A New Algorithm To Find The Best Paths in Intermodal Transportation Systems

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Abstract: An urban transportation network integrating public and private modes of transport is modelled as a Discrete Event System. The behaviour of the model adopted is reproduced by means of a traffic simulator designed on purpose. This paper centres around the functioning of the on-line information service included in the simulator, which is able to provide updated information about the best intermodal paths to reach any destination in the network. Such best paths vary according to the system dynamics, depending essentially on the traffic conditions. Examples of application of the special algorithm implemented to find the best multimodal paths are presented.
Keywords: transportation; urban systems; Discrete Event Dynamic Systems; algorithms; heuristics.

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1. Introduction

A continuously increasing need for efficient transportation services is experienced by anyone living and/or working in a metropolitan area. The satisfaction of such a need is strongly connected with the limitation of the 'private' traffic in the city centers, so as to prevent situations of severe traffic congestion. In turn, this is feasible only in cities where suitably working public transportation services exist. Such services should be widespread, efficient, and sensitive to the changes in users'demand.

To meet these requirements, it is basic to increase the attractiveness of the public means of transport with respect to the private ones. In doing so, researchers have centred in recent years around the integration between public and private transport services, which, in turn, leads to realizing intermodality in transportation. Intermodal transportation can be defined as the serial use of different modes of transport to move passengers and/or freight from a place to another. From the above definition two major research topics result, relevant to the movement of passengers and to the movement of goods (Kondratowicz, 1990; Guelat et al, 1990; Gedeon et al, 1993; Fernandez et al, 1994).

As for the movement of passengers, the realization of efficient intermodal transportation systems is especially intended to lead people not to use their cars to move in the cities, which would result in both an optimization of the travel times and a decrease in air pollution. In this paper, we consider some issues related to the integrated passenger transportation in urban areas. In doing so, first a suitable model has to be chosen. The peculiar characteristics that the considered intermodal transportation network presents, make it suitable to model it as a Discrete Event Dynamic System. In fact, discrete event modelling is appropriate for those systems in which significant changes in their state only occur at discrete time instants. In general, Discrete Event Dynamic Systems are discrete in time and space, asynchronous, and modular. Moreover, they may include control strategies and communication systems capable to make the event flow satisfy the project requirements (Ho and Cao, 1991; Cassandras, 1993). For the transportation system under study, a peculiar discrete event model is proposed. Due to the stochastic characteristics of the transportation system considered, some disturbances are also included.

The behaviour of the discrete event system modelling the transportation network is studied by means of a special-purpose simulation tool. Such a simulator is designed to perform two major functions, almost independent of each other. The first objective is the validation of integrated timetables for different modes of transport, so as to consider the various transportation services to be parts of a whole intermodal transportation system. In this sense, it is possible to evaluate the performance of the system, supporting the user in modifying adaptively a given timetable until reaching the desired performance.

The second objective is to give to the users of the intermodal transportation network some real-time updated information about the state of the network itself. The optimal situation to realize is that in which users can choose, when approaching the area covered by the transportation network, the best path to reach their destinations. Users could enter the on-line information system in the stations of the network, or in a more advanced realization, even from their cars. In particular, if a user approaches the network by car, he can evaluate where it is possible (and convenient) to leave his car and carry on with his journey by public transportation means. To this end, some intermodal stations have a parking area associated with. Such an on-line information system could lead to a fair distribution of the traffic on different (multimodal) paths and modes of transport, resulting in an improvement of the system performance. In this paper, attention is just focussed on the functioning of the information system included in the simulation tool. An evaluation tool which presents some analogy with the one here described, but simulating individual vehicle movement in urban networks to study alternative information supply and traffic control strategies, has been described by Jayakrishnan et al (1994).

The paper has the following structure. Section 2 introduces the considered problem, also giving a brief description of the model adopted to reproduce the behaviour of the intermodal transportation network concerned. Section 3 describes the novel part of the work, i.e. the functioning of the on-line information system. At any time, this information system is capable to provide a customer of the transportation network with information, based on real-time data, about the best multimodal paths to reach his destination, starting from the point where he currently is. The 'quality' of a path is evaluated based not only on its travel time but also on the changes of modes of transport required. The users' requests are satisfied by using a special algorithm, essentially consisting of an ad-hoc modified version of the Dijkstra algorithm (Christofides, 1975). If a user requires to know more than one best multimodal path to reach his destination, the k, $k>1$, best paths required are determined by applying the method proposed by Yen (1971). In Section 4 experimental results relevant to a case study are presented and discussed, showing how the peculiar algorithms implemented behave. Some conclusions end up the paper, also providing some insight into possible further developments of this work.

2.Description of the Operating Framework

As said in the Introduction, a workable approach to achieve the co-operation among the public transport services and private traffic in an urban area is that of considering the whole transportation network to be an intermodal transportation system. In particular, the modes of transport present in the considered system are underground, railway, bus, and private means of transport. As for private transport, we think especially of cars and vehicles similar to cars in their behaviours, since vehicles like motorcycles do not affect significantly the traffic conditions, nor usually give their drivers troubles about parking. For this reason, hereafter we often mean by the term cars all private traffic.

An urban intermodal transportation system of this kind can be represented as an oriented graph, in which the nodes are stations where it is possible to switch from one mode of transport to another. Its fundamental elements are nodes, macronodes, and links. A *node* is a station for a single mode of transport, and it can only exist as a part of a macronode. A *macronode* is an intermodal station, that is, a place where people can enter/leave the transportation network, or change mode of transport. Therefore, a macronode is composed of one or more nodes. A *link* is a unidirectional path which connects two macronodes, and is devoted to a single mode of transport. In this framework, the links which connect the macronodes in the network are divided into three classes: a) bus links; b) railway links; c) underground links. For the sake of simplicity, and without any significant loss of generality, it is assumed that cars use the same links as buses to move in the network. There is also a special class of links, named inner links, which connect two nodes in a macronode and can be traversed only on foot. These links play an important role when evaluating the time elapsed to change the mode of transport in some macronode.

The transportation network above described is modelled as a Discrete Event Dynamic System (Cassandras, 1993) in which some stochastic quantities are introduced to model the nondeterministic nature of the system. We refer the reader to (Di Febbraro et al, 1994, 1995) for a detailed description of the discrete event model. A major module of this tool is the passenger information system, around which the paper centres.

At any time, the passenger information system can receive a request from a user who needs to move from a macronode to another macronode of the network. Then, using real-time data, it provides information about the best multimodal paths between the indicated couple of macronodes, according with the user's preferences given a priori. The algorithms ruling its functioning use both static information about the topology of the network and the travel times on the inner links, and dynamic information such as the link travel times and the expected waiting times at the nodes.

3. The Algorithms To Find the Best Multimodal Paths

In this section, the algorithms which the on-line information system is based on, are described. The very basic algorithm is an adhoc modified version of the well-known Dijkstra algorithm to find the shortest paths in an oriented graph (Christofides, 1975). Before describing in detail the proposed algorithm, it is worth introducing some notation. The transportation network is modelled as an oriented graph $G = \{S, L\}$, where S is the set of macronodes of the network, card $(S) = N_s$, and L is the set of links of the network, card $(L)=N_1$. The origin macronode of a link is identified by the function $O:L\rightarrow S$ such that $O(1)=s, 1\in L, s\in S$, means that macronode s is the origin macronode of link 1. In an analogous way, function $D:L\rightarrow S$ identifies the destination macronode of a link, i.e. $D(1)$ represents the destination macronode of link 1.

As described above, the links are classified depending on the mode of transport to which they are devoted; let M be the set of the modes of transport present in the considered network and card $(M) = N_m$. In the considered case, $N_m=4$ and $M=\{B,R,U,C\}$ where B, R, U, and C stand for bus, railway, underground, and cars, respectively. Moreover, let $M: L \rightarrow M$ be the function such that $M(1)=m,\ 1\in L,\ m\in M$ means that link 1 is devoted to mode of transport m.

Using the variables introduced so far, the topology of graph G is defined by means of an adjacency matrix A, $\dim(A)=N_s x N_s x N_m$ whose elements are:

$$a(i,j,m)=\begin{cases}1 \text{ if } \exists\ 1 \in L{:}O(1)=i, D(1)=j, M(1)=m \\ 0 \text{ otherwise}\end{cases}$$

$$\forall i,j \in S,\ m \in M$$

Other matrices represent the travel times on the links and the waiting times in the nodes. Let T, $\dim(T)=N_s x N_s x N_m$, be the matrix containing the time-varying travel times on the links; if $\tau_{i,j}, i,j \in S$, denotes the present travel time on link $l, l \in L$, $O(l) = i$, $D(l) = j$, the generic element of T can be written as:

$$t(i,j,m)=\begin{cases} \tau_{i,j} & \text{if } a(i,j,m)=1 \\ +\infty & \text{otherwise} \end{cases}$$

$$\forall i,j \in S,\ m \in M$$

Then, let IL denote a matrix, $\dim(IL)=N_s x N_m x N_m$ whose generic element $il(i,m,n)$, $i \in S$, $m,n \in M$, represents the travel time on the inner link within macronode i, and connects the node (station) of mode m with the node (station) of mode n. Of course, $il(i,m,m)=0$. The generic element of matrix WT, $\dim(WT)=N_s x N_m$, namely $wt(i, m)$, $i \in S$, $m \in M$ represents the time to wait presently in macronode i before a means of transport of mode m arrives. Last, to model the time-varying availability of places in the parking areas associated with the nodes, a vector P, $\dim(P)=N_s$ is introduced, and its generic element is defined as

$$p(i)=\begin{cases} 0 & \text{if there is no place in the parking area at node i} \\ 1 & \text{if there is at least one place in the parking area at node i} \\ +\infty & \text{if there is no parking area associated with node i} \end{cases}$$

$$\forall i \in S$$

If macronode i is associated with a parking area, the value of p(i) is randomly determined by taking into account the probability of finding free places in it. Such a probability varies in dependence on macronode i and on the time of the day. It is worth noting that only matrices A and IL have constant elements. Matrices T and WT are updated in real time based on the data provided by the simulation kernel, whereas the components of vector P are stochastic variables.

As said above, the algorithm which solves the problem of finding the shortest path from an origin macronode o to a destination macronode d, $o,d \in S$, considering only public means of transport, is derived from the Dijkstra algorithm. The main difference between the classical version of Dijkstra algorithm and the proposed one lies in the cost function associated with each link. Here, such a cost not only depends on the origin and destination nodes of the link, but also on the mode of transport currently used. Then, the cost associated with link $l, l \in L$, can be expressed by a function $c(i, j, m)$, where $i = O(l)$, $j = D(l)$, and $m \in M \setminus \{C\}$ is the mode of transport used to reach node i, taking on the form:

$$c(i,j,m)= \min_{n \in M \setminus \{C\}} \left\{ t(i,j,n)+il(i,m,n)+wt(i,n) \right\}$$

The basic version of the algorithm has been complicated to include some further possibilities offered to users, who can choose among:

i) the number k, $k \geq 1$, of best paths to find between the indicated pair of nodes;

ii) the maximum travel time T_{max} of the path from node o to node d, i.e. a user can fix the maximum time he is willing to spend to reach his destination;

iii) the modes of transport utilized, i.e. a user can choose which modes he prefers to use to reach his destination.

It is apparent that the introduction of the above possibilities for the user results in additional constraints to be imposed on the solution of the optimization problem. If only point (i) is specified, the problem to solve is that of finding the k best paths to move in the network from node o to node d. We solve such a problem by applying the method proposed by Yen (Yen, 1971), based on the modified version of Dijkstra algorithm that we have proposed above. Almost the same holds if also the maximum travel time is specified. In fact, the structure of the problem to solve is left essentially untouched with respect to the previous case. Only an additional constraint on the travel time has to be taken into account, so that only those paths with overall travel times less than T_{max}, if any, should appear in the list of paths given as an output.

As for decisional aspect (iii), the possibility for the users of neglecting some mode of transport does not lead to a trivial complication of the problem. Actually, if a user chooses to eliminate one or more modes of transport from his path, the resulting graph can become not connected. If so, we would not be able to solve the problem using the algorithm proposed so far, as the Dijkstra algorithm only applies to connected graphs. To deal with this aspect, a special procedure has been implemented to check the type of structure of the graph and act consequently. If the graph does not turn out to be connected, such a procedure removes from it the isolated macronodes. Once a connected topology has been reached again, the algorithm to apply is the same as above, the only difference consisting in the fact that the set of possible modes of transport to consider is now only a subset of M.

The algorithms described so far only work on the public transportation network, without taking specifically into account the presence of private traffic. The primary aim of our work being the integration of the public transportation services with the private traffic, we state now a heuristic algorithm which serves this purpose. It is assumed that this algorithm is run to asnwer a question posed by a user approaching a macronode of the network travelling by car. In fact, the algorithm searches for the best multimodal paths in which the first part (possibly the whole path) has to be covered by car, if any such a path exists.

Heuristic Algorithm

Given an origin node o and a destination node d:

1) define the set of nodes $S' = \{s \in S, s \neq 0$ and $p(s) = 1\}$;

2) $\forall s \in S'$, find, by applying the Dijkstra algorithm on graph $G' = \{S, L'\}$ where L'is the set of links relevant to mode C, the shortest path from macronode o to macronode s and, then, define vector T_c, $\dim(T_c) = \mathrm{card}(S')$ such that $T_c(s)$, $s \in S'$, is the travel time on the path determined;

3) $\forall s \in S'$; find, by applying the modified version of the Dijkstra algorithm described above, the shortest multimodal path from s to d; then, define vector T_{mm}, $\dim(T_{mm}) = \mathrm{card}(S')$, such that $T_{mm}(s)$, $s \in S'$ is the travel time of such a path;

4) determine the best path to move from o to d, by choosing macronode $i \in S'$ such that:

$$T_c(i) + T_{mm}(i) = \min_{s \in S'} \left\{ T_c(s) + T_{mm}(s) \right\}$$

The above algorithm can easily be combined with options (i), (ii), and (iii) described above.

4. Experimental Results

In this section, some experimental results showing the effectiveness of the heuristic algorithm proposed in the previous section, are presented and discussed. The considered case study is relevant to the transportation network depicted in Figure 1.

Such a network is composed of 80 links and 21 macronodes, each of which consists of at least two nodes. In particular, in the network there are 21 bus stations, 13 railway stations, and 11 underground stations. In order to evaluate its behaviour with reference to the case study, the algorithm, which has been implemented in C programming language, has been run with many different input requests. Three significant examples are reported hereafter. The first example consists in a request to find the four best paths to move from macronode 6 to macronode 19 using only the public means of transport. The output of the algorithm is shown in Table 1, in which, for each path found, the following data are reported:

i) the total travel time (TTT) in seconds;

ii) the sequence of the macronodes which compose the path; the nodes where the mode of transport changes are followed by the indication of the new mode adopted;

iii) the number of macronodes (NM) in the path;

iv) the number of mode changes (NMC) in the path.

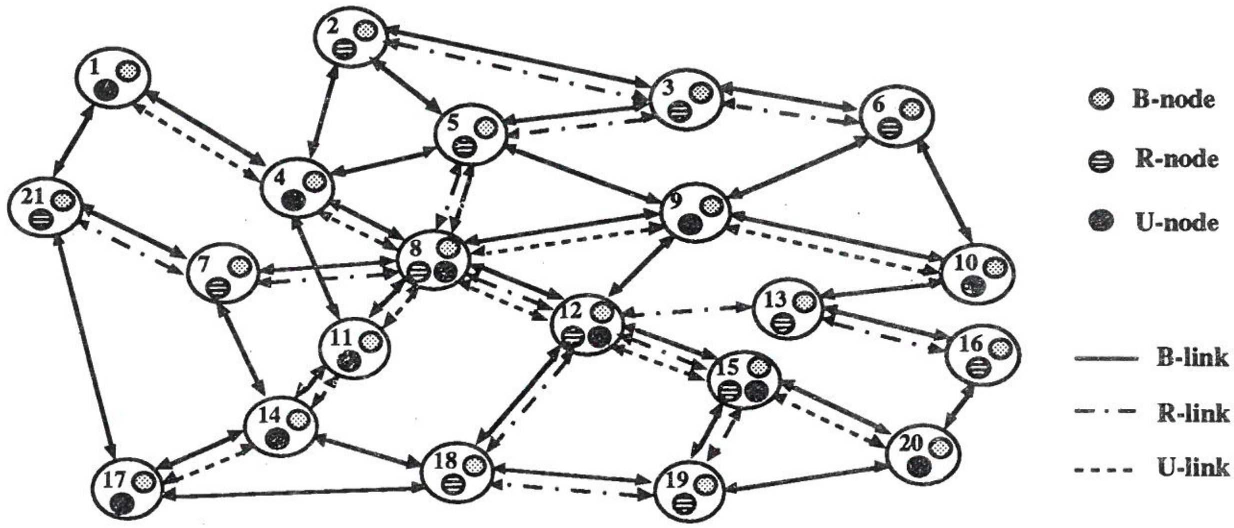

Figure1. The Considered Intermodal Transportation Network

Table 1

	TTT	PATHS	NM	NMC
1	900	6-R-3-5-8-U-12-15-R-19	7	2
2	960	6-B-9-U-8-12-15-R-19	6	2
3	1020	6-R-3-5-8-U-12-15-B-19	7	2
4	1140	6-B-3-5-8-U-12-15-R-19	7	2

The second example refers to a request again regarding the movement from macronode 6 to macronode 19, but this time the use of cars, as well as public transportation facilities, is allowed. The resulting paths are presented in Table 2.

Table 2

	TTT	PATHS	NM	NMC
1	900	6-R-3-5-8-12-U-15-R-19	7	2
2	960	6-C-9-U-8-12-15-R-19	6	2
3	960	6-B-9-U-8-12-15-R-19	6	2
4	1020	6-R-3-5-8-U-12-15-B-19	7	2

Note that in this case there is only one path (path 2) which comprehends the use of private cars, and it has a total travel time very close to the optimum one, as determined by applying the Dijkstra algorithm.

As a third example, we report the case in which the three best paths to move from macronode 2 to macronode 20, with a maximum travel time of 800 seconds and by using only public transportation services, have been asked to the program. The solution is given in the following Table.

Table 3

	TTT	PATHS	NM	NMC
1	720	2-B-5-R-8-U-12-15-20	6	2
2	780	2-B-4-U-8-12-15-20	6	1
3	780	2-B-5-R-8-12-U-15-20	6	2

It is worth noting the importance of the information about the number of changes of modes of transport to be made along a path. In this simple case, the user would probably choose the second path found, because it presents a total travel time very close to the optimum one, and a number of changes of modes less than the one requested by the best path determined.

5. Conclusions

The problems related to the integration of public and private means of transport in urban areas have been dealt with in this paper. The resulting intermodal transportation network has been modelled as a discrete event system, whose behaviour has been studied by means of a special-purpose stochastic discrete event simulator.

The paper focussed on the functioning of the on-line information service included in the simulator, which is able to provide updated information about the best intermodal paths to reach any destination in the network. Such best paths are time-varying as they follow the system dynamics, i.e.the traffic conditions. The heuristic algorithm used to find the best paths, including not only public modes of transport but also private ones, is based on the Dijkstra algorithm and the method proposed by Yen. Examples of application of this special algorithm to best multimodal paths were presented and commented on.

The usefulness of the proposed approach can be improved. In fact, whenever the passenger information system receives a request for a best multimodal path from a user, it provides this path based only on the present state of the transportation network. Actually, this yields good results only if the network is not too big, and the traffic has slow dynamics. Work is in progress to extend the proper applicability of the proposed heuristic algorithm by finding a good technique to estimate the future values of the variables under consideration.

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