A Fuzzy Control Synthesis Using The Variable Structure Approach

Jean-Yves Dieulot, Sonia Hajri, Pierre Borne
Laboratoire d'Automatique et d'Informatique
Industrielle de Lille (LAIL)
URA CNRS D1440
Ecole Centrale de Lille, B.P. 48
F-59651 Villeneuve d'Ascq Cedex
FRANCE

Abstract: A fuzzy control synthesis is proposed along with the theory of sliding motion. Fuzzy control can be seen as saturated control, but also as variable structure control with a continuity in a boundary layer around the state space surface where the membership function of the predicate u is ZERO is different from zero. A guideline for the lookup table design is thus provided. Given certain conditions on membership functions, some stability results can be found and the performance-robustness trade-off can also be analyzed.

1. Introduction

Fuzzy control is sometimes seen as a combination of heuristics and/or the probability theory. One has to keep in mind that fuzzy control is deterministic and that the resulting control u can be looked upon as a non-linear function of variables representing the environment [2] often limited to successive derivatives of the state (the state space equation will then be under the canonical form). One way to determining the stability conditions and the "good" parameters for fuzzy control could be to approximate u with spline functions of the state x. However, this approach is not really easy and it cannot provide methods to design the rules lookup table. The space of the rules, that is to say the lookup table, is a neglected one: very often, a lookup table is not given in the same orientation as the state space plane. The state space is then divided into cells corresponding each to a mesh between the input variables predicates. It is interesting to see that, in this space, the cell containing the point x=0, that is to say all variables have the predicate ZERO, has a control u that is ZERO (when the variables behave "well", the action on the system is zero). Furthermore, the lookup table is generally partitioned into three categories: one part where control is positive (it can be Small Positive, Positive Big, etc.), one part where it is negative, and one part where it is zero. A frontier exists between the positive and negative controls; within this frontier, the predicate for u is often ZERO.

Fuzzy control lookup table thus reproduces some sort of a variable structure control with continuous approximation in a boundary layer. It is then quite interesting to develop this aspect by linking this lookup table space with the state space (often the phase plane; what is interesting is that Utkin notices that choosing Variable structure surface in the phase plane yields better performance [4] , and to use results of smoothed sliding motion to give some idea of fuzzy control stability and performance, and, moreover, to propose a design method for fuzzy control.

Some results for sliding motion with smoothing algorithms will first be recalled. In a second part, fuzzy control is analyzed. In the last part, discussion over stability and performance is undertaken, using results of parts 1 and 2 .

General notations:

x is the state of the system (see eq. (1)).

x is often chosen as being a phase variable, with:

- e is the setpoint-output error, $\dot{e}$ or de is its derivative.
- s is generally defined as a sliding surface.
- u is the control.
- u_+ is the "positive" control, defined as a singleton output predicate in the Fuzzy control case or as the "positive" control in variable structure case.
- u_- is the same as above for "negative" control.
- u_z is the "u is zero" control.
- k is the gain of the sliding controller.

2. Variable Structure Systems

2.1 Structure

Let a system be under the form :

$$\dot{x} = f(x,t) + bu(x,t) + d(t) \qquad (1)$$

The simplest way to have a well-configured system for VSS is to write it under the canonical form.

Variable structure control of system (1) is defined by the following equation:

$$\begin{cases} u = u_i^+(x) & if\ x \in D_i^+ \\ u = u_i^-(x) & if\ x \in D_i^- \end{cases} , \qquad (2)$$

where D_i^+ is defined by $s_i(x) > 0$, and D_i^- is defined by $s_i(x) < 0$, the control values u_i^+ and u_i^- being discontinuous along the surface $s_i(x)$.

When continuous switching between u_+ and u_- occurs around the switching surface $s_i(x)$, the system slides along the surface and remains insensitive to a certain class of perturbations [4].

2.2 Existence, Stability and Design of Sliding Control

There exists a stable sliding motion if the condition

$$s_i(x)\dot{s}_i(x) < 0 \qquad (3)$$

is fulfilled or in case of a varying surface s(t), if there exists η such as:

$$s_i(x)\dot{s}_i(x) < -\eta|s_i(x)| . \qquad (4)$$

(see [4])

Generally, this surface will be taken as:

$s(x)=C^T x$,

and the structure of the law is:

$u=-k^T x$. (5)

Condition (3) often gives minorating conditions on the value of k.

2.3 Approximated Switching Control

Approximated switching control has been described for the first time by Slotine [6], who would basically replace the relay by a saturation. Asymptotic stability is lost in place of a Globally Uniformally Ultimate Bounded Stability [1], that is to say that invariance is lost but within a reasonable level perturbation the control is robust.

We recall without demonstration some results obtained for a class of non-linear systems such that:

$$x^{(n)} = f(x) + bu . \qquad (6)$$

As an example, taken from [Slotine], let us define a switching surface $s = \left(\frac{d}{dt} + \lambda\right)^{(n-1)} x$, a boundary layer depending on x and t such that $|s| < \Phi$ and let in that boundary layer the control u be u=-ksat(s/Φ), the structure of the control remaining as in Eq (5) outside the boundary layer, and k being chosen as to fulfil condition (3). It is shown that this control is asymptotically stable, and (here a particular s is chosen, but the result can be generalized), that, in the boundary layer,

$$\dot{s} = -\bar{k}(k,\Phi)\frac{s}{\Phi} . \qquad (7)$$

It is then shown that this means trade-off between the tracking precision and the boundary layer width, and we can choose the boundary layer width to be $\Lambda = \frac{\bar{k}}{\Phi}$. Slotine ([5] and [6]) shows then that this trade-off can be seen as:

(boundary layer width). (tracking precision) = parameter uncertainty along the trajectory and can be applied to many control problems such as robot control [6]. The main interest of the algorithm is the basic design, that prevents the so-called chattering, which is induced by high-frequency perturbations that affect the state, and is a drawback of pure sliding motion. Chattering consists in high-frequency oscillations affecting the behaviour of the sliding trajectory, and , thereafter, the state x. As placing a first-order just after the relay leads to difficult theoretical problems, many smoothing methods have been proposed.

The method used for fuzzy control synthesis tends to be of the form used in [13] and [14]:

$$u = -k\frac{s}{|s| + \delta} \qquad (8)$$

where δ is a constant positive number that in fact represents the bandwidth, and k is a constant gain. The higher δ, the more robust the system is, but also the less performing. δ cannot be chosen too little [14].

If condition (3) is fulfilled, the system is shown to be Globally Uniformally Ultimate Bounded Stable [1].

3. An Analysis of Fuzzy Control

3.1 The Rule Space

3.1.1 Lookup Table

Fuzzy control with Mamdani model yields that control u can be achieved by rules:

IF x_1 is A_1 AND x_2 is A_2.... THEN u is B.

A_1, A_2,... and B are linguistic predicates, and a membership function for each predicate is attached to the values of the corresponding variables, e.g. $\mu_A(x)$ for x is A.

The operators of inference and composition are the so-called T.norms [15]. Usually, the operators "min" or "multiply" or other monotonous T.norms are employed that is to say μ(x is A AND y is B)=μ(x is A).μ(y is B).

Control u is obtained by defuzzification, for example:

$$u = \frac{\sum \mu_i u_i}{\sum \mu_i}$$

, where the u_i and μ_i values are attached to rule R_i.

An input variable is thus divided into a certain number of predicates along the universe of discourse. It is then possible to draw a lookup table between variables, representing that for instance if x is ZERO and y is BIG then control u is ZERO [2] , [3].

Our fuzzy control examples will use the above formula, that is to say that the output predicates will be reduced to singletons. In practice, this kind of a formula is very common, and our results can be extended to more sophisticated output membership functions, the basic idea being the core of the paper.

3.1.2 Fuzzy and VS Controls

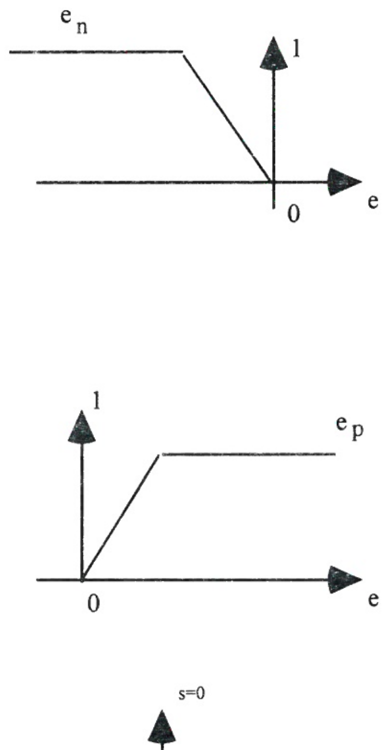
Notations:

It will be supposed first in our study that there are 3 output predicates u_+, u_- and u_z, the latter corresponding to predicate ZERO and having the value zero so that precision can be achieved towards the point x=0, without loss of generality.

The predicates for e and de are Negative, Zero and Positive, the corresponding membership functions are noted e_n, e_z and e_p for e and de_n, de_z and de_p for de.

Figure 1 shows a 2 inputs 1 output lookup table with 2 membership functions. The system is defined as in (1), with x_1=e and x_2=e . One can see easily that there will be a variable structure control around the surface e=0, that is to say u= u_+ if e>0 and u= u_- if e<0.

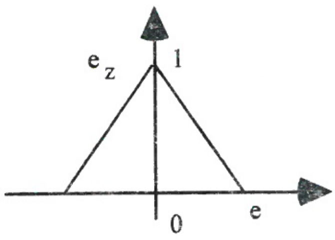
e will also be noted de.



e \ e	N	P
P	U-	U+
N	U-	U+

Figure 1. Fuzzy Control of 2 Variables: Switching Surface

Let now see example in Figure 2 (consider the separation between u_+ and u_-)



ė \ e	N	Z	P
P	U_z	U_+	U_+
Z	U_-	U_z	U_+
N	U_-	U_-	U_z

trajectory in the rule plane

Figure 2. Fuzzy Control with Rule Plane Trajectory

Figure 2 shows a case where the set-point is a unit step and e constantly decreases towards zero.

It is intuitive to see that in the rule space, there exists some sort of variable structure control with a boundary layer in the middle. The problem is that the real space (the state space) is not exactly the same. There can even be a discontinuity as in Figure 3. We can thus guess that there may exist a surface for switching control. We will suppose that membership functions for "NEGATIVE" predicates and "POSITIVE" predicates are disjoint, which is uppermost important. We can see that, outside the area in the state space plane where we may have control u_z (e.g. when membership functions for e Positive and de Negative exist and membership functions for e and de to be zero is zero), control is equal to u_+ and u_-, thus realizing some sort of constrained control. These considerations are true for any inference method and any defuzzification methods. u_+ and u_- are thus being chosen big enough to ensure convergence. That is the reason why some other predicates in output can be chosen, e.g. u_+ and u_{++} meaning Small Positive and Positive, that can achieve convergence and softer dynamics when approaching the surface, thus achieving some sort of "dual mode", or "multi mode".

Our goal is to show that lookup table and membership functions design can provide a switching surface.

Let us choose another example:

de \ e	N	Z	P
P	U+	U+	U-
Z	U+	Uz	U-
N	U+	U-	U-

sliding surface with discontinuity

Figure 3. Switching Surface with Discontinuities

Figure 3 intends to show that in some cases, there is no boundary layer in the rule space (represented by uz) between the two control predicates u+ and u- so that there is a discontinuity in the rule plane.

3.2 Designing a Switching Surface

Let us have a look at Figure 4:

de \ e	N	Z	P
P	U-	Uz	U+
Z	U-	Uz	U+
N	U-	Uz	U+

Figure 4. Classical Example of Fuzzy Control

The above figure shows some sort of "manichean" example of fuzzy control, being not very "soft". Indeed, it is a good "basic" example (the derivative term is not taken into account).

We introduce new variables; μ_+ (resp. μ_- and μ_+) is the SUM of the membership functions of the lookup table cells which correspond to the predicate u is U+ (resp. U_- and U_z), which means that u is Positive (resp. Negative or Zero). We recall that the predicates for e and de are Negative, Zero and Positive, the corresponding membership functions being e_n, e_z and e_p for e and de_n, de_z and de_p for de. For example, in our case, the membership function μ_+ is the sum of the membership functions of the cells "e is positive and de is positive", "e is positive and de is zero","e is positive and de is negative", which leads to: $\mu_+ = (e_p de_p + e_p de_z + e_p de_n)$. We thus have:

$$u = \frac{\mu_z u_z + \mu_+ u_+ + \mu_- u_-}{\mu_z + \mu_+ + \mu_-} \quad ; \qquad (9)$$

if u_z=0 then

$$u = \frac{\mu_+ u_+ + \mu_- u_-}{\mu_z + \mu_+ + \mu_-} . \qquad (10)$$

We recall that $\mu > 0$.

The surface for which u=0 is thus:

$$\mu_+ u_+ + \mu_- u_- = 0 . \qquad (11)$$

We will note Δ_{uz} the surface where u is ZERO.

Here, u=0 yields (from the definition of μ_+ and μ_- and from the lookup table):

$$(e_p de_p + e_p de_n + e_p de_z) u_+ = -(e_n de_p + e_n de_n + e_n de_z) u_- , \qquad (12)$$

so u=0 $\Leftrightarrow$ $e_p u_+ = -e_n u_-$, (13)

so that we define s as $e_p u_+ = -e_n u_-$, which is called "switching surface".

If the positive and negative predicates are zero around zero, there will be a dead zone. Else, in the most common case, if they are just zeroing for e=0, then, the surface will be e=0. On one side, control u is positive, on the other side, it is negative (of course, u_+ and u_- have opposite sign). This explains why "positive" and "negative" membership functions should have a void intersection. If the membership functions are linear, the surface is linear.

Developing (9), from the lookup table, control u is now:

$$u = \frac{de_n e_n u_- + de_p e_n u_- + de_z e_n u_- + de_n e_p u_+ + de_p e_p u_+ + de_z e_p u_+}{de_n e_n + de_p e_n + de_z e_n + de_n e_p + de_p e_p + de_z e_p + e_z de_z + e_z de_p + e_z de_n}$$

, and, factorising:

$$u = \frac{(e_p u_+ + e_n u_-)(de_n + de_p + de_z)}{(de_n + de_p + de_z)(e_p + e_n) + e_z de_z + + e_z de_p + e_z de_n}$$

Supposing that $u_+ = -u_-$, and that e_p and e_n are disjoint (i.e. if $e_p \neq 0$ then e_n=0 and vice versa) we now have one of the two terms e_p and e_n being zero, such that:

$$u = \frac{(e_p u_+)(de_n + de_p + de_z)}{(de_n + de_p + de_z)(e_p) + e_z de_z + + e_z de_p + e_z de_n}$$

if e_n=0 or

$$u = \frac{-(e_n u_+)(de_n + de_p + de_z)}{(de_n + de_p + de_z)(e_n) + e_z de_z + + e_z de_p + e_z de_n}$$

if e_p=0.

We can write: $u = \frac{e_g u_+}{e_g + e_z de_z + \beta}$, where e_g is proportional to e_n or e_p that is to say, s being defined in (13):

$$u = \frac{s u_+}{|s| + e_z de_z + \beta} , \qquad (14)$$

s being defined above. This is the final result for the table defined in Figure 4.

We can then see the parallel with section 1 coming. If membership functions are piecewise linear, then fuzzy control is under a form very close to (10).

Indeed, when looking at the shape of the mapping u, one can notice a behaviour very close to a linear variable structure control.

The example in Figure 4 is rather trivial and we take another more sophisticated one.

Let us now consider example in Figure 2. We have a diagonal of predicates ZERO, so that the control can be seen as switching around a surface for which u=0 , in the area e is ZERO and de is ZERO (we will note this surface by s):

s=0:

$$(e_p de_p + e_z de_p + e_p de_z) u_+ = -(e_n de_n + e_z de_n + e_n de_z) u_-$$

the equation of the surface s can be written as:

$$(e_p de_p + e_z de_p + e_p de_z) - (e_n de_n + e_z de_n + e_n de_z) u_+ = s$$

According to Figure 2, control u can be written as:

$$u = \frac{((e_p de_p + e_z de_p + e_p de_z) - (e_n de_n + e_z de_n + e_n de_z)) u_+}{(e_p de_p + e_z de_p + e_p de_z + e_n de_n + e_z de_n + e_n de_z) + (e_p de_n + e_n de_p) + (e_z de_z)} ,$$

that can be rewritten as:

$$u = \frac{((e_p de_p + e_z de_p + e_p de_z) - (e_n de_n + e_z de_n + e_n de_z)) u_+}{(e_p de_p + e_n de_n) + (e_p de_z + e_z de_n) + (e_z de_p + e_n de_z) + (e_p de_n + e_n de_p) + (e_z de_z)} ,$$

We find s for the numerator and at the denominator, an addition of 3 terms:

$(e_p de_n + e_n de_p), (e_z de_z)$ and $|s|$

So we have:

$$u = \frac{s u_+}{|s| + e_z de_z + e_p de_n + e_n de_p}$$

Suppose that for example e>0 and de<0, we can rewrite the equation of the surface in that quadrant:

$(e_p de_z) u_+ = -(e_z de_n) u_-$.

The switching surface s is defined as $s = ((e_p de_z) - (e_z de_n)) u_+$.

We introduce the new definition of s;. with de<0 we yield de_p=0, and with e>0 we yield e_n=0, so that, replacing the equations in the above formula,

$$u = \frac{s u_+}{|s| + e_z de_z + e_p de_n}$$

If de>0 and e<0, the surface will be different and the same sort of equation as before can be found. For e>0 and de>0, or e<0 and de <0, the same kind of result applies.

We find the same structure again as that in (14) with the difference that the surface is a bit more sophisticated.

Suppose we have linear functions: if we take the expression of s, we can see that the membership functions are:

$$e_p = \frac{e}{k_{ep}} \text{ for } 0 < e < k_{ep}, de_z = 1 - \left|\frac{de}{k_{dez}}\right|$$

$$\text{for } 0 < |de| < k_{dez}, \text{ and } de < 0 \text{ means } de_z = 1 + \frac{de}{k_{dez}},$$

the same applying for e_z and de_n.

Replacing the expressions of the membership functions in the expression of s, we yield:

$$(1 + \frac{de}{k_{dez}})\frac{e}{k_{ep}} = -(1 - \frac{e}{k_{dep}})\frac{de}{k_{den}}$$, and further:

$$\frac{e}{k_{ep}} + \frac{de}{k_{den}} + e\,de(\frac{1}{k_{dez}k_{ep}} - \frac{1}{k_{ez}k_{den}}) = 0$$

This surface is non-linear. Yet, this surface is surprisingly linear-like and even linear with well-chosen parameters. Indeed, these parameters depend only on the membership functions of the positive or negative functions. The "gain" depends only on the value of u_+ and u_-. Further, the smoothing process depends to a great extent on the ZERO membership functions.

We have now a general structure like:

- s=0 is the surface where u=0. It can be taken often as linear.
- the control structure u can then generally be given the form $u = \frac{su_+}{|s| + \mu(u_z)}$, provided that u_+ and u_- are symmetric and that "non-U is ZERO" membership (that is to say the area in the rule plane where u is given a predicate that differs from ZERO) functions are disjoint, which we can also express by:

$$u = u_+ sat\left(\frac{s}{f(s)}\right), \qquad (15)$$

and having s<f(s) while staying in the domain u is ZERO, that is to say $|s| < \Phi$, we can state that f has the general form of:

$$f(s) = |s| + e_z de_z. \qquad (16)$$

More generally, $f(s) = |s| + \mu(\Delta_{uz}) + \beta$, where β is a residual term. The previous term is a function of the membership functions for u to be ZERO.

4. Design of A Fuzzy Controller. Analysis of Stability and Performance

4.1 Stability Proofs

If in the space of rules, there exists a separating surface and on one side u is positive and on the other side u is negative, with membership functions being disjoint for all variables, then there exist a surface where u=0. If u+ =-u-, then u can be given the form $u = \frac{su_+}{|s| + \mu(u_z) + \beta}$ for the area where u is ZERO, where s is the surface, $\mu(u_z)$ is the predicate for u is ZERO, β is a residual term. Outside the area $u = -u_+ sgn(s)$.

Proof: If u has the membership function ZERO control u is written as $u = \frac{\mu_+ u_+ + \mu_- u_-}{\mu_z + \mu_+ + \mu_-}$ (suppose defuzzification is centroid). There exists a surface where u is zero. The form above is also immediate. For other defuzzification algorithms, the same kind of result is expected.

Suppose from above that fuzzy control can be given the form $u = \frac{su_+}{|s| + \mu(u_z)}$.

s is a function of the membership functions. $\mu(u_z)$ is also a function of the membership functions belonging to the predicate u is ZERO. Then the condition of stability is $s\dot{s} < 0$, [5] outside the boundary layer. Inside the boundary layer, the system is stable, if the condition above is fulfilled (provided that in a boundary layer around zero, $\mu(u_z)$ is sufficiently high). Stability conditions depend only on u_+ and the importance of the u is ZERO domain.

We can try to give some stability conditions. If s is found to be linear, then we have s , and we can see this control the same as in (10), with a varying boundary layer.

We will now look for majorating and minorating conditions on the boundary layer so as to render our system dynamic.

4.2 Pole Assignment

From Slotine, a first way to assign fuzzy control dynamics is to choose membership functions so that a number l such as:

$|s| + \mu(u_z) = \lambda$.

should exist.

Then control u can be written as:

$$u = \frac{su_+}{\lambda} \quad (17)$$

Then, from [5], it is possible to represent membership functions as a first order dynamics so that the time constant is $\tau = \frac{u_+}{\lambda}$ and that the trade-off performance-robustness can be chosen. Thus control value u_+ plays an important role and is some sort of "gain" for the fuzzy controller.

If λ cannot be found, then it is important that "u is NON-ZERO" (i.e. the cell in the rule plane where a predicate different from u is ZERO - u is u_z - is given to u) and "u is ZERO" membership functions are not disjoint in the state space so as the denominator of u has a minimum that is non zero. We can then realize why membership functions should have a "good" crossing rate.

If we find 2 numbers Ψ and φ such that $\Psi<\lambda<\varphi$, then we can assume that:

$$\tau_1 = \frac{u_+}{\varphi} < \tau = \frac{u_+}{\lambda} < \tau_2 = \frac{u_+}{\psi} . \quad (18)$$

The dynamics will then be between the dynamics of 2 first order systems with time constants τ_1 and τ_2. These 2 constants depend of course on the membership functions of e and de.

Another way to assign pole dynamics is to consider the system as being :

$u = \frac{su_+}{|s| + \mu(u_z)}$ and then , the value $\mu(u_z)$ is considered as tuning the bandwidth of the system.

4.3 Dual Mode

Suppose now that some more predicates are taken, say without loss of generality u_{++} and u_{--}.

The same conditions for stability apply as above. Outside u is ZERO, control will be:

$$u = \frac{\mu_{++}u_{++} + \mu_+u_+ + \mu_{--}u_{--} + \mu_-u_-}{\mu_{++} + \mu_+ + \mu_{--} + \mu_-} . \quad (19)$$

Outside the zone u is u_+ or u is u_- , the control will be $u=u_{++}$ or u_{--}.

Membership functions should be taken such that the value of u decreases to taking some value when u is ZERO, if possible only depending on u_+ or u_-, so that the value of the control decreases for example from u_{++} to u_+. This realizes some sort of "dual mode", with a high gain depending on u_{++}, and a low gain depending on u_+.

More, there exists a method to ensure convergence:

suppose that above control in (19) is chosen so as to ensure asymptotic convergence and some "performance" (if assimilated to a first order, for example). The u is ZERO and u is u_{++} (and u_{--}) domains are chosen to be disjoint. In the u is ZERO domain, the expression of the control is (10). We can determine from [12] that the domain u is ZERO is an invariant domain for the system (1). The domain of invariance for control structure (1) is $\Delta_l = \Delta_{uz}$. We can now state from [7] , [8] and [9] that the resulting control :

u is (19) outside Δ_{uz} and

u is (10) inside Δ_{uz},

will be a stable one, and will have the dynamics of (10) in Δ_{uz}.

The controller will be globally uniform ultimate bounded stable and the desired dynamics can be chosen in Δ_{uz}: any "far" state will cross the surface Δ_{uz}, that is positive invariant and globally uniformly ultimate bounded stable.

The rules can be adjusted so as to slow down or accelerate the dynamics in some part of the state space.

5. Example

Let us take the following model:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ x_3 = -1.2x_2 - x_1 + u \\ y = x_1 \end{cases} \tag{20}$$

and $s=c^T X$ where $c=[1\ 10\ 0]$ and $X=[x_1\ x_2\ x_3]$.

We then define the variable structure control

$$u = 20\frac{s}{|s| + 0.01}$$

Let us note $e=y^c-y$, where y^c is the set-point.

This control structure is compared with fuzzy control defined with 5 membership functions for x and dx/dt, i.e. Positive (P), Small Positive (SP), Zero (Z), Negative (N) and Small Negative (SN). The same quantization is applied for control u, ++, +, 0, -, - -. The rules for the value of u are listed in the following Table :

e de/dt	N	SN	Z	SP	P
P	+	+	++	++	++
SP	-	0	+	+	++
Z	- -	-	0	+	++
SN	- -	-	-	0	+
N	- -	- -	- -	-	-

Figure 5. Fuzzy Lookup Table

The results are given below.

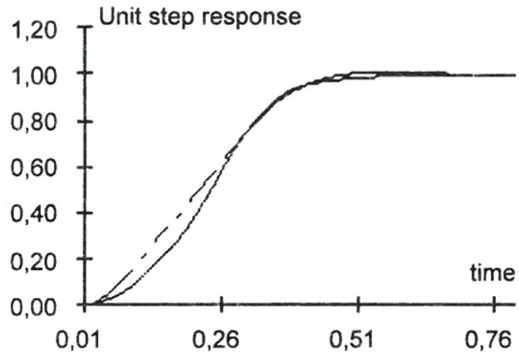


Figure 6. Unit Step Response for Fuzzy and vss Control

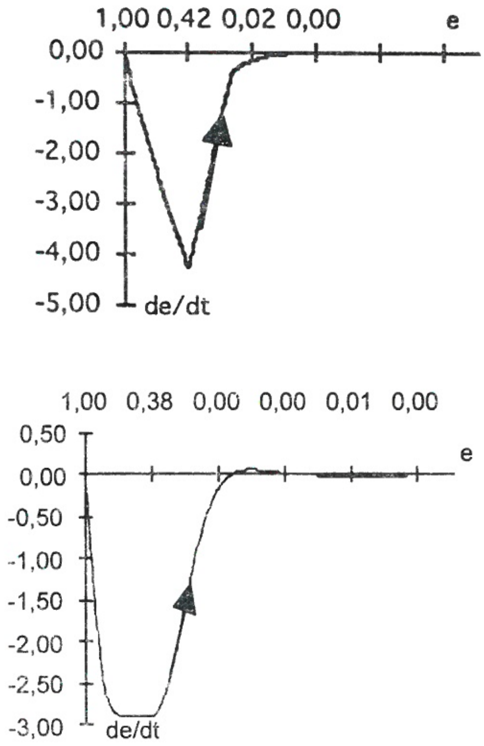


Figure 7. Comparison of State Space Trajectories for vss and Fuzzy Control

One can thus conclude that the vss surface is not linear when near the origin, because of the continuous aspect of the implementation. Fuzzy control trajectory is also not linear. Anyway, both can be approximated by a linear curve when not too near the origin. What seems interesting here is that fuzzy control is a bit smoother.

5. Conclusion

Some stability and performance conditions have been presented for fuzzy control. The control structure is represented as approximated switching control. Stability conditions have been based on the control output membership function values. Switching surfaces being nearly linear, it has been shown that the dynamics around this surface can be represented as a first-order, of which time-constant depends on the equilibrium between the surface value and the value of membership functions for the state space where the predicate ZERO is assigned to the control output.

It is therefore rather easy to develop a design for fuzzy control

- first design the surface with appropriate operators and input-output membership functions;
- then in a second time, design the output membership functions in order to ensure stability;
- finally, design the input membership functions to make pole-placement while respecting robustness with regard to parameter uncertainty.

This method, unlike many others, observes the principles of fuzzy logic which is to avoid the identification of an accurate model. Conditions for stability and performance can be found for coarse control, and fuzzy control tuning can be refined thereafter.

This synthesis method wants to highlight some of the well-known features of fuzzy control:

- it is robust, which is exactly the case for sliding motion, and yet not invariant;
- it shows the need for a good design of membership function, especially for crossing rate;
- it shows that, if the limit values of membership functions are well-chosen and the system is not "too bad" (without limit cycles for example), there is a great chance to have a closed-loop stable system;
- it also shows the importance of the phase plane (as in sliding motion);
- however, it shows the same conceptual drawback as sliding motion with smoothing algorithm: its structure of first-order filter for the bang-bang control means that the dynamics is addicted to be that of the low-pass filter (a high-pass filter in the feedback loop leading to better results); that is the reason for some of the criticisms to the "lack of reactivity" of fuzzy logic (indeed, fuzzy logic tends to be more conservative in the equilibrium area, and even more when the ZERO region is broad). Our approach can provide a good start for a performance-robustness approach which is in fact the crux of the matter in control theory.

Many research topics can be derived from that point:

- more precise conditions for particular cases;
- study of sliding motion with non-linear parameters;
- study of composite fuzzy-conventional controls, i.e. fuzzy control where the input variables are the successive derivatives of the tracking error of a closed-loop system (when a reference model is given).

REFERENCES

1. ESFANDIARI, F. and KHALIL, H.K., **Stability Analysis of A Continuous Implementation of Variable Structure Control**, IEEE TRANS. AUT. CONTROL, T-AC 36, No.5, 1991, pp. 616-619.
2. JAMSHIDI, M. , VADIEE, N. and ROSS, T.J. , **Fuzzy Logic and Control**, PRENTICE-HALL, 1993.
3. LEE, C.-C., **Fuzzy Logic in Control Systems. Fuzzy Logic Controller Parts I and II** , IEEE TRANS. SYSTEMS, MAN, CYBERNETICS, Vol. 20 , 1990, pp. 404-418 , 419-435.
4. UTKIN, V.I., **Sliding Modes in Optimisation and Control**, SPRINGER-VERLAG, 1992.
5. ASADA, H. and SLOTINE, J.J., **Robot Analysis and Control,** WILEY, 1986.
6. SLOTINE, J.J. , **Sliding Controller Design for Non Linear Systems**, INT.. J. OF CONTROL, Vol. 40, No.2, pp. 421-434.

7. BURGAT, C. and TARBOURIECH, S., **Stabilité et commande des systemes linéaires avec saturations, dans les systemes non linéaires, t2, stabilité-stabilisation**, MASSON, 1993.

8. VASSILAKI, M., HENNET, J.C.and BITSORIS, G., **Feedback Control of Linear Discrete-time Systems Under State and Control Constraints**, INT J. OF CONTROL, Vol. 47, No.6, 1988, pp. 1727-1735.

9. CHIANG, H.D., HIRSCH, M.D. and WU, F.F., **Stability Regions of Non-linear Autonomous Dynamical Systems,** IEE TRANS. AC, Vol. AC-33, No.1, 1988, pp.16-27.

10. SLOTINE, J.J., **Applied Non Linear Control**, PRENTICE-HALL, 1991.

11. YING, C., **Practical Design of Fuzzy Controllers**, AUTOMATICA, 1994, pp. 1185-1195.

12. AMBROSINO, G., CELENTANO, G. and GAROFOLO, F., **Variable Structure Model Reference Adaptive Control Systems,** INT..J. CONTROL, Vol. 39, No.6, 1984, pp. 1339-1349.

13. HUNG, J.Y., GAO, W. and HUNG, J.C., **Variable Structure Control: A Survey,** IEEE T-INDUST.ELECTRONIC, Vol. 40, No.1, 1993, pp. 2-23.

14. BRIERLEY, S.D and LONGCHAMP, R., **Application of Sliding Control To An Air-to-air Interception Problem**, IEEE TRANS. AEROSPACE SYST, Vol. 26, No.2, 1990, pp.306-325.

15. WEBER, S., **A General Concept of Fuzzy Connectives, Negation and Implications Based on t-Norms and t-Conorms,** FUZZY SETS AND SYSTEMS, No.11, 1983, pp.115-134.