

# Digraph-Theoretic Approach for Deadlock Detection and Recovery in Flexible Production Systems

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**Abstract:** In flexible manufacturing systems a deadlock arises when jobs in a set are indefinitely prevented from accessing resources because these are taken by other jobs in the same set. This condition is highly unfavourable because it stops the normal flow of parts and can propagate to the entire system. To face this problem, recent literature proposes prevention, avoidance and detection/recovery techniques. This paper introduces a detection/recovery approach based on a simple digraph that characterizes deadlock by describing the current interactions between pieces and resources. The method requires a low computation burden in the detection phase and makes use of a dedicated buffer to activate the recovery phase. Finally, a case study shows the simplicity and effectiveness of the proposed approach.

**Keywords:** Deadlock, Manufacturing Automation, Flexible Manufacturing Systems

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## 1. Introduction

Flexible Manufacturing Systems (FMSs) consist of a set of resources (workcentres, measure and inspection centres, buffers, transport devices, etc.) which the pieces (jobs, items) request service from. In these systems competition of jobs for a limited number of resources can lead to deadlock situations (circular waits). These occur when jobs from a set that holds resources are blocked indefinitely from access to the resources held by other jobs within the same set. Deadlock is

dangerous because it stops production and can propagate to the entire system.

The problem has been extensively studied in computer science for multitasking environments [2]. Only recently have some authors focused on the deadlock in FMSs. Namely there are many peculiarities in deadlock analysis of manufacturing systems, e.g. processing times of pieces are considerably longer than computation times. Moreover the sequence in which each job requests the resources is exactly known in FMSs, so that one can profitably use this information in deadlock-solving algorithms. Hence, due to these peculiarities, it is necessary to develop special techniques for dealing with deadlocks in FMSs.

As in the computer science context, the methodologies to face deadlock in FMSs can be classified in three categories: prevention, avoidance and detection/recovery. The first two classes rule the flow of jobs so as to prevent the system from reaching deadlock conditions [1, 4-7, 9, 13, 15]. In particular, the former utilizes static policies, the latter uses information on the current state of the FMS to limit the freedom in resource allocation. The detection/recovery approaches do not limit in any way the freedom in allocation of resources to jobs [10, 14, 15]. Namely, they only identify current deadlock situations (detection phase) and, if any, start special policies of resource management to break the condition of circular wait and restore normal operating conditions (recovery phase).

Existing techniques of prevention and avoidance impose constraints on resource allocation that can determine reduced utilization of the resources and poor performance of the production system. Generally speaking, even if they involve higher on-line computational burden, the constraints for deadlock avoidance techniques are less restrictive than prevention methods. On the contrary detection/recovery approaches do not constrain the resources during the normal system operation. However they involve additional costs due to both the hardware necessary to activate the recovery policy and the system performance reduction throughout the recovery phase. Consequently the choice of the more favourable method to face

deadlock depends on the peculiarities and on the lay-out of the production system under consideration.

This paper analyzes the deadlock in FMSs by using a simple model of the interactions between jobs and non-unitary capacity resources, i.e. resources with multiple items of the same type. The underlying idea is the definition of a digraph named Transition Digraph that describes the process of resource acquiring and releasing, modelled as a Discrete Event Dynamical System (DEDS). In particular, the Transition Digraph captures the main information of the DEDS state and changes with time as state evolves. The paper shows that there is an equivalence between deadlock and a figure in the transition digraph, named Maximal-weight Zero-outdegree Strong Component (MZSC). This formal characterization of the deadlock allows us to show the effectiveness of a detection procedure identifying the MZSCs by a depth-first search on the Transition Digraph. On the basis of this method and of a recovery mechanism using a reserved central buffer of unit-capacity, we develop an effective detection/recovery policy.

The paper is organized in six Sections. Section 2 describes the peculiarities of the DEDS modelling the production system and defines the Transition Digraph. Moreover it indicates the laws ruling the Transition Digraph modifications according to the state changes and triggered by events involving acquisition or release of resources. Section 3 proves the equivalence between deadlock and MZSC in the Transition Digraph, while Section 4 illustrates the detection/recovery policy. Finally, Section 5 shows the application of such a policy to a case study and Section 6 draws the conclusions.

## 2. The Model

This section describes the peculiarities of the DEDS modelling the production system and defines the Transition Digraph.

### 2.1 The Discrete Event Dynamical System

Let us first look at some background notations that characterize the model of a production system  $S$ . The symbol  $R = \{r_i, i=1, 2, \dots, R\}$  indicates the set of resources, where the first  $R-1$  elements represent multiple slot buffers, machines with identical servers, AGV systems provided with one or several trucks, etc. while  $r_R$  is an additional fictitious resource the jobs acquire as they leave the system. An integer  $C(r_i)$  indicates the capacity

of the resource  $r_i$ , i.e. the maximum number of jobs that can contemporaneously hold  $r_i$ . Thus  $C(r_i)$  is finite for each resource, but for  $r_R$  that can receive jobs without limits. Now if  $J$  is the set of jobs to produce, each element from  $J$  requires a sequence of resources, named Working Procedure. Thus the set  $W = \{w\}$  collects the Working Procedures necessary for processing all the jobs from  $J$ . Obviously the terminal resource of each  $w \in W$  is a fictitious one. With these preliminary notions as background, we refer to a DEDS model to describe the process whereby jobs acquire or release resources [3]. The state of the DEDS model  $q$  contains information on the operating conditions of  $S$ , such as the set  $J_q$  of jobs in process, the resources currently held by each job from  $J_q$ , the Working Procedures associated with such jobs, and, finally, the Residual Working Procedures, i.e. the resources necessary for each  $j \in J_q$  to complete its processing. The set of all the DEDS states is denoted by  $Q$ .

Now, with reference to the current state  $q$ , we introduce some additional notations. In particular  $HR(j)$  denotes the resource currently held by  $j \in J_q$ ,  $SR(j)$  identifies the second resource of the Residual Working Procedure of such a job and  $WP(j)$  indicates its complete Working Procedure. Note that  $SR(j)$  is defined in all the cases. Namely, any Residual Working Procedure contains  $r_R$  and, by assumption, a job leaving the system accedes to  $r_R$ , but, at the same time, it is removed from the set  $J_q$ . Finally we say that a resource  $r_i$  is *empty* (*idle* or *busy*) in the state  $q$ , if  $n_i=0$  ( $0 < n_i < C(r_i)$  or  $n_i=C(r_i)$  respectively) where  $n_i$  denotes the number of jobs holding such a resource.

Obviously, the DEDS must encompass events involving resource acquiring or releasing. We consider two event types:

- a new job enters the system (1-type event). This event is identified by a pair  $(j, w)$ , where  $j \in J$  is the job entering  $S$  and  $w \in W$  is the Working Procedure the job has to follow;
- a job progresses from a resource to another one, or it leaves the system (2-type event). This event is specified by a job  $j \in J_q$  progressing from  $HR(j)$  to  $SR(j)$ , where  $q$  is the current state of  $S$ .

### 2.2 The Transition Digraph

Deadlock detection requires the description of the current interactions between jobs and resources, in each state  $q$  of the DEDS. To this aim, we introduce a digraph, named Transition Digraph



and denoted by  $D_{Tr}(\mathbf{q})=[N, E_{Tr}(\mathbf{q})]$ . The vertex set represents the system resource set. So, for the sake of simplicity, the same symbol indicates the elements of the node set  $N$  and the system resources, i.e.  $N=R$ . Moreover, the edge set changes as  $\mathbf{q}$  is updated: an edge  $e_{im}$  is in  $E_{Tr}(\mathbf{q})$  if and only if (iff for brevity) a job  $j \in J_{\mathbf{q}}$  holds  $r_i$  in the state  $\mathbf{q}$  and requires  $r_m$  as next resource. Therefore the Transition Digraph indicates both the resources currently held by jobs from  $J_{\mathbf{q}}$  and the resources required by the same jobs in the next step of their Working Procedures.

We note that a single edge  $e_{im} \in E_{Tr}(\mathbf{q})$  may represent more jobs detaining  $r_i$  and requesting  $r_m$ . To take into account this situation, we associate the following weight with each edge  $e_{im} \in E_{Tr}(\mathbf{q})$ :

$$a_{\mathbf{q}}(e_{im}) = \text{Card}\{j \in J_{\mathbf{q}}: \text{HR}(j)=r_i \text{ and } \text{SR}(j)=r_m\} \quad (1)$$

where  $\text{Card}(\cdot)$  stands for "cardinality of ...". In this way,  $a_{\mathbf{q}}(e_{im})$  yields the number of jobs which hold  $r_i$  and request  $r_m$  as next resource.

Now, according to Harary *et al* [8], we define Outdegree Value of a node  $r_i$  as the sum of weights of edges outgoing from  $r_i$  in  $D_{Tr}(\mathbf{q})$ , i.e.

$$\text{OV}_{\mathbf{q}}(r_i) = \sum_{m=1}^R a_{\mathbf{q}}(e_{im}) \quad (2)$$

where we consider  $a_{\mathbf{q}}(e_{im})=0$  if  $e_{im} \notin E_{Tr}(\mathbf{q})$ . Hence, the Outdegree Value of a vertex indicates the number of jobs currently using the corresponding resource in the state  $\mathbf{q}$ . In particular, if  $\text{OV}_{\mathbf{q}}(r_i)=0$  then  $r_i$  is *empty*; if  $0 < \text{OV}_{\mathbf{q}}(r_i) < C(r_i)$  then  $r_i$  is *idle*; finally, if  $\text{OV}_{\mathbf{q}}(r_i)=C(r_i)$  then  $r_i$  is *busy*. Obviously, a job  $j \in J_{\mathbf{q}}$  requiring  $r_m$  as next resource, is *blocked* iff  $\text{OV}_{\mathbf{q}}(r_m)=C(r_m)$ . On the contrary, if  $\text{OV}_{\mathbf{q}}(r_m) < C(r_m)$  then job  $j$  is *unblocked*. We indicate by  $J_{\mathbf{q},u} \subset J_{\mathbf{q}}$  the set of jobs unblocked in the state  $\mathbf{q}$ .

Before continuing, let us show how to update the Transition Digraph at each event occurrence. Suppose  $S$  be in the state  $\mathbf{q}$  and consider a Working Procedure  $w \in W$ . In such a condition for a job  $j \in J$  to enter  $S$  and to receive service according to  $w$ , the first resource in  $w$  must necessarily be *idle* or *empty*. On the occurrence of this 1-type event,  $S$  makes transition from  $\mathbf{q}$  to a new state  $\mathbf{q}'$ . The new Transition Digraph  $D_{Tr}(\mathbf{q}')$  is obtained as follows: if  $r_m$  and  $r_p$  are respectively the first and the second resource in  $w$ , then

$E_{Tr}(\mathbf{q}) \cup \{e_{mp}\}$  yields the edge set  $E_{Tr}(\mathbf{q}')$ . Clearly it holds:  $\text{OV}_{\mathbf{q}'}(r_m) = \text{OV}_{\mathbf{q}}(r_m) + 1$ .

Analogously, let  $j \in J_{\mathbf{q},u}$ ,  $r_i = \text{HR}(j)$  and  $r_m = \text{SR}(j)$ . By definition of the set  $J_{\mathbf{q},u}$ ,  $r_m$  is *idle* or *empty* in the state  $\mathbf{q}$ , so that the transition leading  $j$  from  $r_i$  to  $r_m$  can occur. This 2-type event updates the state from  $\mathbf{q}$  to  $\mathbf{q}'$ . In particular, the edge set  $E_{Tr}(\mathbf{q}')$  of  $D_{Tr}(\mathbf{q}')$  and the corresponding edge weights result from the following operations on  $E_{Tr}(\mathbf{q})$ :

- i) put  $a_{\mathbf{q}'}(e_{im}) = a_{\mathbf{q}}(e_{im}) - 1$  and, if  $a_{\mathbf{q}}(e_{im}) = 1$ , remove  $e_{im}$  from  $E_{Tr}(\mathbf{q})$ ;
- ii) provided that  $\text{SR}(j) \neq r_p$ , put  $a_{\mathbf{q}'}(e_{mp}) = a_{\mathbf{q}}(e_{mp}) + 1$ , where  $r_p$  is the third resource in the Residual Working Procedure of job  $j$ , and, if  $a_{\mathbf{q}}(e_{mp}) = 0$ , put  $E_{Tr}(\mathbf{q}') = E_{Tr}(\mathbf{q}) \cup \{e_{mp}\}$ .

In this way,  $r_i$  becomes *empty* if  $\text{OV}_{\mathbf{q}}(r_i) = 1$  or *idle* if  $\text{OV}_{\mathbf{q}}(r_i) > 1$ , while  $r_m$  becomes *busy* if  $\text{OV}_{\mathbf{q}}(r_m) = C(r_m) - 1$  or *idle* if  $\text{OV}_{\mathbf{q}}(r_m) < C(r_m) - 1$ .

Of course all the remaining nodes keep unchanged their Outdegree value and the *busy/idle/empty* condition they had in  $D_{Tr}(\mathbf{q})$ .

### 3. Detecting Deadlock By the Transition Digraph

This Section proves a result that allows deadlock detection on the basis of some particular strong components of the Transition Digraph. For the definition of strong component, walk and other standard figures of digraphs, we refer to [8].

Now, let us begin with some preliminary ideas. As mentioned before,  $\mathbf{q}$  is a deadlock state if each member of a job set waits indefinitely for other jobs in the same set to release resources. The following definition expresses this condition more formally.

*Definition 1:*  $\mathbf{q} \in Q$  is a deadlock state for  $S$  if there exist a non-empty job subset  $J_D \subset J_{\mathbf{q}}$  and a non-empty resource subset  $R_D \subset R$ , satisfying the following properties:

- D1a)  $J_D$  is the maximal subset of  $J_{\mathbf{q}}$  such that  $\text{HR}(J_D) = R_D$ ;
- D1b)  $\text{SR}(J_D) \subset R_D$ ;
- D1c) any resource in  $\text{SR}(J_D)$  is *busy*.

Previous definition has a quite transparent meaning. Namely, by D1c) each job from the set  $J_D$  is *blocked* and, by D1a) and D1b), it requires a resource held by other jobs in  $J_D$ . Deadlock conditions are related to some particular strong components of the digraph  $D_{Tr}(\mathbf{q})$  characterized by the following definition.

*Definition 2:* Let  $\sigma=(N_\sigma, E_\sigma)$  be a strong component of  $D_{Tr}(\mathbf{q})$ . We call  $\sigma$  a "*Maximal-weight Zero-outdegree Strong Component*" of  $D_{Tr}(\mathbf{q})$  (MZSC for brevity) if it enjoys the following properties:

D2a) *Maximal-weight:* all the resources from  $N_\sigma$  are *busy*, i.e.  $OV_{\mathbf{q}}(N_\sigma)=C(N_\sigma)$ ;

D2b) *Zero-outdegree:* all the edges of  $D_{Tr}(\mathbf{q})$  outgoing from vertices of  $N_\sigma$  belong to  $E_\sigma$ , i.e. the elements from  $N_\sigma$  are the only vertices of  $D_{Tr}(\mathbf{q})$  reachable from vertices in  $N_\sigma$ .

In the above statement, symbols  $OV_{\mathbf{q}}(N_\sigma)$  and  $C(N_\sigma)$  indicate the sum of  $OV_{\mathbf{q}}(r_i)$  and  $C(r_i)$  over all the vertices from the set  $N_\sigma$ , and constitute the Overlap Degree and the Capacity of  $\sigma$ , respectively. The following result gives a necessary and sufficient condition for the occurrence of a deadlock state in a system with multiple capacity resources.

*Proposition 1:*  $\mathbf{q}$  is a deadlock state for  $S$  iff there exists at least one MZSC in  $D_{Tr}(\mathbf{q})$ .

*Proof*

Comparing conditions D1a), D1b) and D1c) for the deadlock occurrence (see Section 2) with the properties of an MZSC gives the key to prove this proposition.

*If part*

Let  $\sigma=(N_\sigma, E_\sigma)$  be an MZSC of  $D_{Tr}(\mathbf{q})$  and let  $J_D$  indicate the maximal subset of  $J_{\mathbf{q}}$  such that  $HR(J_D)=N_\sigma$ . The set  $J_D$  certainly exists since, by D2a), all the resources from  $N_\sigma$  are *busy*. On the other hand, by D2b) all the edges outgoing from vertices of  $N_\sigma$  terminate in nodes still belonging to  $N_\sigma$ . This implies:  $SR(J_D)\subset N_\sigma$ . Hence putting  $R_D=N_\sigma$  verifies conditions D1a), D1b) and D1c).

*Only if part*

The proof is in three steps.

**Step 1** By condition D1a) at least one edge of  $D_{Tr}(\mathbf{q})$  starts from each node of  $R_D$ . Moreover, by D1b) all the edges originating from vertices of  $R_D$  still terminate in  $R_D$ . Consequently there exists a walk [8] of infinite length starting from each

vertex in  $R_D$  and touching nodes from  $R_D$  only. Since the cardinality of  $R_D$  is finite,  $D_{Tr}(\mathbf{q})$  must contain at least one non-trivial strong component with all its vertices in  $R_D$ . Moreover each node from  $R_D$  is the starting point of a walk reaching one of these strong components. Obviously such components are disjoint and two vertices from two different components can never be mutually reachable. Hence, at least one of them, say  $\sigma=(N_\sigma, E_\sigma)$ , cannot reach any one of the remaining strong components having vertices in  $R_D$ .

**Step 2** We now prove that all the edges of  $D_{Tr}(\mathbf{q})$  outgoing from  $N_\sigma$  belong to  $E_\sigma$ . The proof is by contradiction. Suppose there exists a vertex  $r_m\notin N_\sigma$ , reachable from some node in  $N_\sigma$ . By step 1,  $r_m\in R_D$  and there exists a strong component of  $D_{Tr}(\mathbf{q})$  having vertices in  $R_D$  and which is reachable from  $r_m$ . Let  $\sigma^*$  be such a component. If  $\sigma^*=\sigma$  then  $r_m$  and vertices from  $N_\sigma$  are mutually reachable. But this conclusion leads to the contradiction:  $r_m\in N_\sigma$ . On the other hand, if  $\sigma^*\neq\sigma$ , the fact that vertices from  $\sigma^*$  are reachable from nodes of  $\sigma$  contradicts the definition of  $\sigma$  given at Step 1. To sum up, this step proves D2b) for  $\sigma$ .

**Step 3** By condition D1a), if for any job  $j\in J_{\mathbf{q}}$  it holds  $HR(j)\in N_\sigma$ , then  $j\in J_D$ . Moreover, since each vertex from  $N_\sigma$  is adjacent from another vertex of  $N_\sigma$ , we get:  $N_\sigma\subset SR(J_D)$ . Hence condition D1c) implies that each vertex from  $N_\sigma$  is *busy*. This proves D2a).

*Example 1*

To illustrate Proposition 1 and clarify the notations, we consider a system with  $R=4$ , producing a job mix  $J$  according to the following Working Procedures:

$$\mathbf{w}_1 = (r_3, r_1, r_3, r_2, r_4)$$

$$\mathbf{w}_2 = (r_3, r_2, r_3, r_4)$$

Let the capacities of the resources, but the fictitious one, be  $C(r_i)=2$  for  $i=1, \dots, 3$ . Moreover let the system be in a state  $\mathbf{q}$ , with:  $J_{\mathbf{q}}=\{j_i; i=1, 2, \dots, 6\}$ ;  $HR(j_1) = HR(j_2) = r_3$ ,  $SR(j_1) = r_2$  and  $SR(j_2) = r_1$ ;  $HR(j_3) = HR(j_4) = r_1$  and  $SR(j_3) = SR(j_4) = r_3$ ;  $HR(j_5) = HR(j_6) = r_2$  and  $SR(j_5) = SR(j_6) = r_3$ .

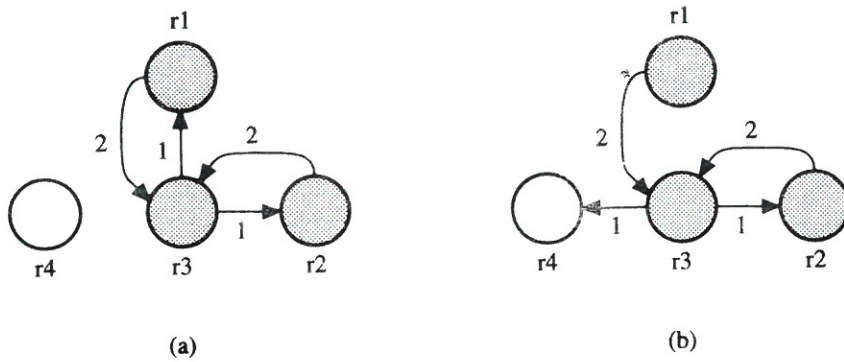


Figure 1. (a):  $D_{Tr}(q)$  exhibits a deadlock condition; (b):  $D_{Tr}(q')$  contains no MZSC

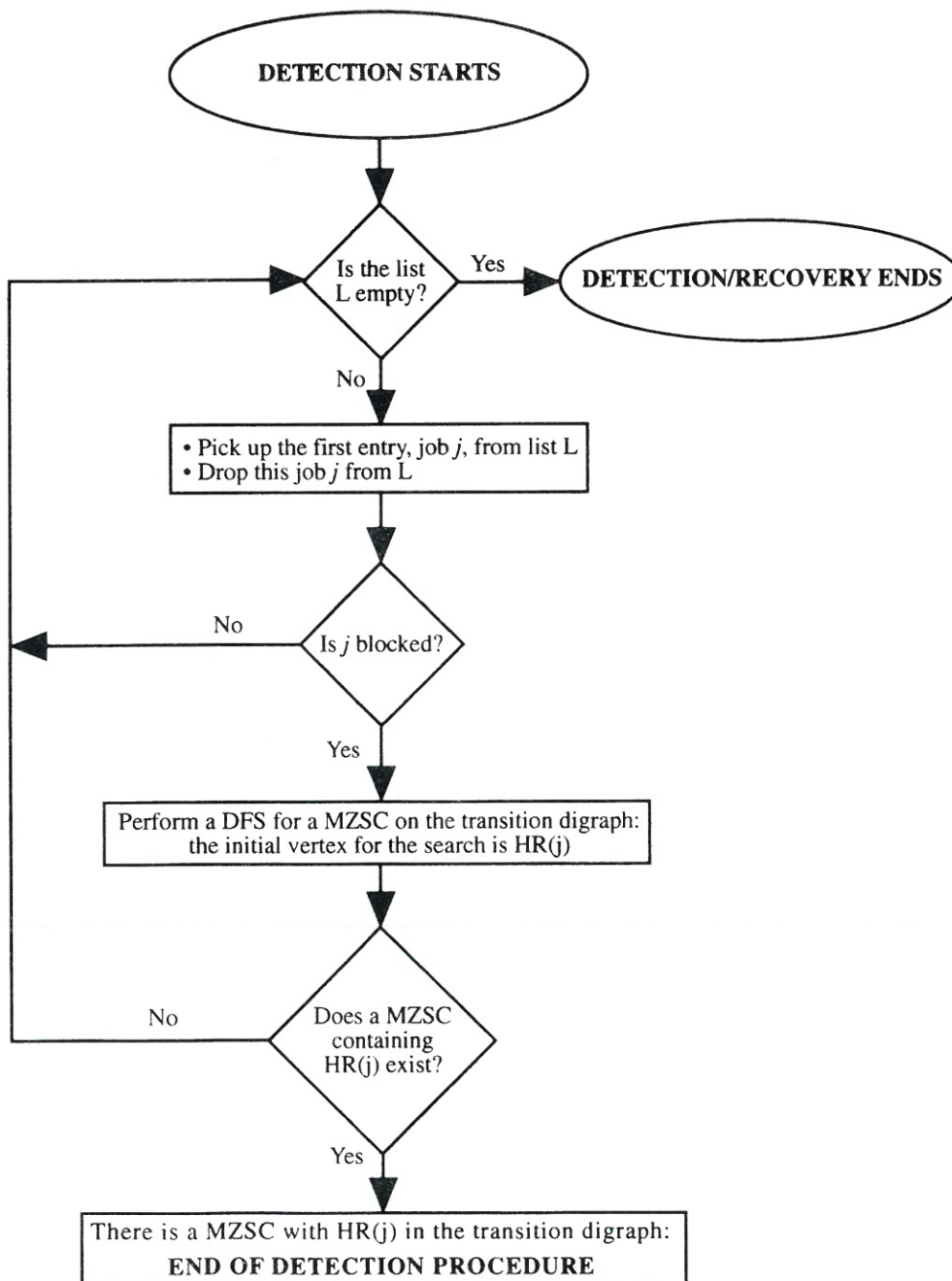


Figure 2. Detection Procedure Flow-chart



Figure 1a shows the Transition Digraph  $D_{Tr}(q)$  where edges are labelled by the corresponding weights and dark nodes indicate *busy* resources. The strong component  $\sigma = (\{r_3, r_1, r_2\}, \{e_{31}, e_{13}, e_{32}, e_{23}\})$  is a MZSC in the deadlock state  $q$ . It is easy to verify that all the jobs in  $J_q$  are permanently blocked, i.e.  $q$  is a deadlock state. On the contrary, for the state  $q'$  equal to  $q$  but for  $SR(j_1)=r_4$ , no MZSC exists in  $D_{Tr}(q')$  (see Figure 1b). State  $q'$ , indeed, is not a deadlock state because no job from  $J_q$  is indefinitely blocked on the resource it currently holds.

Proposition 1 suggests an easy, real-time deadlock detection procedure. To apply it, the controller has to keep memory of the Transition Digraph and to update it according to the rules described at the end of the previous Section. In particular, after each 1-type or 2-type event occurrence, the controller must check if the Transition Digraph contains any MZSC. If this is the case, the controller determines the sets of parts and of resources involved in the deadlock. Then it triggers the recovery procedure that moves a deadlocked job into a special storage buffer  $B$  of unit-capacity. This action breaks the circular wait condition and, if combined with a proper policy for resource allocation, resolves the deadlock. The following Section describes these concepts in detail.

## 4. Detection/Recovery Algorithm

As mentioned in the previous section, the algorithm is in two phases called Detection Procedure and Recovery Procedure, respectively.

### 4.1 Detection Procedure

When a job requires a *busy* resource, its identifier is added to a list ( $L$ ) containing all the blocked jobs. Then, if the Detection/Recovery algorithm is still in execution no more actions are required at the moment. On the other hand, if the algorithm is not in execution, the Detection Procedure gets started following the steps below (see Figure 2).

- 1) If  $L$  is empty, the Detection/Recovery algorithm comes to an end. On the contrary, if  $L$  is not empty, then its first entry (say  $j$ ) is considered and dropped from the list.
- 2) If  $j$  is no longer a blocked job, step 1) is executed again. On the other hand, if  $j$  is still blocked, a Depth-First Search (DFS for brevity) on  $D_{Tr}(q)$  begins to check if a MZSC containing the vertex  $HR(j)$  exists.

- 3) If no MZSC containing  $HR(j)$  exists, the procedure executes step 1) again. On the contrary, if the DFS finds such a component  $\sigma$ , detection ends while the Recovery Procedure starts.

Obviously, the DFS is the core of the detection procedure. In particular, to determine the MZSC we use the algorithm proposed in [12] that finds strongly connected components. Such an algorithm is suitably adapted to our specific problem as described in the schema of Figure 3. Indeed it searches for a strong component of  $D_{Tr}(q)$  that must: (a) have zero outdegree, (b) contain vertex  $HR(j)$ , (c) contain only busy vertices.

It is well-known that in each digraph there is always a strong component (eventually trivial) with zero outdegree. Moreover it can be easily verified that the first strong component determined by the algorithm proposed in [12] has zero outdegree. So, choosing  $HR(j)$  as starting vertex for the DFS, the first strong component the algorithm finds (say  $\sigma$ ) is just the zero-outdegree strong component containing  $HR(j)$ , if it exists. In this case,  $HR(j)$  coincides with the root of  $\sigma$  [12]. So, when the DFS determines the first strong component, the algorithm checks if  $HR(j)$  is its root: if this is the case, conditions (a) and (b) are enjoyed. Moreover, condition (c) is checked for each new vertex considered by the DFS.

We remark that the proposed depth-first search algorithm is performed in  $O(e)$  time, where  $e$  is the number of edges in the Transition Digraph.

Figure 4 depicts a DFS example: a job holding  $r_2$  requests the busy resource  $r_3$ , so that  $r_2$  is the initial vertex for the DFS (see Figure 4(a)). The steps of the procedure are indicated by the vertices and the edges examined in the search, up to the MZSC identification (see Figure 4(i)). We suppose that all the five resources are busy. All *tree* edges (continuous lines), *back* edges (dashed lines) and *forward* edges (short dashed lines) [12] are labelled by their weights. Figure 4 also shows the DFS numbers (bold numbers) and the lowlink numbers (bold numbers in parentheses) [12]. In the final step the MZSC itself is indicated.

### 4.2 Recovery Procedure

Once the detection finds an MZSC, the Recovery Procedure starts according to the following steps (see Figure 5).

```

S ← empty stack
i ← 0
for  $x \in V$  do  $num(x) \leftarrow 0$ 
r ←  $HR(j)$ 
[r is the initial vertex for the Depth-First Search]
STRONG(r)
procedure STRONG(v)
  if v is empty or idle then { [There is no MZSC]
    Stop the algorithm
  }
  else
    {
      i ← i + 1
       $num(v) \leftarrow i$ 
       $lowlink(v) \leftarrow i$ 
      S ← v
      for  $w \in Adj(v)$  do
        {
          if  $num(w) = 0$  then { [(v, w) is a tree edge]
            STRONG(w)
             $lowlink(v) \leftarrow \min\{lowlink(v), lowlink(w)\}$ 
          }
          else if  $num(w) < num(v)$  then { [(v, w) is a back edge or a cross edge]
            if w is on S then { [w is in the same strong component as v,
              since  $w \in Adj(v)$  and since w on S implies
              that there is a path from w to v]
                 $lowlink(v) \leftarrow \min\{lowlink(v), num(w)\}$ 
            }
          }
          if  $lowlink(v) = num(v)$  then { [There is a strong component:
            its root is v]
              if  $v = r$  then { [r is the root of the strong component
                and this component is a MZSC]
                  while x, the top vertex on S, satisfies:
                    {
                       $num(x) \geq num(v)$  do { Add x to the MZSC
                        Delete x from S
                      }
                    }
                }
              else { [r is not the root of the strong component,
                so there is no MZSC containing r]
                }
            }
          }
        }
    }
  return

```

Figure 3. DFS Algorithm

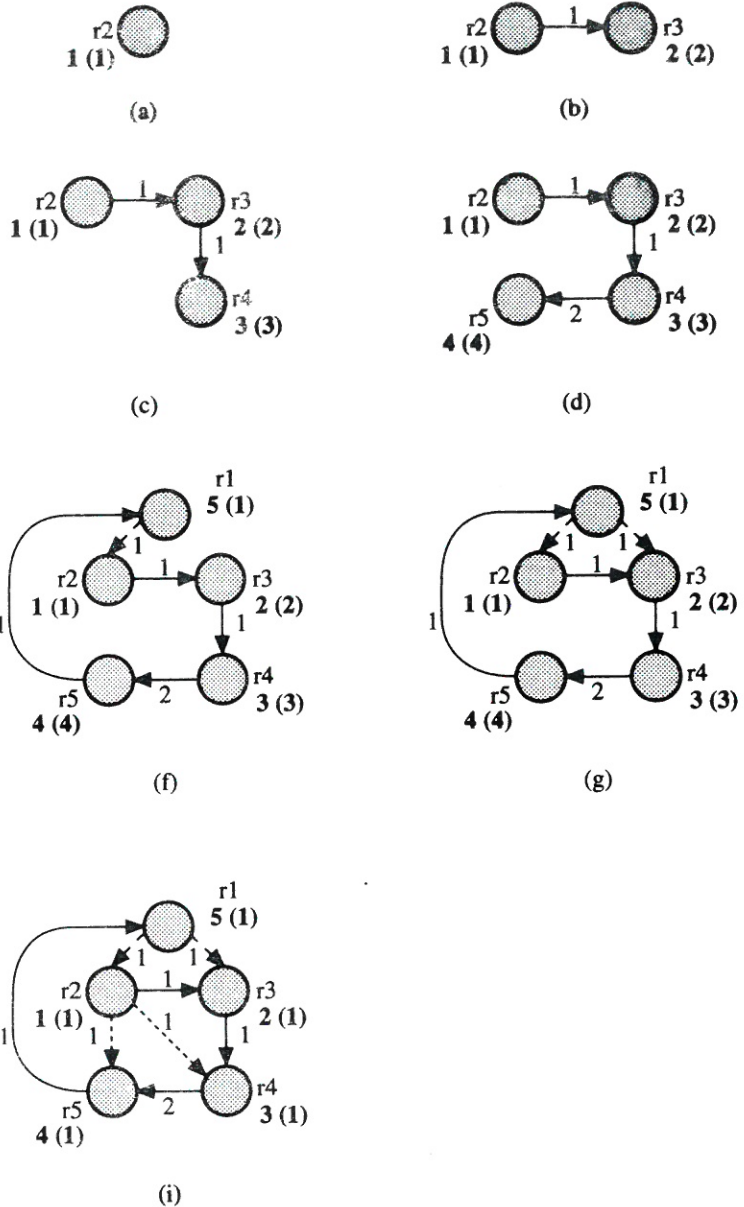
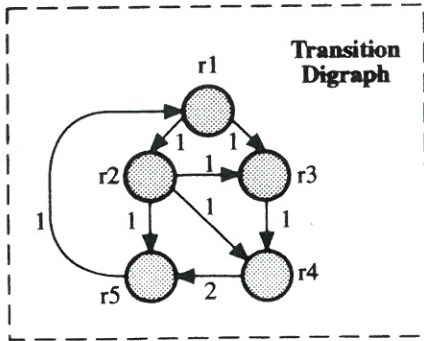


Figure 4. A DFS Example

- 1) Identify a cycle  $\gamma_n = (N_n, E_n)$  in the MZSC  $\sigma$ , containing edge  $e_{im}$ , where  $r_i = HR(j)$  and  $r_m = SR(j)$ . A DFS inside the MZSC performs this identification, by choosing  $e_{im}$  as first edge. A DFS algorithm generating all the cycles of a digraph, performs this operation in time  $O[(e+v)(c+1)]$ , where  $v$  is the number of nodes in the digraph and  $c$  is the number of generated cycles [12]. However, the algorithm has to determine one cycle only, by using  $r_i$  as first vertex and  $e_{im}$  as first edge in the search. So, it is executed in time  $O(e+v)$ .
- 2) Move job  $j$  to the buffer  $B$ . In this way the system state becomes  $q'$ , with:  $OV_{q'}(r_i) =$

$C(r_i) - 1$  and  $a_{q'}(e_{im}) = a_q(e_{im}) - 1$ . Obviously  $D_{Tr}(q')$  does not contain  $\sigma$  as an MZSC so that a transition of any job to  $r_i$  becomes feasible.

- 3) Impose a restriction on resource allocation preventing any arrival of additional jobs to resources in  $N_n$ . This policy must inhibit both new jobs from entering the system to reach resources in  $N_n$  and all job transitions corresponding to edges  $e_{kp}$ , with  $r_k \notin N_n$  and  $r_p \in N_n$ . Obviously, in this way jobs holding resources from  $N_n$  and requiring resources in the same set can go one step ahead in their Residual Working Procedures.



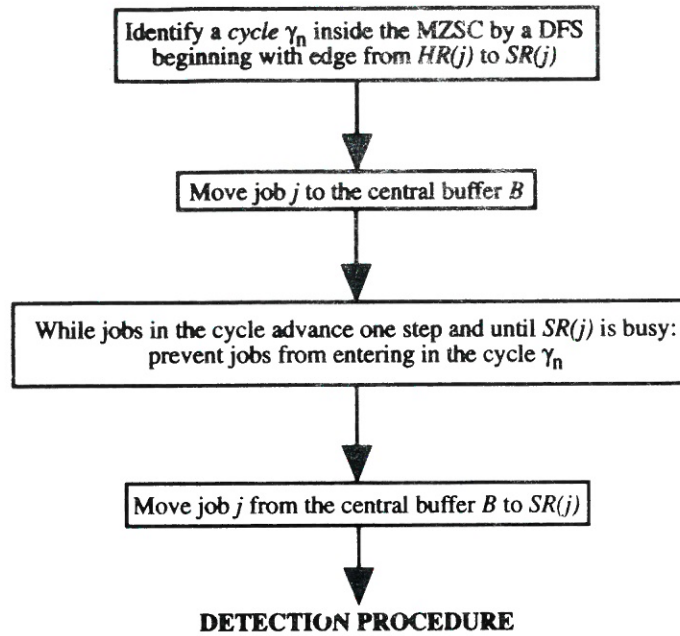


Figure 5. The Deadlock Recovery Procedure Flow-chart

- 4) When the system reaches a state  $q^*$  such that  $OV_{q^*}(r_m) = C(r_m) - 1$ , job  $j$  is transferred from  $B$  to  $r_m$  and the restriction policy is removed. So, the Recovery ends and the Detection Procedure re-starts.

## 5. A Case-Study

This section applies the deadlock detection/recovery technique described above to the flexible manufacturing cell shown in Figure 6.

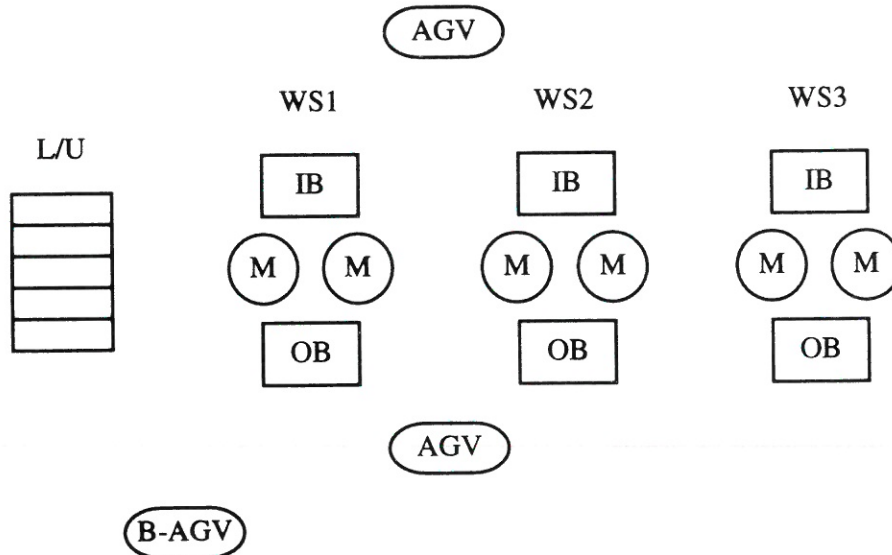


Figure 6. System in the Case-Study

Note that at the end of the recovery there may still be a MZSC in the resulting Transition Digraph. In this case a new execution of the Detection/Recovery algorithm is necessary. This allows some jobs involved in the deadlock to proceed one step more ahead in their Residual Working Procedure, so that no piece can remain indefinitely blocked.

The system consists of a load/unload station (L/U for brevity), three workstations (WSs) and an AGV system with two trucks. Each WS is composed of two identical machines (M) and has an input buffer (IB) and an output buffer (OB), both of unit-capacity. The load/unload station can take no more than five pieces. Thus Table I lists  $R=12$  distinct resources, including the fictitious one.

**Table I. Resources and Their Capacities**

	<i>resources</i>	<i>capacities</i>
r <sub>1</sub>	load/unload station	C(r <sub>1</sub> )=5
r <sub>2</sub>	input buffer of WS1	C(r <sub>2</sub> )=1
r <sub>3</sub>	input buffer of WS2	C(r <sub>3</sub> )=1
r <sub>4</sub>	input buffer of WS3	C(r <sub>4</sub> )=1
r <sub>5</sub>	machines of WS1	C(r <sub>5</sub> )=2
r <sub>6</sub>	machines of WS2	C(r <sub>6</sub> )=2
r <sub>7</sub>	machines of WS3	C(r <sub>7</sub> )=2
r <sub>8</sub>	output buffer of WS1	C(r <sub>8</sub> )=1
r <sub>9</sub>	output buffer of WS2	C(r <sub>9</sub> )=1
r <sub>10</sub>	output buffer of WS3	C(r <sub>10</sub> )=1
r <sub>11</sub>	AGV system	C(r <sub>11</sub> )=2
r <sub>12</sub>	fictitious resource	∞

There are two job types to produce: the former requires a machine of WS1 and, successively, a machine of WS2; the latter receives service from a machine of WS1 and then from a machine of WS3. The AGV units transfer pieces from the L/U to the IBs and from the OBs to either the IBs or the L/U. Moreover a third AGV truck, denoted by B-AGV, plays the role of the reserved buffer B necessary to carry out the recovery procedure. Going into detail we get:

$$w_1 = (r_1, r_{11}, r_2, r_5, r_8, r_{11}, r_3, r_6, r_9, r_{11}, r_1, r_{12})$$

$$w_2 = (r_1, r_{11}, r_2, r_5, r_8, r_{11}, r_4, r_7, r_{10}, r_{11}, r_1, r_{12})$$

We implement the described detection/recovery procedure by a SIMAN discrete-event simulation model [11] assuming the system throughput as performance index. The following conditions rule the simulation. Job types enter the system according to a randomly generated sequence, with equal probability for each type. Since each simulation is performed with a constant number N of jobs in process, a new piece is loaded as soon as a completed job leaves the system. Service times for machines and AGV are generated by a gamma distribution with mean *m* (reported in Table II) and standard deviation *s*=40% of *m* (case A) or *s*=80% of *m* (case B). Finally, the law "First In First Out" rules the priority setting.

**Table II. Mean of Service Times**

<i>resources</i>	<i>means</i>
r <sub>1</sub>	10
r <sub>5</sub>	40
r <sub>6</sub>	30
r <sub>7</sub>	30
r <sub>11</sub>	10

Table III shows the throughputs (jobs per time unit) resulting from each simulation of 1000 completed parts, for different values of number N. The second column of Table II reports the number of times (R) the recovery procedure is triggered in each simulation.

**Table III. Simulation Results**

N	Case A		Case B	
	R	Throughput	R	Throughput
3	0	.0245	0	.0244
4	0	.0319	0	.0308
5	0	.0383	0	.0368
6	7	.0434	24	.0404
7	36	.0459	63	.0436
8	<b>86</b>	<b>.0470</b>	149	.0433
9	213	.0462	<b>195</b>	<b>.0440</b>
10	317	.0455	290	.0416
11	439	.0448	400	.0412
12	551	.0443	570	.0403
13	749	.0430	770	.0400
14	1017	.0447	1037	.0386
15	1389	.0440	1371	.0397
16	1729	.0446	1678	.0398

The results show that for N=3, 4, 5 the recovery policy is not invoked because no deadlock occurs (R=0). The throughput reaches the highest values in Case A for N=8 and in Case B for N=9 (shown in bold by Table III). For N>8 in Case A and N>9 in Case B, throughput decreases because of congestion conditions due to the high work in process. The results of Table III confirm that the deadlock/recovery technique allows us to increase the number of pieces in the system and to obtain better performance indices.



## 6. Concluding Remarks

The detection/recovery method proposed in Section 4 appears to be simple and effective. In particular, the detection phase requires low computational burden because the method developing the idea of MZSC and using an DFS-like algorithm involves linear complexity.

The recovery phase is also very simple and it only requires a dedicated unit-capacity buffer.

These characteristics make the proposed approach easy to apply at low costs. On the other hand, as shown by the case-study, the technique allows us to increase the system utilization, thus improving its performance indices and avoiding the unfavourable effects of deadlock at the same time.

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