetilde{?}^{\ell} \leftrightarrow 0^{\ell}, \pm\widetilde{\lambda}^{\ell}$ -real coordinate, ${}^{\pm}\widetilde{\varphi}^{\ell}, {}^{\pm}\widetilde{\chi}^{\ell}, {}^{\pm}\widetilde{\psi}^{\ell}$ - imaginary coordinates.

All Tables are derived from $\{\widetilde{T}^{\beta\ell}\}$ in the following way:

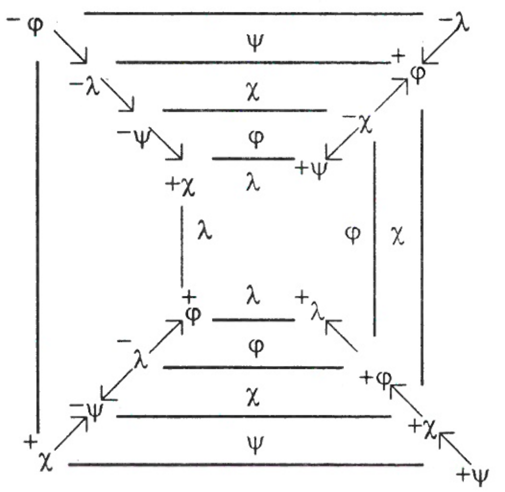

Figure 5. The Diagram of Quaternion Multiplication

- the index $i_c \in {}^{\beta}L$ of diverse states is the sign of ${}_{ic}\tau^{\ell}$ in the initial state of space $\widetilde{T}^{\ell}$: in this state ${}_{\lambda}\tau^{\ell}$ is a center of coordinates;

- the index $i_x \in {}^{\beta}L$ of input units (coordinating signals) is the sign of level i_x; i_x becomes the new center of coordinates;

- the changing of index in the new space of coordinates cannot repeat the results of other lines in the Tables.

This way algebra systems and automata are defined by means of L^S, but the adverse act is impracticable because of L^S as a hierarchical multilevel unit with interlevel connections. The identification of the number code with polynomial is a very serious and common error of set theory based mathematics. The point is that the addition of numbers in L^S leads to the event when the power of result exceeds the powers of addenda (a new level unit arises), but in the polynomial group (and in algebra in general) this event is forbidden. The named error is one of the main reasons for the existing arithmetics systems incoherence with each other and with the laws of real level space. Similar errors occur on account of the absence of a number code theory. The practical significance of this theory set up was justified by Lebeg (Lebeg, 1938).

The images of integer, real, complex, hypercomplex numbers have been described above by the strata of the outlook in the level space $(\lambda, \varphi, \chi, \psi)$ of the numbers power (the coordinator of the number space). This is congenial with the history of numbers creation. Real numbers were created in the uncertain field of the results of the acts with integer numbers. Complex numbers appeared when the results of the acts with real numbers were found beyond the boundaries of real numbers. The appearance of every new stratum of the number space has changed all aforecreated strata: the numbers images acquired new signs in their constructions.

Geometry unit $^{\ell}$ in aed theory is also a hierarchical unit S^{ℓ} and it has its own construction $\sigma^{\ell-\tau}$, an aggregated image ω^{ℓ} and the environment $\sigma^{\ell+\tau}$ (other units beyond the boundaries ${}_{\omega}\gamma^{\ell}$ of $^{\ell}$); for measuring of $^{\ell}$ the metrical characteristic μ^{ℓ} has been used.

The connections ${}_{\omega}\gamma^{\ell}$ of ω^{ℓ} with other units are its coordinates in $\sigma^{\ell+\tau}$. The constructions have two basic characteristics: $\xi^{\ell\pm\tau}$ (connection break) and $\delta^{\ell\pm\tau}$ (constructive dimension); μ^{ℓ}, $\delta^{\ell\pm\tau}$ and $\delta^{\ell\pm\tau}$ are connected and described in the number code of L^S arithmetics.

The metrical characteristic $\mu^{\ell} \in M^{\ell}$ is constructed from the units ${}_{\tau}\eta^{`} \in H^{`} \leftrightarrow \{{}_{\psi}\eta^{`}, {}_{\chi}\eta^{`}, {}_{\varphi}\eta^{`}, {}_{\lambda}\eta^{`}\}$ with coefficients from L^S :

$${}_{\tau}\widetilde{\mu}^{\ell} \leftrightarrow_{\tau} \left({}^{-}\widetilde{\mu}^{\ell}, {}^{0}\widetilde{\mu}^{\ell}, {}^{+}\widetilde{\mu}^{\ell}\right),$$

$${}^{0}_{\tau}\widetilde{\mu}^{\ell} \leftrightarrow_{\tau} \left({}^{-0}\widetilde{\mu}^{\ell}, {}^{00}\widetilde{\mu}^{\ell}, {}^{+0}\widetilde{\mu}^{\ell}\right) \leftrightarrow {}_{\tau}\widetilde{\mu}^{\ell-1},$$

where $\tau \in_{\psi} L$, ${}^{-}\widetilde{\mu}^{\ell}$ - the negative, ${}^{0}\widetilde{\mu}^{\ell}$ - the neutral, ${}^{+}\widetilde{\mu}^{\ell}$ - the positive component of $\widetilde{\mu}^{\ell}$; ${}^{0}\widetilde{\mu}^{\ell}$ is a unit of level $\ell - 1$ and so on.

The numerical characteristic of S^{ℓ} - the connection break $\widetilde{\xi}$ and the constructive dimension $\widetilde{\delta}$ are constructed this way.

Let the object Ξ^{ℓ} and the reflection ${}_{\xi}\rho : {}_{\sigma}\gamma^{\ell} \to \Xi^{\ell}$, be such that:

$$\Xi^{\ell} \leftrightarrow \{\xi^{\ell}_{\sigma} \leftrightarrow (\xi, n)^{\ell}_{\sigma}, (\xi, n)^{\ell}_{\sigma} \in L \times N \,\&\, \xi \in L \,\&$$

$$\&\, n \in N \,\&\, N \leftrightarrow N^{+} \cup \{0\}\}$$

$$\left[{}_{\xi}\rho({}_{\sigma}\gamma^{\ell}) \leftrightarrow (\xi, \nu)^{\ell}_{\sigma}, \xi \leftrightarrow \ell - \hat{\ell}\right] \Leftrightarrow$$

$$\Leftrightarrow [\left(\exists \overline{\omega}^{\ell} \subset \sigma^{\ell}\right)({}_{\omega}\gamma(\overline{\omega}^{\ell}) \leftrightarrow {}_{\omega}\gamma^{\hat{\ell}} \,\&\, {}_{\omega}\gamma^{\hat{\ell}} \subset {}_{\sigma}\gamma^{\ell} \,\&\, {}_{\omega}\gamma^{\hat{\ell}} \leftrightarrow \{{}_{\omega}\gamma_{i}$$

$${}_{\omega}\gamma_{i}^{\hat{\ell}} \leftrightarrow {}_{\omega}\gamma(\omega_{i}^{\ell}) \,\&\, {}_{\ell}\rho({}_{\omega}\gamma_{i}^{\hat{\ell}}) \leftrightarrow \hat{\ell} \,\&\, {}_{n}\rho({}_{\omega}\gamma^{\hat{\ell}}) \leftrightarrow n\})],$$

${}_{\ell}\rho({}_{\omega}\gamma_{i}^{\hat{\ell}}) \leftrightarrow L, {}_{n}\rho({}_{\omega}\gamma^{\hat{\ell}}) \leftrightarrow N$ and n - the cardinality of ${}_{\omega}\gamma^{\ell}, {}_{\omega}\gamma_{i}^{\ell}$ - the interactions of S_i^{ℓ} in σ^{ℓ}; N^{+} - the natural numbers space, $L \in L^{s}$.

Then ξ^{ℓ}_{σ} is the connection break of σ^{ℓ} with ξ (degree) and n (cardinality).

For a 3-D space:

$$(\forall \ell \in L) \Rightarrow (\xi_{\sigma} \in I_{\xi}, I_{\xi} \leftrightarrow \{0,1,2,3\}).$$

The locations of the connection breaks in σ^{ℓ} are defined by $\xi^{\ell}_{\sigma,\gamma}$:

$$\xi^{\ell}_{\sigma,\gamma} \leftrightarrow (i,\tau)_{\xi} \ldots (i,\tau)_{0}:$$

$$(\forall \xi^{\ell} \geq 0) \Rightarrow [(i,\tau)_{\xi} \leftrightarrow \{(i,\tau) : (i,\tau) \in I^{\ell} \times I^{\ell} \,\&\, \tau \neq i \,\&$$

$${}_{\omega}\rho(\gamma^{\hat{\ell}}_{i,\tau}) \leftrightarrow \ell - \hat{\ell} \leftrightarrow \xi^{\ell}\}].$$

For each unit S^{ℓ} of level $\ell \in L$, the connection break ξ^{ℓ}_{ω} in ${}_{\omega}S^{\ell}$ is the contraction of the connection break $\xi^{\ell+1}_{\sigma,\gamma}$ of $S^{\ell+1}$ unit with ${}_{\omega}\gamma^{\ell}$. In number code the connection break is defined as:

$$\widetilde{\xi}^{\ell} \leftrightarrow (n_3 \ldots n_0)_{\xi}, \widetilde{\xi}^{\ell} \in \{\xi_{\sigma}{}^{\ell}, \xi_{\omega}{}^{\ell}\}.$$

The constructive dimension definition and its calculation method derive from the definition of the connection break $\widetilde{\xi}^{\ell}$.

The constructive dimension $\delta^{\ell} \in \Delta^{\ell}$ of the unit S^{ℓ} is a number characteristic of S^{ℓ} as described in L^S code:

$\widetilde{\delta}^{\ell} \leftrightarrow (n_3 \ldots n_0)_{\delta}, \widetilde{\delta}^{\ell} \in \{\delta_{\sigma}{}^{\ell}, \delta_{\omega}{}^{\ell}\} (n_i)_{\sigma} \leftrightarrow (n_{3-i})_{\xi}$, where $(n_i)_{\sigma} \in N, i \leftrightarrow 0,1,2,3$; $\delta_{\sigma}{}^{\ell}$ and $\delta_{\omega}{}^{\ell}$ - the constructive dimensions of ω^{ℓ} and σ^{ℓ}.

Notice that Euclid, Lebeg-Brauer, Uryson, fractal and parametric dimensions come out as details of the constructive dimension δ^{ℓ}, and the known graph classes could be defined by means of ξ^{ℓ}.

L^S may be regarded as a new coordinate space (hierarchical coordinates) which is subject not only to habitual transformations of coordinates, but also to dimension changes (in line with real, complex and hypercomplex numbers interlevel connections).

The representation of $\widetilde{\xi}^{\ell}$ and $\widetilde{\delta}^{\ell}$ in L^S code gives the possibility of carrying out all L^S calculations with $\widetilde{\xi}^{\ell}$ and $\widetilde{\delta}^{\ell}$, and one may change the units dimensions and connections by changing their scales in ${}_{\psi}\widetilde{T}^{\ell}$. The $\mu^{\ell}_{\sigma,\gamma}$, $\xi^{\ell}_{\sigma,\gamma}$ and $\delta^{\ell}_{\sigma,\gamma}$ characteristics of the construction σ^{ℓ} and the aggregated dynamical realization ω^{ℓ} are calculated one after another according to ${}_{\psi}\widetilde{T}^{\beta\ell}$ laws.

All the geometric characteristics are changeable and coherent with the principal law of the hierarchical space. The changes of construction connections in $\sigma^{\ell-\tau}$ (changes in $\xi^{\ell-\tau}$ and $\delta^{\ell-\tau}$) cause an alteration of the coordinates ω^{ℓ} (the movements of $^{\ell}$ in σ^{ℓ}) and thus, a change in the construction $\sigma^{\ell+1}$.

A more detailed description of L^S and $^{\ell}$ units, the dynamical system $(\overline{\rho},\overline{\varphi})$ and sound language constructions are given in other papers of the authors.

4. Aed Processor Realizations

Cybenetics technologies in single processors and their networks keep the restrictions of the mathematics means to the extent to which they realize them. One-level theories still take the lead in the space of cybernetics technologies. Actual coherence with hierarchical mathematics will only be attained by next generations of cybernetic means.

Nevertheless, when learning about the existing processors and networks, we discover their great abilities which are not perceivable in one-level descriptions. These abilities are enforced not by abstract (one-level) constructions, but by real processors and networks constructions, to be revealed by aed symbols and to bear signs of hierarchical constructions.

Due to that the sample realizations of aed processors were made on a standard element base and on widespread computer architectures.

The general processor for integer, real, complex and hypercomplex numbers was developed as a device. The processor calculations concurrently run on diverse levels and level bases (radices) which are not necessarily identical . (Bases inequality is a constant occurrence in nature strata and measuring units: time, length and other sizes have unequal bases at various levels; it depends on the signs proper to diverse strata of the real hierarchical space).

The below described software version (IBM PC/AT, PASCAL language) opens up a new way to carrying out tasks which are either very difficult or perishable in widespread technologies. Among them there are the design&control (with new changing graphical images) of several real units and processes in diverse connected strata.

4.1 Changeable Graphic Symbols of Hierarchical Units and Actions

The symbols of real units and processes in an aed processor have their changeable graphic images, which are coherent to their constructions and activity. These images belong to the aed graphic language. Color movies with regulated rate belong to an external aed graphic image (the user connection) and keep direct contacts with the internal images of real units and actions in an aed processor. They are created in the following way.

Aed geometry considers all the known dimensions as constructive ones. 0-D units have their own changing construction with power strata and outlook in the level space. Given this they are connected in higher level constructions. 0-D unit becomes 1-D, 2-D or 3-D and vice versa when the scales or points of view change, that is when its connections with certain higher units modify. 3-D units uniting (synthesis) strategy gives an idea of the general way of creating any dimension unit .

Once created, the 3-D unit is constructed as the union of any form details which developed independently; they can have incoherent scales (sizes) and locations.

The uniting technology in this case only requires a hint at common (for the resulting construction) details of the units being united; other details are to be restored from the indicated ones (thanks to their already established connections). Consequently the uniting technology creates a connected unit with coherent scales and locations of its details. This unit can be reconstructed by any of its details.

The uniting process could take place at random or on purpose - depending on the connections uncertainty in the units being created. The connecting process rate is very high and user convenient.

The units created as shown above will connect the movements (changes) in their constructions at various levels. The laws of connected changing of diverse levels constructions (the hierarchical mechanics laws) are set up in line with the real behavior of imaged units. In this way the real processes get their moving graphic symbols in processors. Changing images in aed geometry differ from the known (one-level) visualization and animation means by nature and they work without an unwise waste of resources for their creation and activity.

It is easy to understand that the uniting technology in an aed processor is like in chemical, biological and other level units creation when the memory of the whole construction is with all construction details and the whole unit can be restored by one detail (with the individual signs defined during the adaptation to a new environment).

Geometry design in an aed processor has main advantages in comparison with other well-known methods: the convenience of Constructive Solid Geometry for a user, Boundary Representations fitness to engineering analysis, neuronet and holographic memory capabilities for connecting parallel calculations with learning and other positive signs. At the same time aed geometry is free from certain drawbacks of current technologies.

4.2 Instances of Real Processes in Aed Processor

The moving graphic images of real constructions of diverse strata are defined in an aed processor by the concrete signs indication in the basic symbol image. The images of physical, chemical, biological, technical units and processes are displays of one and the same construction&technology. Generally, movements are of a uniting and multiplying strategy realization type.

The general strategy for a large number of details was set out as the synthesis of polymers with a real number of chemical units, technical and biological membranes growth. Computer movies allow us to see, for instance, how single molecules from a molecular bunch stick to the silicon layer and how this layer structure gets changed; then clusters come up; their interactions make the membrane layer be smoothed.

The unit movements modeling in heterogeneous environments, where movements were caused not only by external forces but also by construction changes, was carried out for:

- running waves of changes in bodies which lead to mass movement;
- autowaves spreading and uniting (with a new grasp of the Belousov-Gabotinsky reaction);
- polymers movement through the membrane canal;
- snakes, caterpillars and wheels mechanisms movements.

The rate of unit movement in a changeable environment and its construction changes in an aed processor may prove considerably higher or smaller than the changes occurring under real conditions; for the video control convenience, the user can speed up or slow down the graphical images movement on the screen (and in the processor) to the desired state.

Known mathematics technologies (integral and differential equations, the finite elements method and others) are not suited for the defined tasks since they are based on theories which practically do not take into account the interlevel connections and the coherent changes of constructions of diverse levels. Besides, such technologies require much more time and computer memory resources for much simpler tasks than the above considered technologies in an aed processor.

5. Hierarchical Knowledge Networks : Highway of Arising Time

In accordance with aed statute, the new power of the hierarchical space arises in the connections of knowledge units. The connections of the main knowledge units (States) are made by all their strata (natural, demographical, engineering and knowledge) and must be designed and controlled by key constructions - knowledge networks. In the course of time these networks acquire the signs of cybernetics units which settle all the directions of diverse States activity. They turn out to be a real key to this activity (and thus to all States strata constructions) changing. However the cybernetic realizations of knowledge networks are practically possible now without any theoretical support: one-level mathematics theories cannot describe their constructions and activity, the laws of their design&control in line with the laws of other strata. The new networks, as well as all strata ignored by the power, have (increasingly) bent to infringe the laws of all layers - scientific exchange, engineering units control et al. Besides, for lack of a knowledge networks theory, their development does not in many cases produce the expected results. Moreover even huge outlay for strategic networks (as the networks for processing the Earth data and space measurement by means of space stations) developed by the best forces of science and engineering state, does not lead now to achievements in the State control. The measurement data are mainly a dead weight and this outlay destroys the States (and first of all their science) rather than strengthens them.

Aed statute gives a chance to elaborating new highways of knowledge which will have direct contacts with all known and arising strata. It is the key to design&control of knowledge networks which will be capable of understanding and relating the history, the current states and arising strategies of the knowledge units.

Hierarchical Multilevel Systems Laboratory in collaboration with the UNIBEL (Belarus network connected to Internet), Belarus State University, Belarus Academy of Sciences and other scientific and applied research institutes, is drawing up the aed statute as a knowledge network. It is coherent to real State construction and activity, and unites the networks of diverse State strata; these networks contain a changing image of the higher unit, and are able to restore the whole unit. Several among them (for microelectronics, laser raster and other technical device design, for design&control in learning, art, science, currency institutes, engineering units, for oncological profilaxy under radioactive conditions and other contaminations of the environment) have to maintain the State power. The network for a new State Statute coherent to aed symbol realization (which connects all known strata in the State own construction and its interactions with other States) is supported by the arising State strata.

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