Model Based Predictive Control for Managerial Planning in A Composite Marketing-Production Problem

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Abstract :This paper treats the managerial planning problem of a composite marketing-production process in an uncertain environment through the Model Based Predictive Control (MBPC) approach. For completeness, a review of the literature concerning the application of stochastic control methodologies to dynamic operational research and managerial planning problems under uncertainty is first provided. Next, an outline of the Model Based Predictive Control methodology is given including all key elements required for our study. The composite marketing-production problem that involves uncertainties is then fully formulated. The MBPC approach is tested through simulation in a number of nontrivial example cases. In all cases the performance of the resulting controlled system was shown to be excellent a fact that verified our original feeling about the suitability of the MBPC method for complex managerial decision making.

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1. Introduction

The dominating problem of production management is uncertainty. Uncertainty about market demand, uncertainty about the current status of the production and inventory systems, uncertainty about the cost -effectiveness of production-inventory control policy in meeting demand, uncertainty about the effectiveness of advertisement expenditures for the firm's product promotion and so on. One possibility to allow for management decisions under such uncertainty conditions is to introduce stochastic elements in the problem. There are at least two ways to do this : (i) use a known probability distribution function as state variable within a deterministic optimal control problem [1-3] (the difficult task in this approach is to formulate the model so that it can be solved within a deterministic framework), (ii) use a stochastic/adaptive control approach introducing the uncertainty explicitly into the equations of the process. The problem becomes more difficult if one considers that the parameters of the model are also unknown. In this case, the computations become very complex and so approximation techniques have to be employed [4-5]. The problem considered in this work falls within this framework.

Consider a firm that wishes to determine joint decisions of how much to produce and how much to spend on advertising, so that total cost is minimized while operational constraints and management goals are satisfied. This problem is solved here using the technique of Model Based Predictive Control (MBPC) which is well suited for such an environment.

In Section 2 the relevant literature on applying stochastic control methods to Operational Research problems is reviewed. In Section 3 the key ideas of MBPC are briefly presented within a framework that satisfies the needs of managerial planning under uncertainty. In Section 4 the joint problem of production-advertisement control is presented and in Section 5 its application is demonstrated through a set of simulated examples for some cases. Finally, some concluding statements are given in Section 6.

2. Literature Review

Although the mathematical theory of stochastic systems has started its development at the beginning of our century in Economics and Management Science, deterministic models have prevailed until recently. Therefore it is not astonishing that management applications of stochastic control theory are relatively few. Some surveys and further references can be found in [4-8].

Regarding the marketing (advertisement) process, Tapiero (1977) gives some examples of how to use stochastic control theory [9]. He considers (1975) [10], a stochastic version of the Vidale - Wolfe advertising policies using filtering theory. Open-loop advertising strategies, with the filter equations as certainty equivalents, are used as approximations in determining the optimal strategy. Later on (1978) [11], he presented feed-back type policies and open-loop policies which involve goodwill. Furthermore, Tapiero (1982) [12] ,compares optimal advertising policies under two different objective functions, namely expected profit maximization and constant risk aversion utility, and shows that adopting the latter means that there will be less advertising.

An example for a solution of a discrete-time advertising problem using stochastic dynamic programming is provided by Monahan (1983) [13]. Another variant of the Vidale-Wolfe model is optimized under a quadratic cost function by Sethi (1983) [14]. He compares a deterministic version of this model to a stochastic one, where an additive white noise process is present in its dynamics. He shows that for a class of variance functions of the stochastic process ,the maximum value of the objective function as well as the form of the optimal feedback advertising control are identical in the deterministic and the stochastic environment. This result can be seen as an extension of certainty equivalence beyond the LQ case.

However it is the problem of production planning that has been addressed extensively in the literature in an assortment of its stochastic variations. Simon (1956) and later Holt et al (1960) [15] considered the certainty-equivalence principle for a linear system with a quadratic criterion, and examined various applications in production and workforce scheduling. Schneeweiss (1971,1974,1975,1977) [16-19], studied several non-linear cost functions (criteria). In particular he considered piecewise linear cost functions including set-up costs. Assuming a linear feedback control, stationary policies and normality, he approximated the non-quadratic costs with quadratic ones such that the best linear policy with respect to the original costs, is obtained. This approach is essentially maintaining the certainty equivalence principle and turns out to be more efficient than the more common technique of postulating certainty - equivalence (and in this way reducing the stochastic problem to a deterministic one). This is also the case if safety stocks are introduced in the latter approach.

Using the approach of Schneeweiss, Gaalman (1976) [20] considered the general multi-dimensional case. He derived necessary conditions and suggested methods for solving them. Approximation of optimal policies by LQG approach is used by Baetge and Fischer (1982) [24], to determine the control policy for a linear dynamic production, inventory and pricing model, where unknown future demand varies stochastically and has to be estimated. Here overlapping plans are modelled by recomputing the optimal decision rule periodically, using an ARIMA model to predict the exogenous variable.

O'Grady and Bonney (1981) [22] have studied a production system affected by random variations in the manufacturing process. The production system includes several workstations and it is not possible to obtain measurements at each station. Their paper is related to some other works of Koivo and Hendricks (1973) [23], and Stohr (1979) [24]. The above work of Koivo and Hendricks is among the very few works that consider marketing variables (namely advertising expenditures) explicitly into the production process and determine optimal decisions.

A simple but instructive example of how to apply stochastic differential equations (Ito calculus) to the optimal control of a production-inventory model is given by Sethi-Thompson (1980) [25]. The production of one good with inventory is considered. The demand rate is constant but inventory changes are distributed stochastically. The problem is to minimize a quadratic objective function over time,which includes terms of deviations of the production and inventory levels from their factory-optimal policies. It is shown that if the inventory level is very high, optimal control may be negative, which amounts to disposing part of it, to save holding costs. Necessary and sufficient conditions for this type of policy are derived. An explicit consideration of a non-negativity constraint for the production rate enormously complicates the stochastic optimization problem.

Another interesting application of continuous time stochastic optimal control theory using impulse control, has been examined by Vickson (1982) [26]. The problem here is to obtain average cost minimizing policies for a sequential production model with random variation in completion times of successive jobs in the long run. Jammernegg [27] has investigated another stochastic dynamic production-inventory model with incomplete information about demand.

Also the applicability of linear-quadratic stochastic control theory to production and inventory control has also been illustrated by Bensoussan (1974) [28], Mohapatra and Sahu (1977) [29], Willke and Miller (1978) [30]. Kleidorfer and Glover (1973) [31], proved some general results for linear convex stochastic control problems and discussed their impact on production planning. Baker and Peterson (1979) [32], and Baker (1981) [33] used a discrete linear-quadratic model for evaluating rolling schedules.

Stochastic control theory is also particularly useful for dealing with problems of the information flow within the firm. The question of information control for inventory monitoring as a combined estimation-identification problem has been addressed by some researchers. The interesting point is that the value of information itself enters the objective function. This aspect becomes explicit in the works of Deissenberg and Stoppler (1982,1983),[34-35], where information gathering can be influenced by management. Costs of observation need to be taken into account. They formulate a stochastic optimal control problem of the LQG type and show that a separation property holds for the closed -loop optimal control policy.

The above list of works is by no means exhaustive but simply refers to the most important works in the field. Recently authors have pointed out the importance and usefulness of using MBPC as a decision making tool for production-inventory planning problems [36]. The present paper extends our previous results to considering the composite production-marketing problem and demonstrating its effectiveness through simulated examples. In the following the basic ideas of MBPC are given and its functionality as a decision tool within the framework of managerial planning is outlined.

3. Model Based Predictive Control

Model Based Predictive Control (MBPC) is a control strategy based on the explicit use of a process model to predict the process output over a long-range time period. The key elements characterizing MBPC can be summarized as follows:

- Prediction model. It is the model by which the real output of the plant/process is predicted for a time period of T samples. This is an internal model "running" in parallel with the real one which is supposed to be partially unknown.
- Reference Trajectory. The purpose of the control is to lead the output vector along a desired and generally smooth path r(t) to a final set point d(t). Such a path r(t), is called reference trajectory. In many cases it is assumed that the future desired process output is unknown. If the set-point is not prespecified in the future one can use a predictor in order to predict the desired trajectory that the output of the system is forced to track. This is a very important feature for handling complex managerial goals that may vary dynamically as a response to some unpredictable events.
- Structuring the control law. The predicted output $y_p(t+j/t)$ j=1,...,T depends on the postulated control input u(t+j) j=0,1,...,T. An essential task of the prediction procedure is the calculation of the prediction errors. An effective prediction error plays the vital role of self-compensating the cumulative effect of model mismatches, external disturbances and additive noise that deteriorate the real process operation. A very flexible and efficient way was pointed out in [37] for handling this problem. Having the predicted values of y_p the MBPC algorithm is trying to minimize the distance between $y_p(t+j)$ and r(t+j). Whatever form of performance criterion is involved, one has always to minimize a function of the T independent future control variables. So the number of calculations required to solve the problem depends on the length of the planning predictive horizon T. This is one reason for putting some structure into the control scenario u(t+j). Control structuring essentially means reducing the number of degrees of freedom by specifying a priori some relationship among the future control variables. This can be done mainly by using a control horizon ($T_u < T$) which reduces the dimension of the control vector. In addition, "structuring" has been found to enhance the robustness and the performance of the control system.
- Algorithmic Calculation. Generally, the objective function can be of any form. However in our case (as it is always the case in the literature) the objective is of the quadratic form and contains three terms : (a) deviations of the output from a nominal trajectory, (b) magnitude of the control signal, and (c) differences of the control signal at time k, minus control value at time k-1. Weight matrices are adjoined to each of the above terms that can be viewed either as design parameters or as cost elements. Their choice affects the accuracy, robustness and dynamic performance of the plant output.

Solving at time t the resulting equations, one determines all elements of the control signal. However only the first one is actually applied to the system. At time (t+1), as new measurements are obtained, the whole procedure is repeated, after an appropriate backward time shifting, and a new control input is determined. This is the well-known receding horizon strategy,, a common feature in managerial planning. In fact if there are no constraints on control or state space variables, the MBPC approach is equivalent to the classical LQG which is the most frequently encountered technique in the literature.

Given the key ideas of MBPC let us now clarify some points and discuss the framework within which it can be used as a decision making tool. As we have argued above, MBPC closed-loop control yields a linear feedback decision rule. Given that as in many real systems the production management process is expected to be subject to noise and/or non-linearities (stemming from simplifying assumptions), the appearance of feedback terms is shown to attenuate all these

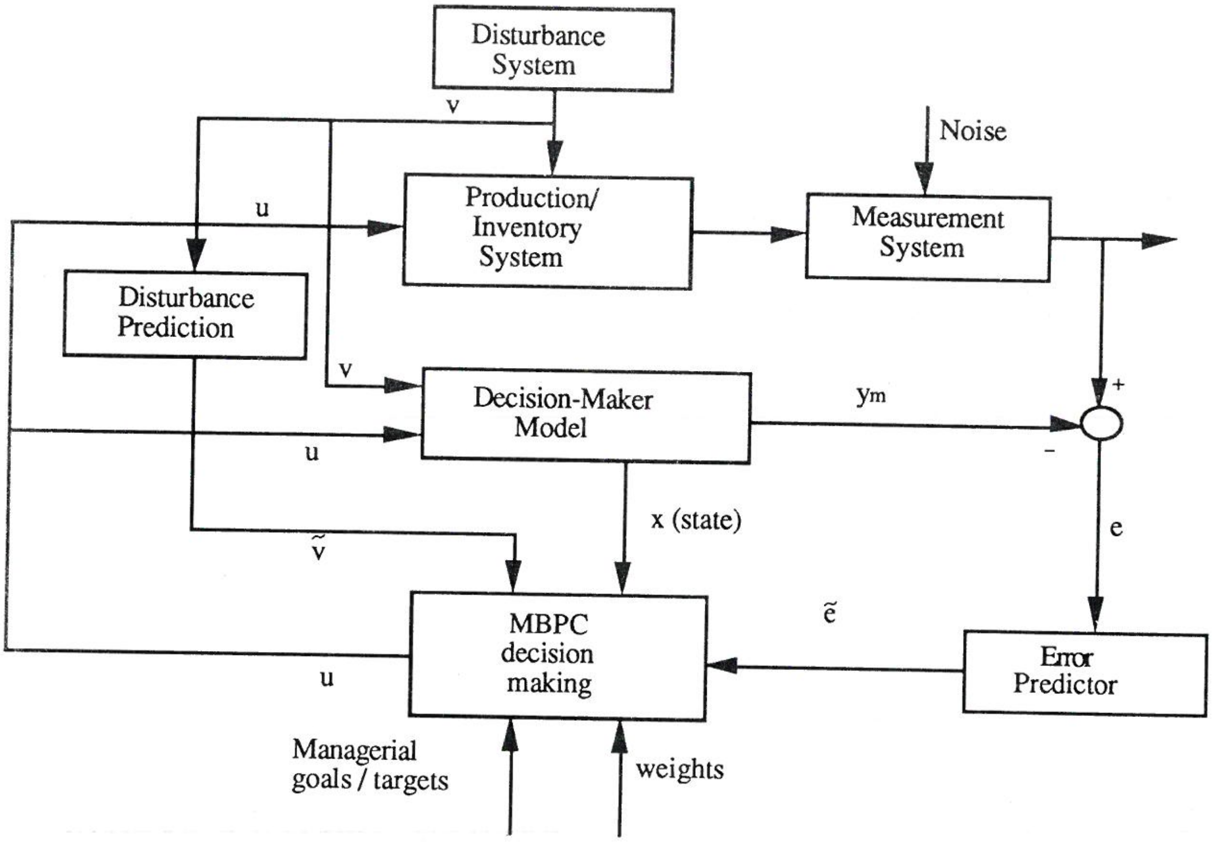

Figure 1. MBPC Decision Making Scheme

factors and thus it is very desirable. Another key feature is the requirement for feedforward control action counteracting mainly the disturbances effect. For managerial planning this notion is considered to be of equal importance, since it represents the activity of forecasting, which is viewed as a common element of most managerial jobs. For the decision makers ,planning, and hence forecasting, is fundamental to the management of the enterprise; often the role of feedback is less immediately obvious to them. In practice however forecasting methods usually involve considerable errors, hence the recognition in the Management Science of the need for control systems that can monitor performance in the light of plans, and adjust as necessary. MBPC introduces all these concepts in a straightforward manner providing a great deal of flexibility to the decision maker. The functional structure of the MBPC approach is shown in Figure. 1. The decision maker takes action based on his (her) model description of the process. It is highly unlikely that this model will be an exact model of the process. Optimal/stochastic control methods are very sensitive to such model mismatches. The MBPC approach is proven to be efficient even with these mismatches, mainly through the corrective action of the error predictor. More than that, even if the model is under- or over-parameterized, something very common in Management Science, MBPC can produce satisfactory results. This is very important since in inventory-advertisement process there are distributed time lags which are rarely known exactly. For such difficult dynamics this approach can constitute a very effective tool. The same is true if non-linearities exist in the real process while the decision maker model is assumed to be linear. In fact, this is one of the most powerful arguments to use this approach, and will be demonstrated through several simulation examples.

The information available in every time period is used in a consistent manner to continuously update all predictors involved in the control scheme. It is the philosophy of control that makes it superior to such problems. Disturbance prediction is used in other approaches too. Then problems of shorter time periods are solved. This idea is best exploited in the MBPC scheme. The flexibility offered by this approach is reflected by the fact that we can handle uncertain set-points too. This is of fundamental importance for managerial planning where the management rarely knows well in advance the ideal functionality point of the enterprise, due to unpredictable exogenous events. The decision maker has the option of handling some uncertainty in the reference signal and can take action in advance, so that he can smooth out these sources of difficulty.

In the next section we are dealing with a composite production marketing problem for a single product, and highlight some of the key-points mentioned in the previous discussion.

4. A Composite Marketing-Production Problem

Marketing and production policies in a firm are interdependent. Marketing policies are normally designed to generate demand for the firm's products, whereas production policies are usually designed to meet that demand. However, although marketing and production policies are interdependent, most of the marketing and production models assume that the decision making in the two areas is separate [14-15, 38-43]. It is only in the recent years that models incorporating interdependencies between the two areas have been proposed [23, 44-47]. Most of these works assume that the inventory level at any time responds immediately to the difference between the production and sales rate. But some products are depleted not only by sales but also by decay, such as spoilage in fruit, physical depletion in highly volatile liquids or deterioration in electronic components. Also advertising outlay has effects on sales that carry over into future periods, and thus there exist some time delays between advertisement and increases in sales rate. Inherited to this already highly complex problem is the uncertainty introduced since neither the exact values of the parameters involved are known nor the acquired measurements are free of errors. Koivo has studied a stochastic model incorporating an arbitrary relationship of sales advertising response. However the decision maker is still assumed to have at his disposal the exact values and time delay of the marketing model (which is the most difficult to be estimated). All the other works treat the same problem within a deterministic framework except for [44], where distributed time lags are assumed for the marketing model and the duration of the customer's memory of the product, generated by advertisement, is a random variable (generalization of the Vidale-Wolfe model).

Let us now formulate our problem. We assume that a firm produces a product and then,, using advertisement strategies, tries to promote and distribute it to end-customers. The price is fixed and the firm influences the potential market by using appropriate levels of advertisement. The underlying sales-advertisement is not exactly known, but assumed to have the following form :

$$s(k+1)=(1-\lambda)s(k)+\sum_{i=0}^{m}a_i A(k-i) \quad (1)$$

where λ is the decay of the sales or forgetting coefficient, a_i is the efficiency of advertisement at time k-i, m is the time lag introduced in the process, s(k) is the sales level, and A(k) is the advertisement effort.

The advertising response models that have been built over the last years fall in two general classes [42-43]: (i) a priori models that provide a more conceptually sound set of characteristics, and (ii) econometric models that are better related to available data [48-50]. The model above (1), falls within the second category, but it can be used as a normative one for some range of the life cycle of the firm's product. Note that although we assume "some" econometric estimation or calibration of the model, it is preasumptuous to claim exact values of the parameters involved in the process.

To demonstrate the flexibility of our approach in the simulation examples, we investigate a case where although the real process follows a non-linear form, namely the Vidale-Wolfe model, the decision maker still employs a linear model. It is shown that after some "learning" period the MBPC approach performs well.

The production department is modelled by the well -known inventory balance equation :

$$I(k+1) = (1-r)I(k) + \sum_{i=0}^{n} r_i p(k-i) - s(k) \qquad (2)$$

where I(k) is the inventory level at time k, r is the inventory decay coefficient, p(k) the production capacity, r_i is a parameter denoting which part of the production enters the warehouse due to possible production delays. Without affecting the generality one can assume $r_0=1$, $r_i=0$ for $i \neq 0$ as it is done in most models in the literature.

The objective of the firm is to achieve a desired sales level assigned by some central authority and to keep a specified inventory level while using minimum advertising and production effort. The costs involved are assumed to be quadratic (although they can have any other non-linear form with an increase in the computational load) and the initial values of the problem variables are assumed as known.

The mathematical form of the objective function is :

$$J = \sum_{k=0}^{k_f} h(x(k)-x_d)^2 + rp^2(k) + \qquad (3)$$

$$+ w\left(s(k)-s_d\right)^2 + dA^2(k)$$

where x_d and s_d are the desired inventory and sales level respectively, m, r, d, are appropriate unit costs and h,w are coefficients which are chosen by the management depending on how severe are the desired sales or inventory level achievement (or it can be regarded as a cost parameter expressing unfulfilled/lost sales, market share, in equivalent cost).

An alternative cost formulation is to introduce into the objective function the changes of the production control variable, a feature that is known in the production planning literature as "production smoothing problem". This alternative is also examined through a simulation example in the next section.

Often, management receives information concerning the values of the states of the system only at discrete time instants (e.g. weekly) and the management decisions are applied at discrete time instants too. Frequently, at the time an optimal decision is to be made the measurement of some of the states is not available or it is inaccurate. In order to include such a possibility in our model, the relation between the observations and the state values is described (in vector form) by :

$$y(k) = z(k) + w(k) \quad , \quad z(k) = (x(k), s(k))^T$$

where y(k) is the observed output of the system inventory level and sales level, and w(k) is the measurement error that can be approximated by white Gaussian noise. Note that such a white noise can be present at the state equations (1) and (2) of the system describing random changes during successive transitions of the production marketing process.The management may use some previous historical data to get " a first idea" of the model parameters and then to use it for decision making process. The block diagram of the process is given in Figure 2.

Note that in our case we have disregarded autonomous demand which cannot be controlled by advertisement strategy. One can use existing forecasting techniques to identify its level and consider it in the design process. However, for our purposes it is assumed that either this segment of the market is too small or can it be identified efficiently by some appropriate method.

5. Simulation Examples

Extensive simulation studies have been carried out to demonstrate the effectiveness of the MBPC approach. The results are presented in Figures 3 through 11. In all cases zero initial conditions (start-up of the operation) are assumed and use was made of the constrained MBPC version, in order to take into account physical constraints on the control variables of the problem. The lower bounds were set to zero while upper bounds are 200 and 100 units for the production and the advertisement control variables, respectively.

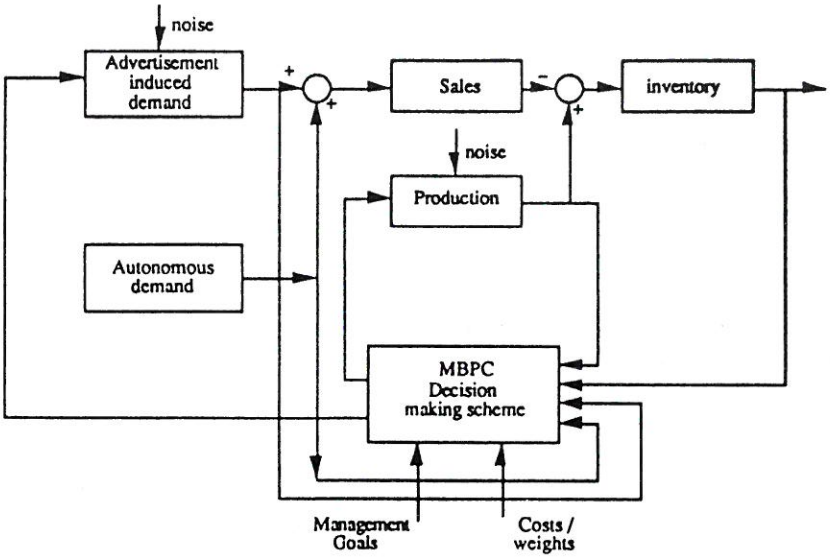

Figure 2 . Block Diagram of Production-Marketing System with MBPC Scheme

In Figure 3 exact model measurements are assumed i.e. there is no noise corrupting the process, and only a slight misspecification of the decaying factor _ of the marketing model exists. The real value is 0.02 while the decision maker uses its estimated value 0.03. The objectives of the process are attained without any difficulty, besides the previous mismatch. All other parameters are identical. The form of the employed model is :

$$s(k+1) = 0.98\, s(k) + 0.1\, A(k) + +0.01\, A(k-1) + 0.001\, A(k-2)$$

$$I(k+1) = 0.97\, I(k) + p(k) - s(k)$$

In Figure 4 the same system is simulated. Here the decision maker model also uses a wrong value for the inventory decay coefficient (r=0.03 instead of its real value 0.01). The results are excellent again.

In Figure 5 the decision maker model is assumed to have the following form :

$$s(k+1) = 0.97\, s(k) + 0.1\, A(k) + +0.02\, A(k-1) + 0.0001\, A(k-2)$$

$$I(k+1) = 0.97\, I(k) + p(k) - s(k)$$

i.e. there are differences in the parameters λ r, a_2 and a_3 . Note the occurrence of a large overshoot in the inventory which is due to untuned model used for the decision purposes. After a short learning period the operation of the process is quite satisfactory.

In Figure 6 the production model is as in the previous case but the decision maker uses the following advertisement response relationship:

$$s(k+1) = 0.97\, s(k) + 0.3\, A(k) + 0.15\, A(k-1)$$

Both model mismatch and time delay underparameterization exist in the decision maker model. The learning period takes more time but satisfactory tuning is achieved after some time period.

Figure 7 depicts the case where the changes of the control signal are punished instead of its magnitude, i.e. a type of production smoothing problem is considered. The production control is somewhat smoother and the results are similar.

In Figure 8 the main disturbance corrupting the system performance is assumed to be white Gaussian noise. The process parameters are known to the central authority but random changes affect its operation. The error predictor involved counteracts this source of disturbance and yields a quite satisfactory operation.

In Figure 9 the "worst" case is considered to be that where the decision maker faces both model mismatch and noise corruptive action. The overall operation is degraded but still the performance of the system is satisfactory.

Finally, the last cases shown in Figures 10 and 11 demonstrate the MBPC approach under non-linear process behaviour. The real system evolves under the non-linear model of Vidale-Wolfe. The decision maker still uses for his planning procedure the previous linear delayed model. Figure 10 shows the trajectories when the advertisement effectiveness of the Vidale-Wolfe model is 1, while in Figure11 its value is 0.7. In both cases the assumed value was 1 (this value is the parameter a_0 according to the notation of Section 4). Observe the occurrence of a small wave effect probably stemming from the inherited non-linearity of the latter case.

6. Conclusions

A firm must combine marketing and production information, use it to modify its conception of the market, use the revised conception to make marketing decisions and production decisions, and finally arrange for gathering of new information. Briefly speaking, a firm needs an efficient control system for a successful and profitable operation in a rapidly changing marketing environment. The MBPC approach discussed here is very suitable for this problem, namely for setting the advertising level and production capacity level. The concept of such a flexible and powerful tool seems virtually incontestable: a company should learn from experience in an organized way. The model studied can be useful as it stands and is certainly capable of extension. The results presented in the paper demonstrate the potential usefulness and applicability of such an approach in solving management planning and decision problems. Although every model is only an approximation of the real technical and economic processes, computer simulation provides some insights into the dynamic interactions between these processes. Although decision maker models are not the same as the real ones, MBPC was shown to offer a valuable tool in successfully testing and implementing efficient strategic corporate plans.

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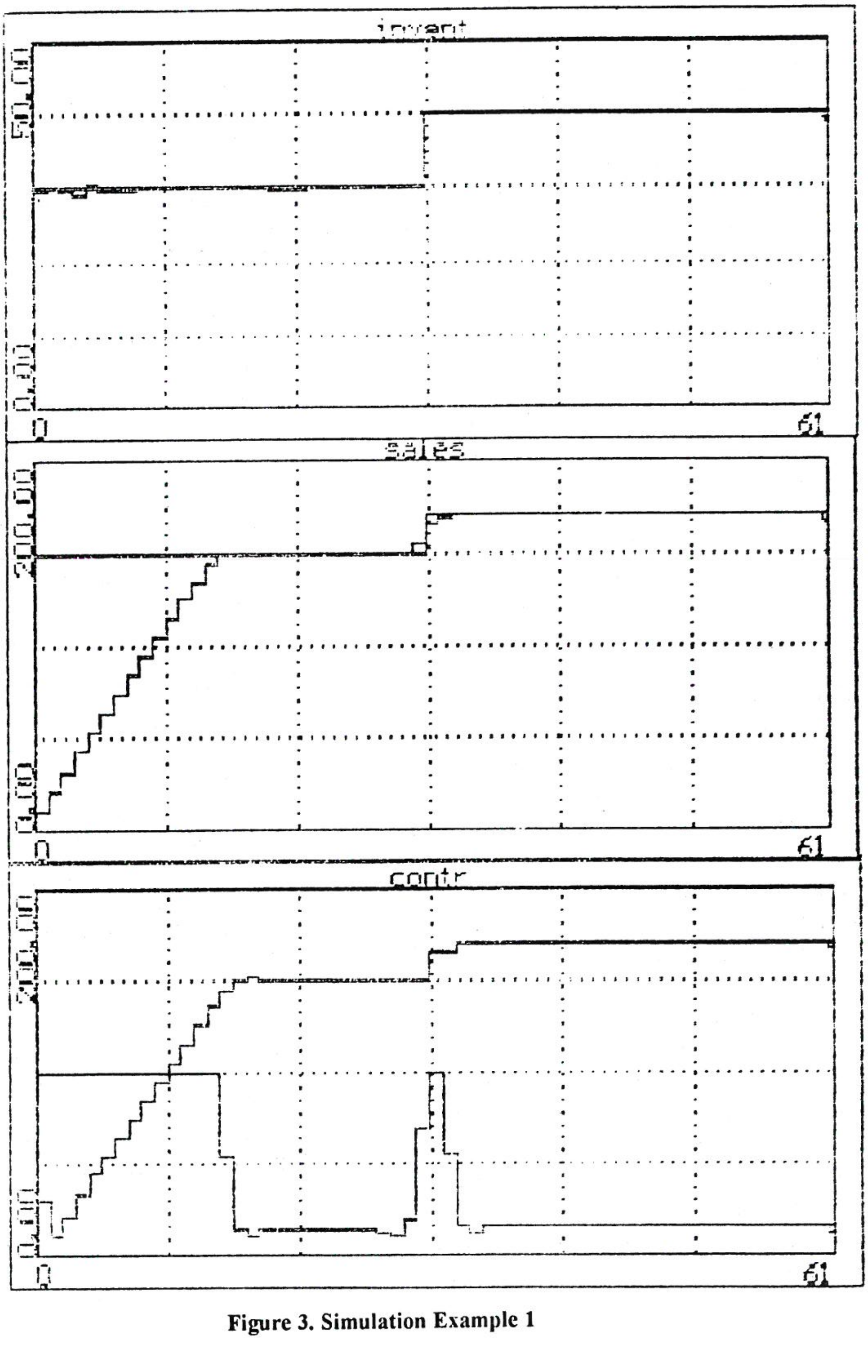

Figure 3. Simulation Example 1

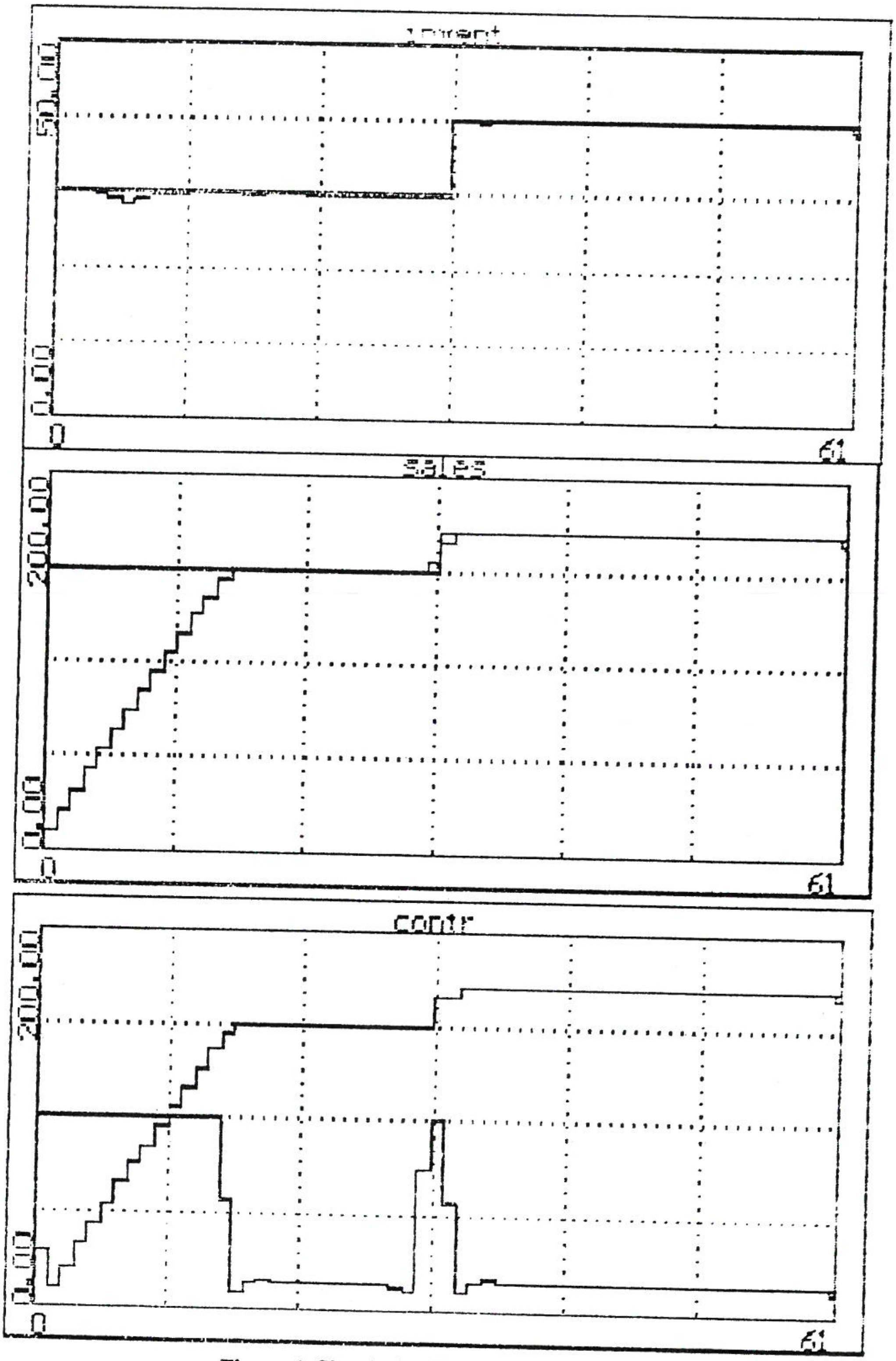

Figure 4. Simulation Example 2

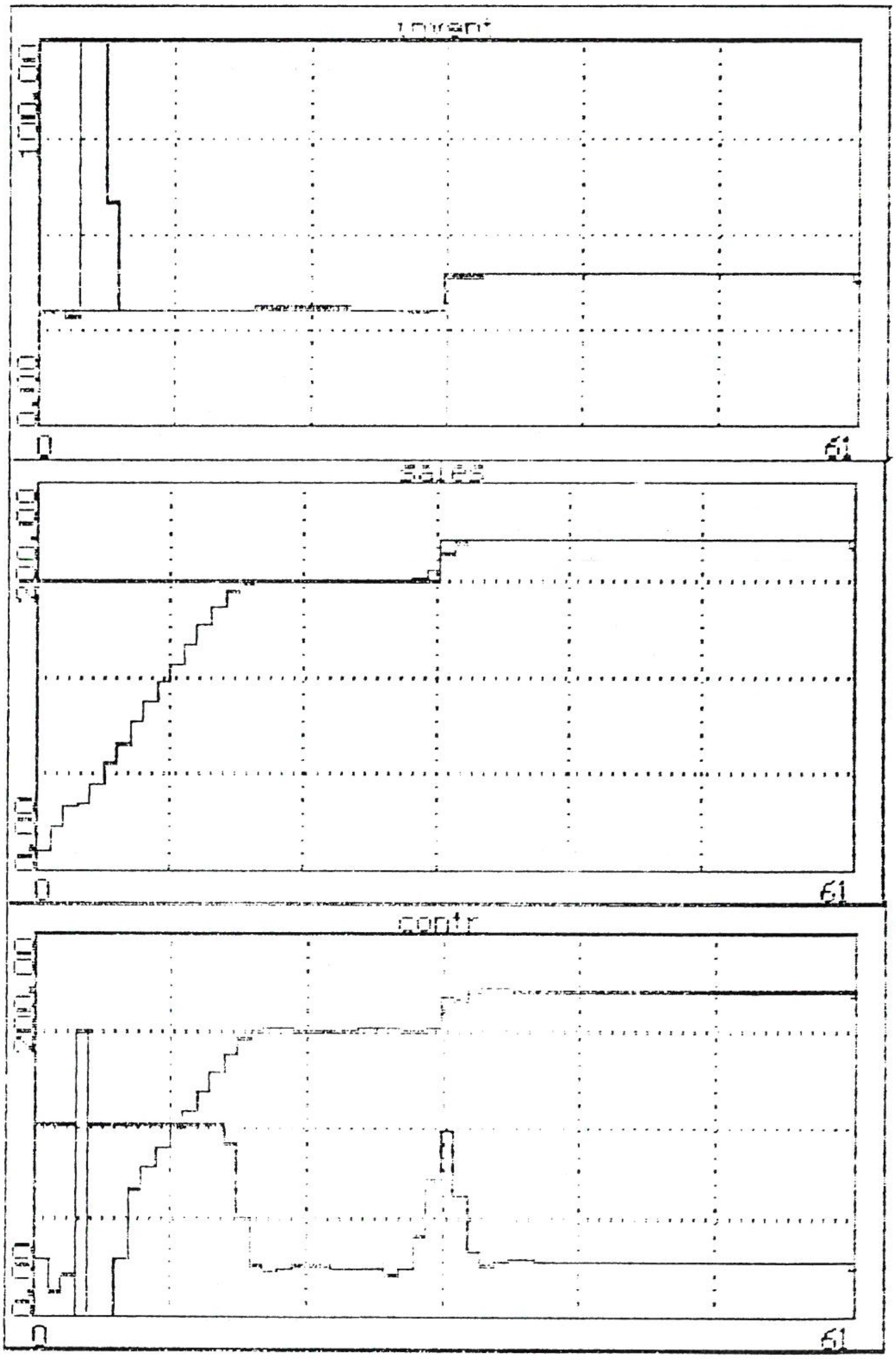

Figure 5. Simulation Example 3

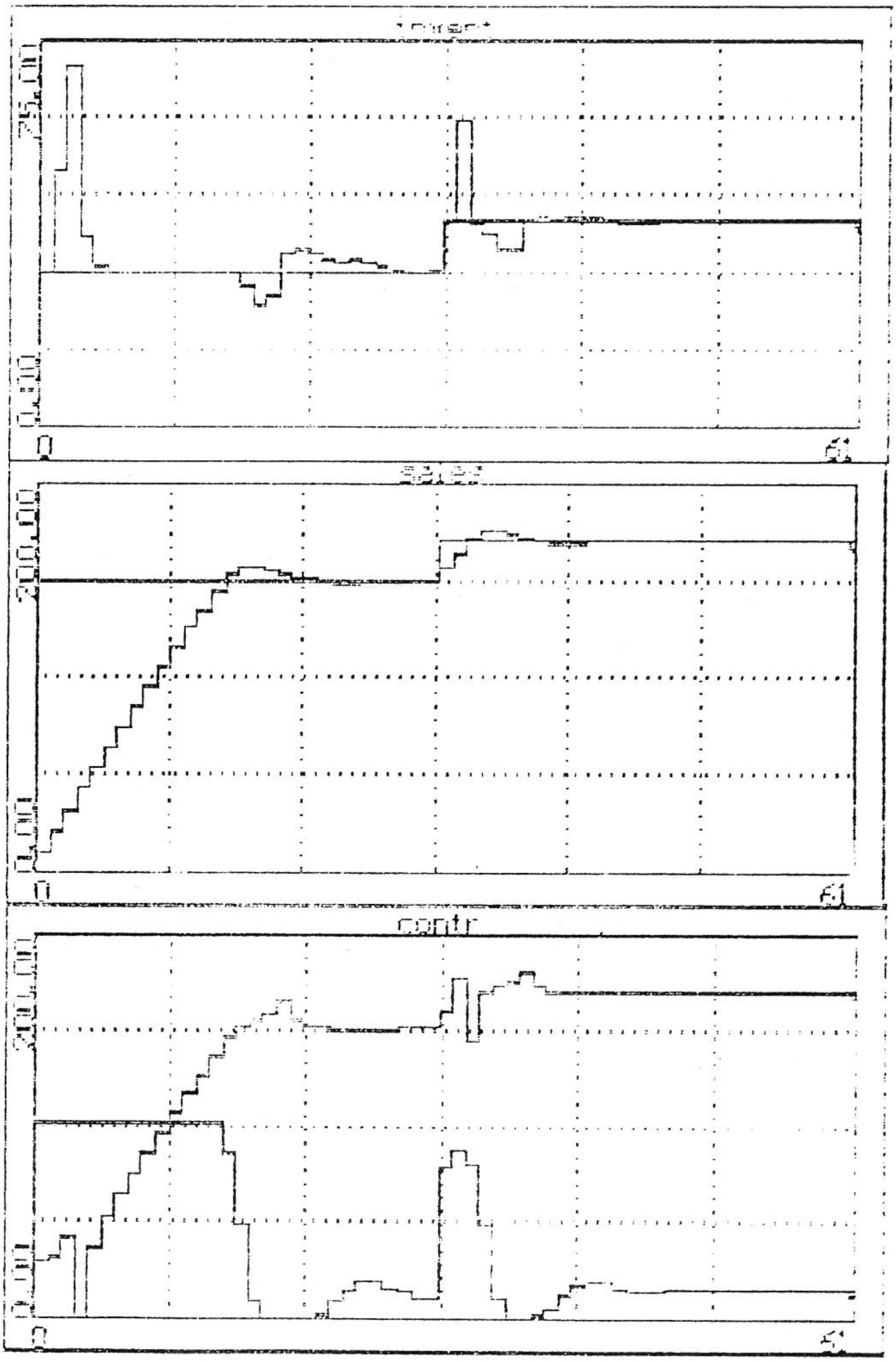

Figure 6. Simulation Example 4

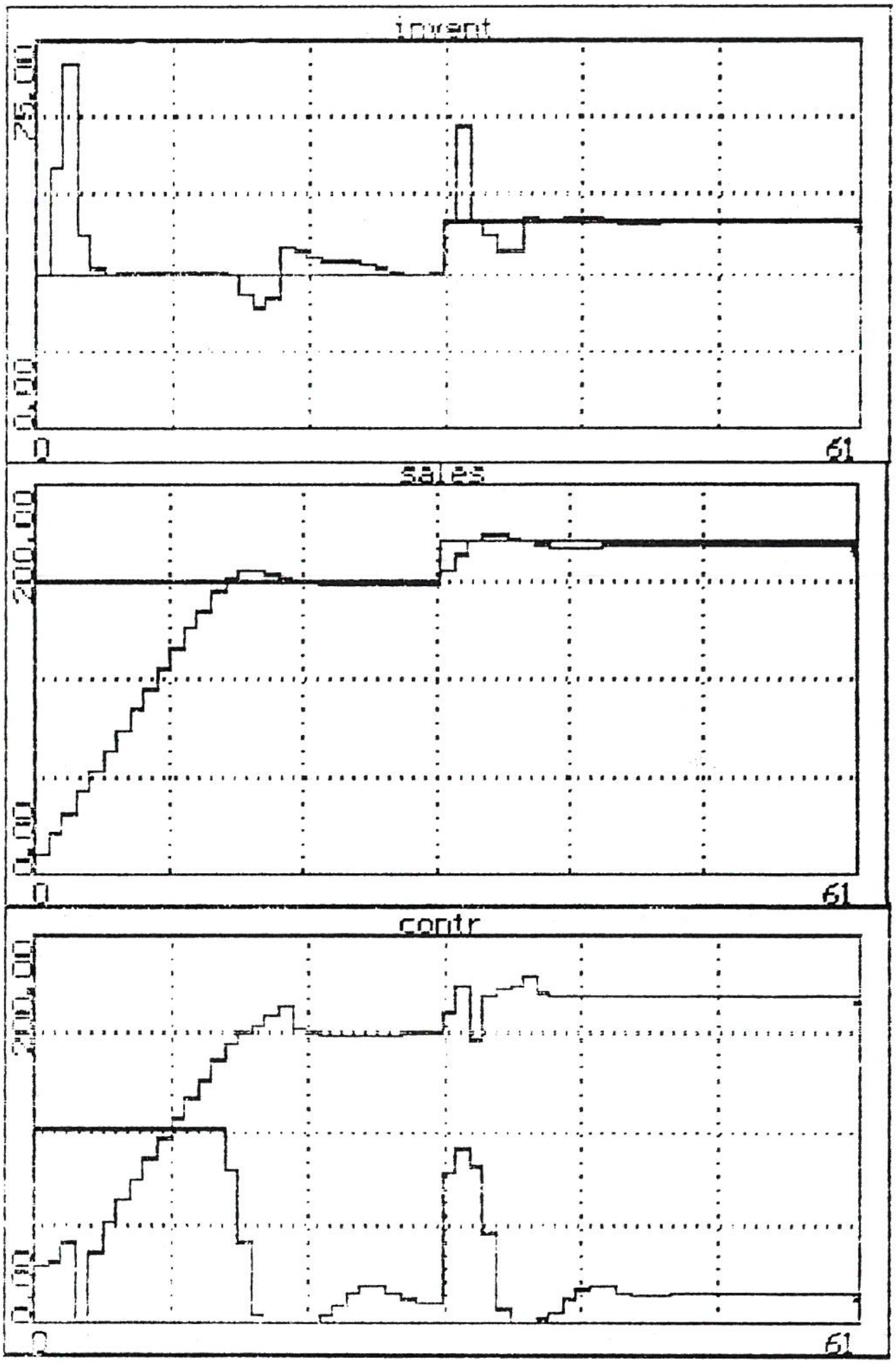

Figure 7. Simulation Example 5

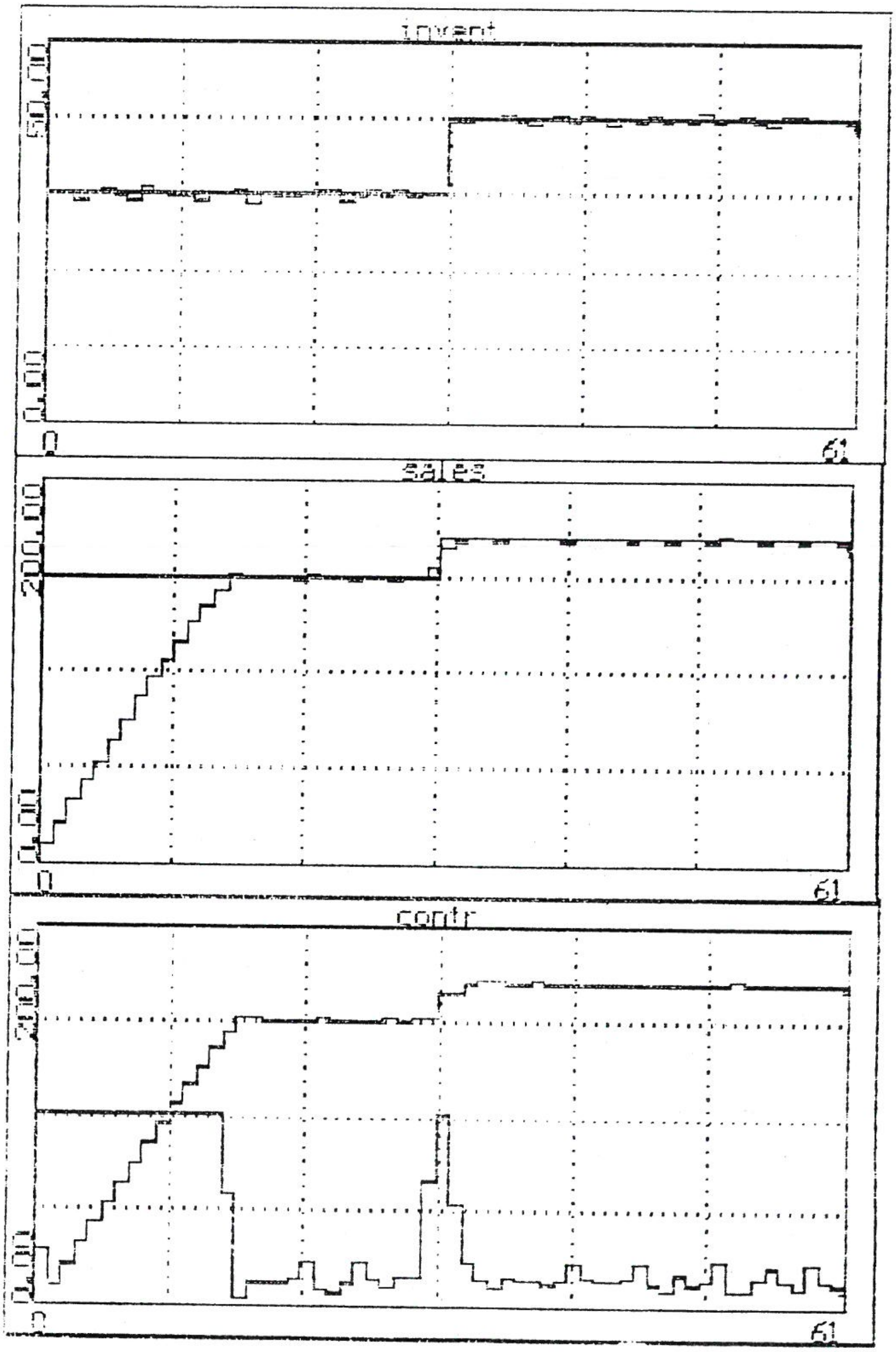

Figure 8. Simulation Example 6

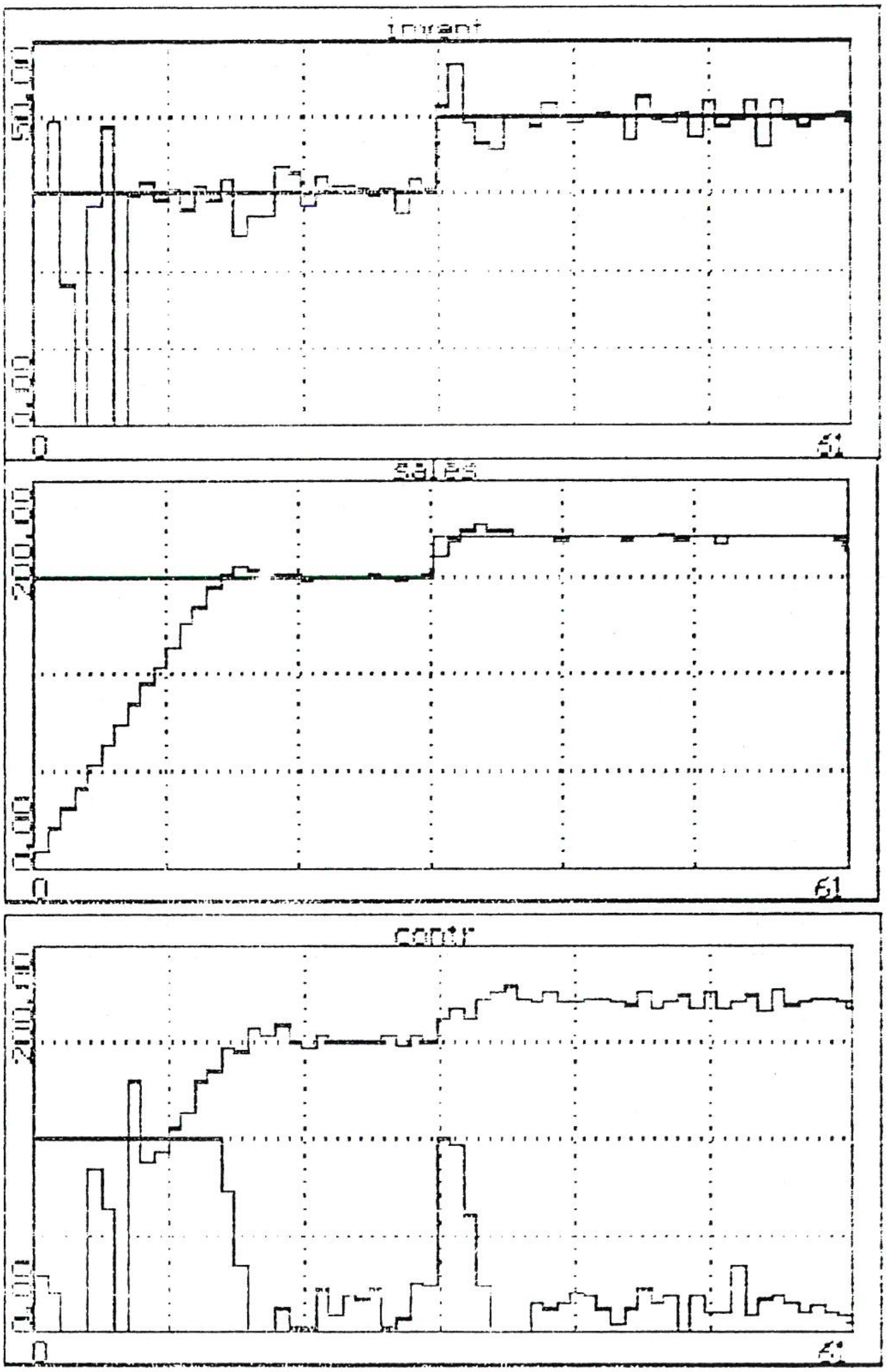

Figure 9. Simulation Example 7

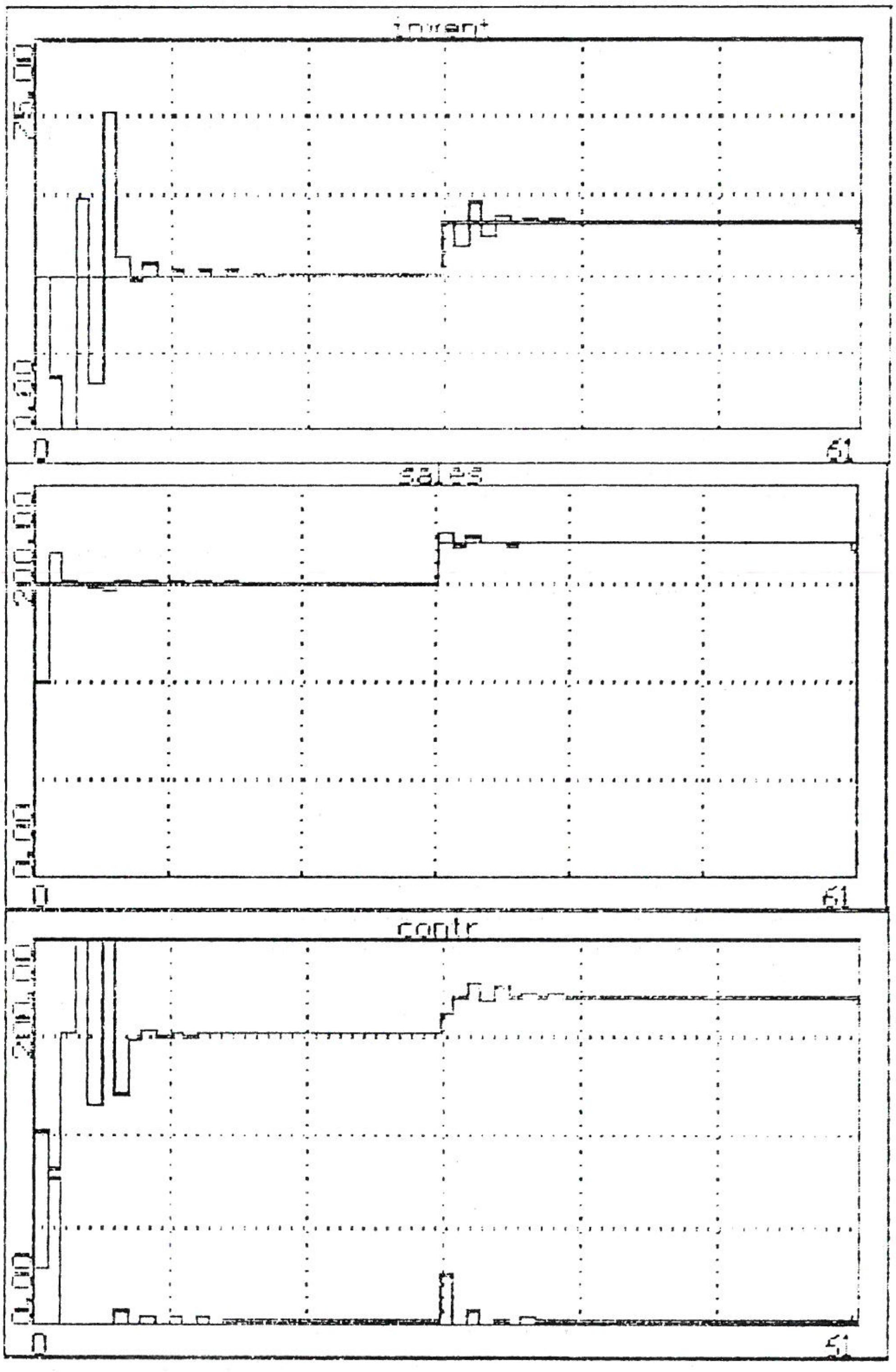

Figure 10. Simulation Example 8

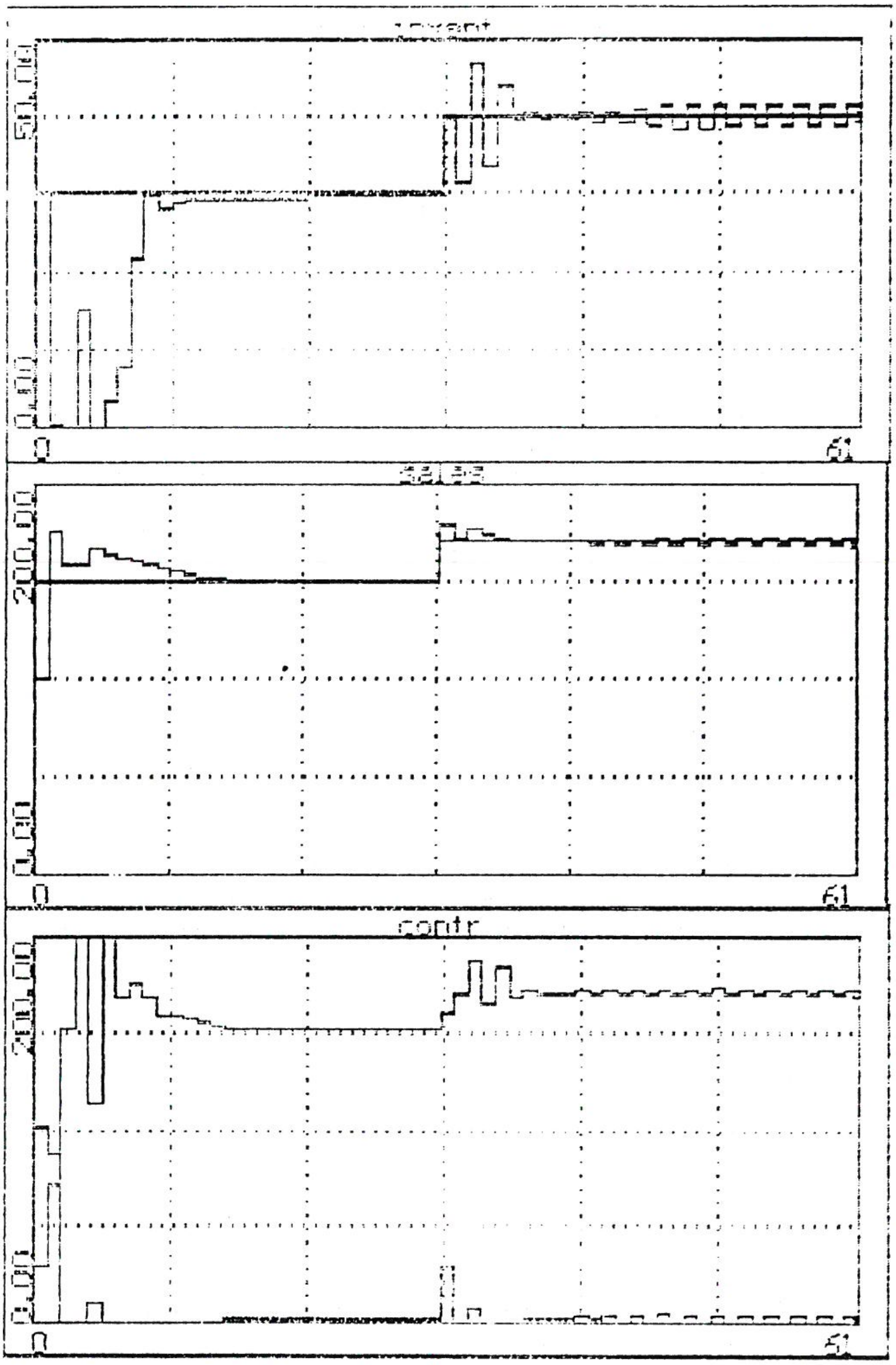

Figure 11. Simulation Example 9

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