Fuzzy Systems: The Connectionist Approach

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Abstract: The main concern is the difference between two approaches in fuzzy sets theory. Also considered briefly is the similarity between fuzzy systems and neural networks.
Keywords: The theorem of representation, Neural network implementation of fuzzy systems.

Constantin Virgil Negoita is professor of Computer Science at the City University of New York (Hunter College). He received a Ph D in Information Science from the University of Bucharest. Invited by the Royal Society in England, in 1972, he started an international seminar devoted to Fuzzy Systems. The main results of this seminar were published in Applications of Fuzzy Sets to System Analysis (Wiley, New York, 1975), Management Applications of System Theory (Birkhäuser Verlag, Basel, 1979), and Fuzzy Systems (Abacus Press, London, 1981). In 1985, the Institute of Electrical and Electronic Engineers awarded him " for his significant work and publications in Fuzzy Logic, and for his development of an Algebraic Theory of Fuzzy Systems with Applications to Expert Systems". After the publication of Pullback (Vantage Press, New York, 1986), Simulation, Knowledge-Based Computing, and Fuzzy Statistics (Van Nostrand, New York, 1987), Cybernetic Conspiracy: Mind over Matter (Falcon Press, Phoenix, 1988), and Connections (Carlton Press, New York, 1990), he chaired the 8th International Congress of Cybernetics and Systems (Hunter College, New York, June 1990). He is an editor of Fuzzy Sets and Systems, Kybernetes, and other journals.

1. Introduction

The history of Fuzzy Systems is marked by two different visions of what a fuzzy set is, each having its correlated research program.

For a long time one vision saw a fuzzy set as a function. The other vision saw it as a family of crisp sets. One took the logical implication as its paradigm, the other the associative memory. One spoke about fuzzy logic. The other spoke about system thinking (Negoita, 1981).

Recently, the latter vision was markedly emphasized. What follows is an attempt to explain this trend.

2. The Representation of Fuzzy Sets

Consider a fuzzy set

$$f: x \rightarrow [0,1]$$

and let

$$L_a f = \{x \in x \mid f(x) \geq a\}$$

be the a-level set of f. It is easy to show the following properties:

(1) $L_0 f = x$

(2) $a \leq b \Rightarrow L_a f \supseteq L_b f$

Property (2) shows that the level sets $(L_a f)_{0 \leq a \leq 1}$ form a nested family of sets.

Many constructions involving fuzzy concepts have to start with the following question: **given a family** of subsets of X $(A_a)_{0 \leq a \leq 1}$ **does there exist a** fuzzy set f: X → [0,1] such that $A_a = L_a f$ **for each** $a \in [0,1]$

Unfortunately, (1) and (2) are not enough for the existence of f. The extra property for the family $(A_a)_a$ is

(3) If $a_1 \leq a_2 \leq \ldots$ and $\lim a_n \neq a$, then $A_a = \bigcap_{n=1}^{\infty} A_{a_n}$

Various operations with fuzzy sets can first be defined on level sets and then extended by using the representation theorem (Negoita and Ralescu, 1975).

3. Neural Network Implementation of Fuzzy Systems

Any Fuzzy System can be expressed as a set of implications

$$\left(V_k, W_k \Rightarrow Z_k\right)$$

An implication (or rule) k, for example, can be written in the form:

if V is V_k and W is W_k then Z is Z_k

where V,W, and Z are linguistic variables and V_k, W_k, Z_k are linguistic values, which are represented by fuzzy sets. The fuzzy set for a given variable is characterized by a membership function defined over its measurement space. For example a fuzzy set V_k is defined as

$$V_k=\left\{\left(v, f_{V_k}(v)\right)\right\}$$

where $f_{V_k}(v)$ is a membership value for V_k defined over a measurement space or universe $\{v\}$ (Negoita, 1985).

The very existence of the representation theorem makes possible the implementation of the implication by a neural network. The logic implication of the Fuzzy System is replaced by a single hidden layer feedforward network. The input and output fuzzy sets are expressed in terms of level sets. The network is trained using the back-propagation algorithm to establish fuzzy associations between the input and output fuzzy sets. Each input and output unit of the network corresponds to a level in the universe of the input and output variables.

4. Conclusions

It may be worth going a bit deeper into the ideas behind the neural network implementation of a fuzzy system. If used as a Rosetta Stone, one can start thinking again about a categorial interpretation of the learning process (Negoita, 1992).

Objections to artificial intelligence are often heard, to the effect that work in this field is ad hoc and incapable of exhibiting a sustained and meaningful growth. This has been true of some past work but the situation seems to be changing.

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