Mathematical Control Theory: An Introduction

by Jerzy Zabczyk
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Dr. Vasile Sima was born at Lita, Romania, on the 21st of October, 1949. He graduated from the Polytechnical Institute of Bucharest in Control Engineering in 1972, and from the Department of Mathematics at the University of Bucharest, in 1978. He obtained his doctoral degree in Control Engineering (adaptive control) from the Polytechnical Institute of Bucharest, in 1983. Since 1972 he has held several research positions at the Research Institute for Informatics in Bucharest. He is senior research worker and vicepresident of the Scientific Board of the institute. Dr. Vasile Sima has published more than 80 scientific papers (about 35 of them were published in international journals and symposia proceedings). He is co-author of two books (in Romanian), and author of other two books. His research interests include control theory, adaptive and optimal control, computer-aided control systems design, nonlinear programming, numerical linear algebra and scientific computations.

Well-written, this book is a self-contained and concise presentation of modern mathematical control theory for deterministic systems, focussing on typical and characteristic results. Linear and nonlinear systems, as well as infinite dimensional linear systems are dealt with. Some of the overlooked topics are discrete-time systems, and stochastic systems. This option made it possible to unify presentation and style.

Mathematical control theory is a distinct branch of mathematics, which has recorded an explosive development in the last decades. One of the main points of the reviewed book is its impressive ability to get together a huge amount of theoretical results and offer the reader a valuable guidance in effectively using these results.

The book consists of an introduction and four distinctive parts. The introduction summarizes the basic concepts of mathematical control theory and presents several interesting and suggestive examples (an electrically heated oven, a rigid body model, Watt's governor, an electrical filter, a soft landing problem, an optimal consumption model, and an infinite dimensional model for heating a rod).

Part I, entitled *Elements of Classical Control Theory*, deals with structural properties of linear systems, discussing basic results on controllability and observability (**first chapter**), stability and stabilizability (**second chapter**), realization theory (**third chapter**), and systems with constraints (**fourth chapter**). Algebraic characterizations of controllable and observable systems, as well as a complete classification of controllable systems with one input are given. Stability is discussed in terms of associated characteristic polynomials (Routh theorem) and Lyapunov equations. Luenberger's observer is introduced to illustrate the dual concept of detectability. Realization theory is based on the input-output map generated by a linear control system, and characterized in terms of the impulse response function and the transfer function. Beside the usual topics, the fourth chapter discusses more special classes of systems, such as linear systems with bounded sets of control parameters, and the so-called positive systems.

Part II, entitled *Nonlinear Control Systems*, reformulates the structural properties and realization theory for nonlinear systems. The topics are covered by **three chapters**, which parallel those corresponding to linear systems. Two approaches to controllability and observability are discussed: one based on linearization, and the other one on concepts of differential geometry, using Lie brackets. Exponential, asymptotic and Lyapunov stability are discussed, using either linearization or Lyapunov's function approaches. Topological methods are used for analysing a relationship between controllability and stabilizability. Concerning the realization theory, the impulse-response function of a linearization of the input-output map in a control system is determinant. A control system with a given input-output map is also constructed.

Part III, entitled *Optimal Control*, is organized in **four chapters** which cover: dynamic programming, dynamic programming for impulsive control, the maximum principle, and the existence of optimal strategies, respectively. The main objective is to find optimal controls. The topics include: Bellman's optimality principle, its typical application to the linear regulator problem and to impulse control; Pontriaghin's maximum principle for classical problems with fixed control intervals, as well as for time-optimal and impulse control problems; and Fillipov's basic theorem for existence of optimal controls. The derived Bellman's equations are used to solve the regulator problem on a finite time interval. The essential role of the Riccati matrix differential equations (for instance, for assessing stability), is pointed out. The existence of optimal impulse control strategies is derived from general results on fixed points for monotonic and concave transformations. The key role played by the separation theorems for convex sets, when solving time optimal problems by the maximum principle, is emphasized.

Part IV, entitled *Infinite Dimensional Linear Systems*, also includes **four chapters**, devoted to linear (infinite dimensional) control systems, controllability, stability and stabilizability, and linear regulators in Hilbert spaces, respectively. The discourse is restricted to linear systems and the so-called semigroup approach. The theory of semigroups of linear operators on Banach spaces—which applies to linear uncontrolled systems—is summarized in the first chapter. The Hille-Yosida, Phillips, as well as Lions theorems are proven, and illustrated by self-adjoint and differential operators. The controllability analysis is based on images of linear operators in terms of their adjoint operators. Specific descriptions of approximately and exactly controllable systems modelled by parabolic and hyperbolic equations are given. It is shown that asymptotic stability does not imply exponential stability, and is not determined by the spectrum of the generator. Stable systems are characterized by their corresponding Lyapunov equations. Null controllability implies stabilizability and, under appropriate conditions, the converse is also true. Finally, the existence of a solution to an operator Riccati equation related to the linear regulator problem in separable Hilbert spaces is determined, and a formula for the optimal solution on an arbitrary finite time interval is given. The existence of an optimal solution on the infinite time interval is also discussed, and applications to the stabilizability problem are included.

An appendix collects the fundamental results (without proofs) for metric, Banach and Hilbert spaces, Bochner's integral, and spaces of continuous, or measurable functions. The references chapter includes 70 titles (mainly books), well-appreciated by the control theory community. The book ends by an one-page notation section and a four-page subject index.

The book covers several less traditional topics. Such examples are: the realisation theory and geometrical methods for analysing the structural properties of linear and nonlinear systems; the theories for positive, impulsive and infinite dimensional systems; or, stabilization of nonlinear systems using topological methods.

All chapters contain mathematical treatment (with proofs for essential facts), interesting examples, and many proposed exercises (some of them, with keys). Each chapter begins with a short presentation of the topics covered, and ends up with a section, titled *Bibliographical Notes*, that gives a brief account of the recommended ancillary literature (mainly textbooks), directing the reader to the sources used, where additional details can be found.

By its concise, but comprehensive coverage, extremely good organization of the material, the readability of the exposition, the included theoretical results, and by its challenging examples and exercises, the reviewed book is highly recommended for beginning a graduate course in mathematical control theory or for self-study by those professionals willing to know about the theory underlying the control theory applications. It is a valuable reference for control theorists, mathematicians, and all those who use mathematical control theory in their work. The assumed background includes basic linear algebra, differential equations, and calculus. The final part of the book requires previous knowledge in semigroups of linear operators theory.

Vasile Sima