BOOK REVIEW

Learning and Geometry
Computational Approaches

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The present book represents a partial record of the 1991 Workshop on "Learning and Geometry" hosted by the University of Maryland and the Center for Night Vision.

As the fields of Geometry Theorem Proving and Computational Learning are at present quite mature, being able to tackle real- life problems, it has become an interesting research topic to try to integrate them, especially in order to deal with applications in which no one of these two approaches has been successful on its own. Such applications include interpreting data produced by a variety of sensors (current vision techniques based on computational geometry being able to extract features from data, but failing in the task of recognition of the sensed objects).

It seems that human high-level vision could be studied within the context of learning from geometrical examples. On the other hand, the lower level representation of features, and classes of geometrical shapes is more amenable to geometric reasoning.

The point of view of the organizers of the workshop is that "further progress in computer vision requires a careful reexamination of vision fundamentals along the interpretation towards discovering the appropriate mathematical foundations and semantics which best fit vision problems".

The papers presented in the volume approach the problem of integrating learning and geometry using the machinery provided by various disciplines, such as computational learning, model- theoretic semantics, computational linguistics, geometry theorem proving as well as synthetic, foundational and algebraic geometry. The task of integrating these approaches with the goal of improving the current computational vision systems is by no means an easy task. It involves deepening the research performed in various fields as well as trying to unify the results obtained.

For example, computational learning models (which address the problem of learning from examples) such as the Valiant model of the learnable have been successful in several problems but have not been extended or modified to cope with the specific types of examples encountered in vision. Special-purpose reasoning taking into account the phenomenology specific to vision seems to be needed in order to obtain better results.

Also, although geometry theorem proving has been very successful lately (as opposed to automatic theorem proving which has had a much more modest success), a lot more work has to be done in order to address the problem of the most suitable representation of geometrical reasoning as well as of learning.

The papers in the volume have been grouped in two main sections: "*Learning*" and "*Geometry*".

The **first section**, devoted to **Learning**, includes three papers mainly concerned with MDL and PAC learning.

The first paper entitled "**Learning by MDL**" by J.Rissanen and Bin Yu tries to formalize machine learning as the problem of estimating a conditional distribution of the "concept" to be learned. The Minimal Description Length (MDL) principle is used to quantify the amount of learning from data.

It is argued that an adequate theory of learning should extend Valiant's model by allowing noise in the data. Also, while polynomial restrictions on the readiness with which the estimation is to be made for a concept to be learnable, are desirable from a practical point of view, the authors stress the fact that useful information can be extracted from data even if the entire "concept" is too complex to be learned in polynomial time.

The second paper, "**PAC Learning, Noise and Geometry**" by Robert H. Sloan, describes Valiant's probably approximately correct (PAC) model of concept learning, especially in the case where instances are points in an Euclidean space. Finally, the problem of dealing with noisy training data is considered.

The next paper, "**A Review of Some Extensions to the PAC Learning Model**" by Sanjeev R. Kulkarni, considers several extensions of the basic PAC model focusing on the information complexity of learning. The extensions discussed are:

- learning over a class of distributions
- learning with queries
- learning functions and
- generalised samples.

Learning over a class of distributions (as opposed to distribution-free learning) deals with cases in which the learner has prior knowledge of the distribution used to draw the samples.

Learning with queries involves models of learnability in which the learner has more powerful information gathering mechanisms than in the basic PAC model (in which the examples are provided to the learner according to some probability distribution over which the learner has no control). Thus, instead of passively receiving examples, the learner can use various types of queries to an "oracle": membership queries, equivalence queries, subset, superset and disjointness queries as well as exhaustiveness queries. It turns out that, as expected, the use of oracles can often aid in the learning process. However, considering information complexity only, the use of oracles does not enlarge the set of learnable concept classes. On the other hand, from the viewpoint of computational complexity, allowing oracles can enlarge the set of learnable concept classes.

Learning functions (rather than just membership functions of examples in classes) represent another natural extension of the PAC model.

Another extension may consist in providing **generalised samples** to the learner. More precisely, rather than receiving random samples of some unknown function, the learner receives a generalised function - essentially a functional assigning of a real number to each concept. These numbers are not necessarily point values; instead, they can be other attributes of the unknown concept (such as the integral over a given region).

Although learning with generalised samples is just a transformation of a standard learning problem, it is extremely important since it allows the application of the learning framework to a much wider range of problems, in particular to signal/image processing and geometric reconstruction, which are very important from the point of view of vision.

The three papers concerned with learning present results which are very useful from the point of view of computational vision. However, only a very limited number of direct references to learning in the geometrical domain are made.

This is probably due to the small amount of interdisciplinary work carried out up to now in this very challenging field of "geometrical learning". Rather than being regarded as a drawback, this should be considered an incentive to deepening our understanding of this domain.

The **second section** of the book groups five papers dealing with **Geometric Reasoning**.

The first, entitled "**Finite Point Sets and Oriented Matroids - Combinatorics in Geometry**" by Jürgen Bokowski, is concerned with the topological invariants of sets of points in the Euclidean space (or equivalently, of matrices over the reals). **Oriented matroids are useful** for geometrical reasoning in cases in which only the topological properties of the geometrical objects are of concern. The paper illustrates the basic ideas in the theory of oriented matroids by using various examples.

The second paper, **"A Survey of Geometric Reasoning Using Algebraic Methods"** by S.C. Chou and X.S. Gao analyses the most successful algebraic methods for mechanical geometry theorem proving.

Typically such methods start by assigning co-ordinates to key points of the problem and proceed by translating the hypotheses and the conclusion of the problem into multi-variable polynomial equations and inequalities.

The resulting algebraic statements are subsequently proven using various algebraic techniques such as the Ritt-Wu characteristic set (CS) method, the Gröbner basis (GB) method or Collin's quantifier elimination method for real closed fields of Tarski's type.

The present survey concentrates on the applications of the CS and GB methods to automated reasoning in elementary and differential geometry, as well as in mechanics.

Walter Whiteley's paper on **"Synthetic vs Analytic Geometry for Computers"** tries to extend computer geometry beyond the traditional approach based on analytic geometry. Alternatives to Cartesian analytic geometry such as projective geometry and co-ordinate - free analytic geometry (synthetic geometries) are considered.

The author describes translations from co-ordinate analytic geometry to co-ordinate-free "invariant" analytic geometry using classical invariants and the Cayley (extended exterior) algebra. This co-ordinate-free "invariant" representation is not only more amenable to theorem proving and learning, but also it produces more readable proofs.

It seems that this approach better captures the precise layer (in a hierarchy starting with topology, going through projective and affine geometry and ending with Euclidean geometry) at which reasoning should take place.

The next paper, also written by Walter Whiteley, deals with **"Representing Geometric Configurations"**. The importance of "knowledge representation", initially advocated by the Artificial Intelligence community, is considered here from the point of view of finding the adequate representation of a discrete geometric set. Dependence on the level of geometry (Euclidean, similarity and projective) is also analysed. The paper shows that finding a "good representation" is a unifying theme for many deep geometric results. This observation should have an important impact on the design of more performant vision systems.

The last paper of the volume, **"Geometry Theorem Proving in Euclidean, Descartesian, Hilbertian and Computerwise Fashion"** by Wu Wen - Tsun, presents the evolution of geometry theorem proving starting with Euclid up to now. Some recent achievements (mainly due to the Mathematics-Mechanisation Group of the "Academia Sinica"), are also briefly presented.

Although computational learning is hardly mentioned in the five papers of the "Geometry" Section of the book, the results and ideas herein presented are extremely significant w.r.t. the ultimate goal of computer vision. First, it is shown that geometry theorem proving can be done effectively and efficiently on a computer (as opposed to general automatic theorem proving which appeared as much more difficult). Second, a number of various approaches to geometric reasoning is made and the importance of an "adequate representation" (for reasoning as well as learning) is underlined.

A large number of open problems and incentive research topics is presented or just hinted at.

In conclusion, the book comprises a set of very interesting papers in the area of interference between learning and geometric reasoning. Hopefully, it will spawn new researches in this very promising field and lead to a deeper understanding of the relationships between knowledge representation, learning, representation change and visual understanding.

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