Comparative Study Of Some Sequential Detection Methods for Digital Signals

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Abstract: The paper deals with performance evaluation of some methods for sequential detection of changes in non-stationary digital signals. Methods are "exactly" valid under the assumption that the signal under study is an autoregressive process. However, they appear to be robust to this assumption and can, therefore, be applied to other signals as well. The detection algorithms herein considered are based on the quadratic forms of Gaussian random variable, which are χ^2 distributed under the null hypothesis (no change); random variables used to construct the quadratic forms include AR parameters, estimated residual variance and sample and partial residual correlations. The presented methods combine sequential and sliding block analyses.The considered methods performances are evaluated by simulation. Also, the methods' robustness as to the assumption of autoregressive data and to the model structure, is investigated.

Keywords: Hypothesis testing, detection of changes, autoregressive modelling, non-stationary time series, quadratic forms, simulation

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1 Introduction

Modelling and processing of non-stationary digital signals stands for the core issue of various application areas such as speech processing, image processing, automatic analysis of biomedical signals, geophysics, fault detection and isolation in automated processes. Such problems have been given considerable attention during the last two decades, in both research works and the area of application analyses on real processes. Surveys of and contributions to this subject, revealing its importance and describing different applications can be found in Bodenstein and Praetorius (1977), Ishii, Iwata and Suzumura (1979), Appel and Brandt (1983), Gersh and Kitagawa (1983), (1985), Basseville and Benveniste (1986), Stoica (1990), Popescu and Demetriu (1990), Popescu (1993), Isermann (1991), Basseville and Nikiforov (1993), Ruokonen (1994), etc.

The paper makes a presentation and evaluation by simulation of some techniques for change detection in non- stationary scalar signals. Some extensions to vectorial case are direct. The following problem is addressed: Let $\{Y_1\}$ and $\{Y_2\}$ be two sets of stationary data. Testing the null hypothesis is aimed at.

H_0 : $\{Y_1\}$ and $\{Y_2\}$ are from the same generating mechanism

H_1 : $\{Y_1\}$ and $\{Y_2\}$ are from different data generating mechanisms

A solution to the above stated problem asks for an assumption on the data generating mechanism: it is assumed that under H_0, data sets $\{Y_1\}$ and $\{Y_2\}$ are generated by an autoregressive $AR(p)$ process, whose parameters may jump at some unknown time, i.e.

$$y_n + \sum_{k=1}^{p} a_k^{(n)} y_{n-k} = \epsilon_n, \quad \mathrm{var}(\epsilon_n) = \sigma_n^2 \quad (1)$$

where

$$a_k^{(n)} = a_k^{(1)}, \quad 1 \le k \le p, \quad \text{for} \quad n < \tau$$

$$\sigma_n^2 = \sigma_1^2, \qquad \text{for} \quad n < \tau$$

$$a_k^{(n)} = a_k^{(2)}, \quad 1 \le k \le p, \quad \text{for} \quad n \ge \tau$$

$$\sigma_n^2 = \sigma_2^2, \qquad \text{for} \quad n \ge \tau$$

and ϵ_n is a white noise sequence.

This is not a too restrictive assumption if agreeing that practically many stationary processes can be closely approximated by AR models. Such an assumption is worth- making thanks to the computation easiness of the resulted test procedures. More general models could be handled by methods similar to those presented, but the corresponding test procedures would then be more computationally engaged.

Quite often, in practice, the true parameter values of the AR models before and after the change are still unknown. Moreover, the structure of the true underlying models may keep unknown, and then the AR models are only used as a tool for change detection in the process.

The change detection problem consists in the sequential detection of the change, and the estimation of the change time, τ, with few false alarms, short delay for detection and symmetrical detection (comparable performances when detecting a change from model (1) to model (2), or vice versa).

There are various algorithms for change detection in AR or $ARMA$ models, which operate in a sequential or nonsequential way or involve a sliding-block analysis.

In this paper performance evaluation of some methods for sequential detection of changes in non-stationary digital signals, based on quadratic forms of Gaussian random variables, which are χ^2 distributed under the null hypothesis (no change) is discussed; two models and different statistics for comparing them, are being used. The paper is organized as follows. In Section 2 the methods concerned and their properties in ideal situation are presented. The problem of practical implementation of these statistics in real conditions is dealt with in Section 3. In Section 4 a numerical evaluation of the methods performance is reported, including a comparison of the test statistics under normal conditions, and provided the hypothesis of the assumptions on autoregressive data and model structure, is violated.

2 Change Detection Algorithms

The basic idea of these algorithms is to compare two AR models determined for different data sets of non-stationary time series, to see whether they differ significantly or not.

These algorithms are based on quadratic forms of Gaussian random variables, which are χ^2 distributed, under H_0, and which are expected to get larger values under H_1. Let x denote such a quadratic form and assume that, under H_0, $x \sim \chi^2(m)$ (i.e. x has a χ^2 distribution with m degrees of freedom). The current method used to test H_0 against H_1 is to let α be a probability value close to zero and to define the threshold $\chi^2_\alpha(m)$ by

$$\text{Prob}(x > \chi^2_\alpha(m)) = \alpha \qquad (2)$$

Then

$$\begin{array}{lll} \text{Accept } H_0 \text{ if} & x \le \chi^2_\alpha(m) & \text{, unknown risk} \\ \text{Reject } H_0 \text{ if} & x > \chi^2_\alpha(m) & \text{, risk equal to } \alpha \end{array} \qquad (3)$$

This section conceptually describes three of the methods which will be investigated by simulation.

The first method has been proposed in several alternative forms by Söderstrom and Kumamaru (1985) and the AR models estimated from two data sets $\{Y_1\}$ and $\{Y_2\}$ are compared to see whether they differ significantly or not. In summary, this method consists of the following steps:

1. Fit an AR model to $\{Y_1\}$ with a given order p.
2. Determine the parameters of the AR model of the same order, which fits $\{Y_2\}$.
3. Compute the quadratic form (Stoica, 1990):

$$x = \frac{N_1 N_2}{\hat{\sigma}_1 (N_1 + N_2)} [(\hat{\theta}_1 - \hat{\theta}_2)^T \hat{R}_1 (\hat{\theta}_1 - \hat{\theta}_2) + \qquad (4)$$

$$+ \frac{(\hat{\sigma_1}^2 - \hat{\sigma_2}^2)}{2\hat{\sigma}_1}]$$

and perform the test (3) with $m = p + 1$.

In (4), $\hat{\theta}_i = [\hat{a}_1^i, \dots, \hat{a}_p^i]$ and $\hat{\sigma}_i^2$ are the estimates of the model parameters and residual variance obtained for data set i; N_i denotes the number of available data points in data set i, and

$$R_1 = \begin{bmatrix} \hat{r}_0 & \hat{r}_1 & \cdots & \hat{r}_{p-1} \\ \hat{r}_1 & \hat{r}_0 & \cdots & \hat{r}_{p-2} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \hat{r}_{p-1} & \hat{r}_{p-2} & \cdots & \hat{r}_0 \end{bmatrix} \qquad (5)$$

In (5), $\hat{r}_k$ is a consistent estimate of the covariance $\hat{r}_k = E[y(t)y(t+k)]$, for the first data set $\{Y_1\}$.

The second method, originally proposed by Quenouille (1958), entails fitting an AR model to the data sets $\{Y_1\}$ and $\{Y_2\}$ put together, and comparing the partial residual correlations obtained from the fitted AR scheme in each separate series $\{Y_1\}$ and $\{Y_2\}$. The method is extended by comparing the residual serial correlation made by Stoica (1990). In summary, the method consists of the following steps:

1. Fit an AR model to the concatenated data sets $\{Y_1, Y_2\}$ with a given order p.

2. Determine the residuals of the AR scheme provided by 1 in each separate series $\{Y_1\}$ and $\{Y_2\}$. Compute the sample serial and partial correlation vectors $\{\hat{\delta}^{(1)}, \hat{\delta}^{(2)}\}$ and $\{\hat{\mu}^{(1)}, \hat{\mu}^{(2)}\}$:

$$\hat{\delta}^{(i)} = [\hat{\delta_1}^i, \hat{\delta_2}^i, \ldots, \hat{\delta_s}^i], \quad i = 1,2 \tag{6}$$

$$\hat{\mu}^{(i)} = [\hat{\mu_1}^i, \hat{\mu_2}^i, \ldots, \hat{\mu_s}^i], \quad i = 1,2 \tag{7}$$

where s is some positive integer.

3. Compute x_δ and x_μ :

$$x_\delta = \frac{N_1 N_2}{N_1 + N_2} ||\hat{\delta}^{(1)} - \hat{\delta}^{(2)}||^2 \tag{8}$$

$$x_\mu = \frac{N_1 N_2}{N_1 + N_2} ||\hat{\mu}^{(1)} - \hat{\mu}^{(2)}||^2 \tag{9}$$

where $||\cdot||$ denotes the vector Euclidean norm. Accept H_0 if $x_\delta < \chi^2_\alpha(s)$ and $x_\mu < \chi^2_\alpha(s)$; otherwise accept H_1. Concerning the choice of s, we simply set $s = 2p$ (Stoica, 1990).

This method based on a model determined by the whole data set $\{Y_1, Y_2\}$ is equally advantageous and disadvantageous. With a larger number of data points being used in the estimation stage, one can better approximate the asymptotic theoretical results which the testing procedure is based on. On the other hand, if the data sets $\{Y_1\}$ and $\{Y_2\}$ correspond to different AR processes, the estimation procedure executed on the concatenated set results in an "average AR model". The sample correlations of the residuals obtained from the application of that model on each data set might differ less than if the model had been determined from one data set only. To fight this drawback a new method was proposed (Stoica, 1990). It consists of the following steps:

1. Fit an AR model of order p to the data set $\{Y_1\}$.

2. Identical to step 2 of the previous method.

3. Identical to step 3 of the previous method.

A similar but not identical approach has been made by Bodenstein and Praetorius (1977).

3 Implementation Aspects

In practice, the AR models of the signal before and after changes have to be identified. When using two AR models, identifiable at different places in the signal, the problem of where to locate them comes up. Three possible approaches can be distinguished, concerning the positions of data blocks to be used for change detection purposes.

A first approach, A1, is due to Bodenstein and Praetorius (1977), and was used for EEG signal segmentation. The scheme is presented in Figure 1: A first model AR, M_1, is identified in a fixed data window and is used as a reference model, and the second model AR, M_2, is identified in a sliding data window having the same dimension with the fixed data window. When the models sufficiently differ, the signal is segmented, the second model becomes the reference model and the procedure goes on. A disadvantage of this approach is the risk of accepting H_1 under H_0, which is called "false alarm", because some of the information concerning the data previous to the change instant is lost, the M_1 model being identified in a limited data window.

Such a situation can be resolved using M_1 as a reference model, a global one (long-term model) instead of a local one (short-term model), see Figure 2. If the model M_1 is estimated with a filter having a reduced forgetting capability, it will not be affected by change and will be more precisely identified than in the former case. Concerning the position of the data windows for the long- and short-term model there can be distinguished 2 situations, corresponding to the second and the third approaches, respectively (see Figure 1 and Figure 2).

The latter approaches have been concurrently used by Appel and Brandt (1983) and by Basseville and Benveniste (1982), (1983). They also used different test statistics in change detection problem: Chernoff distance and Kullback's divergence.

The filters used for identifying M_1 and M_2 models in the investigated change detection methods resort to lattice implementation of the approximate least square method (Makhoul, 1977) for the long-term filter (M_1 in A2, A3) and

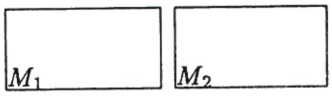

Figure 1. First Approach - A1

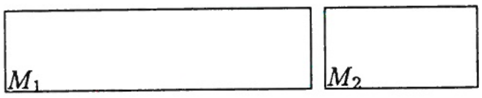

Figure 2. Second Approach - A2

Figure 3. Third Approach - A3

to the co-variance method (Markel and Gray, 1976) for the local filter (M_1 in A1 and M_2 in A1, A2, A3).

4 Performance Evaluation

This section is devoted to some experimental results, obtained via simulation, for the test statistics described in Section 2. Also, the robustness of the methods, as to the assumption of autoregressive data and to the model structure, is discussed.

Table 1. The Cases Considered in Simulation

Case	Generation of $\{Y_1\}$ and $\{Y_2\}$
C1	$y_n = 0.6y_{n-1} + \epsilon_n; \sigma^2 = 1.$ $y_n = 0.1y_{n-1} + \epsilon_n; \sigma^2 = 1.$
C2	$y_n = 0.3y_{n-1} + 0.5y_{n-2} + \epsilon_n; \sigma^2 = 4.$ $y_n = 0.3y_{n-1} + 0.5y_{n-2} + \epsilon_n; \sigma^2 = 0.25$
C3	$y_n = 0.3y_{n-1} + 0.5y_{n-2} + \epsilon_n; \sigma^2 = 0.09$ $y_n = 0.5y_{n-1} - 0.3y_{n-2} + 0.6y_{n-3} -$ $-0.5y_{n-4} + \epsilon_n; \sigma^2 = 0.16$
C4	$y_n = \sqrt{2}\sin(0.2\pi t) + \epsilon_n; \sigma^2 = 0.64$ $y_n = 0.7y_{n-1} + 0.5y_{n-2} - 0.56y_{n-3} +$ $+\epsilon_n; \sigma^2 = 1.$
C5	$y_n = \sqrt{2}\sin(0.2\pi t) + \epsilon_n; \sigma^2 = 0.64$ $y_n = \sqrt{2}\sin(0.23\pi t) + \epsilon_n; \sigma^2 = 0.64$
C6	$y_n = \sqrt{2}\sin(0.2\pi t) + \epsilon_n; \sigma^2 = 0.64$ $y_n = \sqrt{2}\sin(0.23\pi t) + \epsilon_n; \sigma^2 = 1.$

The methods described in Section 2 have been applied to the cases shown in Table 1. In each case were generated one realization of $\{Y_1\}$ and 100 independent realizations of $\{Y_2\}$, of 500 sample points each. Using multiple simulation runs, we can evaluate the probability of accepting H_1 under H_0 (first type of risk), which is also called "false alarm", and the probability of accepting H_0 under H_1 (second type of risk) for the testing methods under consideration. Note that these cases are grouped into two classes: for the first 3 cases in Table 1, the assumption on the autoregressive data is satisfied, while for the latter 3 cases it is not.

In all cases, at the beginning, only the filter which identifies the model M_1 is activated. After 200 sample points the second filter (sliding block) and the test will get activated. If the size of the window used for identifying model M_1 is too small, false alarms may occur due to poor estimation of AR coefficients. For this reason the window size has been chosen as of 200 samples. As the number of sample points used for the second filter is 200, it results that two successive changes occurring within less than 200 sample points could not be detected by the investigated methods. For all the methods, the critical probability value α was set to $\alpha = 0.05$.

4.1 Test Statistics Comparison

The results obtained for C1,C2 and C3 are given in Table 2. As one can see the combination MIII-A3 has no sense. The model order used was: $p = 1$ for C1, $p = 2$ for C2 and $p = 4$ for C3.

*Remark 1.*It can be noted that the first type of risk for MI is greater (for A1 and A2 approaches) than that of MII and MIII. At the same time, MI implies the smallest risk of second type in all cases considered.

Remark 2. Initially, the data window for the reference model will contain only data from $\{Y_1\}$. When the data window used for the current model includes enough data from $\{Y_2\}$, a change is detected. Afterwards, the data window for the reference model will contain data from $\{Y_1\}$ and $\{Y_2\}$ and the data window for the current model will include only data from $\{Y_2\}$. Sometimes, in this case a second change is detectable. This depends on the number of data samples from $\{Y_1\}$ for which the reference model is computed. Thus, the real change instant will appear between two successive change detection instants. Table 3 presents for C1, the number of cases with a single and double change, in the analysed realizations. Obviously, the number of double change detections reduces for A3 approach, in comparison with the A1 and A2 approaches, as well as for MI and MII. It results that for MI-A3 and MII-A3 the change detection instant will be very close to the real change instant.

Table 2. Results for C1,C2,C3 Cases

Case	Testing method	Estim. first type of risk	Estim. second type of risk
C1	MI-A1	0.10	0.00
	MI-A2	0.08	0.00
	MI-A3	0.00	0.00
	MII-A1	0.03	0.00
	MII-A2	0.04	0.00
	MII-A3	0.00	0.00
	MIII-A1	0.02	0.00
	MIII-A2	0.05	0.00
C2	MI-A1	0.06	0.00
	MI-A2	0.00	0.00
	MI-A3	0.01	0.00
	MII-A1	0.00	0.18
	MII-A2	0.01	0.04
	MII-A3	0.00	0.00
	MIII-A1	0.02	0.24
	MIII-A2	0.02	0.00
C3	MI-A1	0.22	0.00
	MI-A2	0.23	0.00
	MI-A3	0.01	0.00
	MII-A1	0.10	0.00
	MII-A2	0.15	0.00
	MII-A3	0.00	0.00
	MIII-A1	0.10	0.00
	MIII-A2	0.14	0.00

Table 3. No. of Cases with Single and Double Change for C1

Testing method	No. cases with single change	No. of cases with double change
MI-A1	0	100
MI-A2	3	97
MI-A3	83	17
MII-A1	3	97
MII-A2	7	93
MII-A3	88	12
MIII-A1	15	85
MIII-A2	10	90

Remark 3. MII and MIII are not sensitive to a scaling of data. More exactly, MIII is all through insensitive to scaling (it is based on correlations that are not affected by scaling) and MII is hardly sensitive (due to a slight modification of the AR model fitted to the concatenated set $\{Y_1, Y_2\}$, produced by a "reasonable" scaling of $\{Y_2\}$).

Remark 4. MI is very sensitive to scaling (it has been designed to detect this type of change, due to the second term in (4)).

Remark 5. Concerning the computational burden involved, MI is comparable to MIII.

4.2 Assumption of Autoregressive Data

The assumption on autoregressive data is not satisfied for C4, C5 and C6. The results obtained in these cases are given in Table 4, the same as for the cases C1,C2 and C3. The model order chosen in all cases, was $p = 3$.

Remark 6. The results obtained for C4, where $\{Y_2\}$ data are generated by an AR process, are similar to the previous results. For C5, where only a small change occurs (the angular frequency jumps from 0.2π to 0.23π) all the methods and approaches point to a high second type of risk. The insignificant change in C5 shows an increase of variance, while the second type of risk decreases, especially for MI.

Table 4. Results for C4,C5,C6 Cases

Case	Testing method	Estim. first type of risk	Estim. second type of risk
C4	MI-A1	0.07	0.00
	MI-A2	0.13	0.00
	MI-A3	0.00	0.00
	MII-A1	0.11	0.00
	MII-A2	0.12	0.00
	MII-A3	0.00	0.00
	MIII-A1	0.14	0.00
	MIII-A2	0.18	0.00
C5	MI-A1	0.00	0.87
	MI-A2	0.00	0.40
	MI-A3	0.00	0.67
	MII-A1	0.00	0.53
	MII-A2	0.00	0.49
	MII-A3	0.00	0.97
	MIII-A1	0.00	0.54
	MIII-A2	0.00	0.45
C6	MI-A1	0.00	0.10
	MI-A2	0.00	0.08
	MI-A3	0.00	0.68
	MII-A1	0.00	0.14
	MII-A2	0.00	0.22
	MII-A3	0.00	0.92
	MIII-A1	0.00	0.29
	MIII-A2	0.00	0.33

4.3 The Importance of Model Order

In the cases where the AR model order is not known, for the investigated methods, the underestimation of this order can cause poor detection. The results obtained for C3 case with a filter of order 3 and respectively 2, instead of real order 4, are given in Table 5.

Table 5. Results for C3 Case, p=3, p=2

p	Testing method	Estim. first type of risk	Estim. second type of risk
3	MI-A1	0.25	0.00
	MI-A2	0.37	0.00
	MI-A3	0.03	0.00
	MII-A1	0.34	0.00
	MII-A2	0.39	0.00
	MII-A3	0.05	0.00
	MIII-A1	0.32	0.00
	MIII-A2	0.35	0.00
2	MI-A1	0.37	0.00
	MI-A2	0.50	0.00
	MI-A3	0.07	0.00
	MII-A1	0.25	0.18
	MII-A2	0.37	0.04
	MII-A3	0.02	0.00
	MIII-A1	0.26	0.24
	MIII-A2	0.34	0.00

Table 6. Results for C5 Case, p=5, p=10

p	Testing method	Estim. first type of risk	Estim. second type of risk
5	MI-A1	0.00	0.02
	MI-A2	0.01	0.00
	MI-A3	0.00	0.06
	MII-A1	0.03	0.62
	MII-A2	0.07	0.12
	MII-A3	0.01	0.76
	MIII-A1	0.06	0.09
	MIII-A2	0.21	0.00
10	MI-A1	0.01	0.00
	MI-A2	0.02	0.00
	MI-A3	0.00	0.00
	MII-A1	0.02	0.25
	MII-A2	0.01	0.01
	MII-A3	0.00	0.79
	MIII-A1	0.02	0.00
	MIII-A2	0.02	0.00

*Remark 7.*One can notice that the behaviour of the detector, especially for the second type of risk, is not affected by an underestimation of the model order. It seems that the practice of identifying AR filters in lattice form may prevent this fact (see A2, A3 for all methods). The first type of risk will however be affected by an underestimation of the order.

*Remark 8.*A considerable improvement in change detection of the second type risk can be noted for C5 (non autoregressive data), when the model order increases from $p = 3$ to $p = 5$ and respectively $p = 10$. The results are given in Table 6. This improvement brings about a slight increase of the first type of risk.

5 Conclusions

The performance evaluation problem of some methods for sequential detection of changes in non-stationary signals has been addressed. The detection algorithms, considered in the paper, are based on quadratic forms of a Gaussian random variable (estimated AR parameters, estimated residual variance and sample serial and partial residual correlations). The robustness of these algorithms is also investigated. A final conclusion is that if any of the methods is to be preferred in most practical cases, it is MI, A2 and A3.

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