Structural Properties of a Class of Two Level Hierarchical Systems

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Abstract: In this work, the structural properties of a class of two level hierarchical systems where N subsystems at the lower hierarchical level are connected through a coordinator in the upper hierarchical level, are studied. The results from this type of analysis can be used in order to determine controllability of the overall system by the coordinator or more generally by any one of the subsystems. A special case where the interconnection pattern is such that the optimum quadratic local control equals the optimum quadratic global control is presented.

Keywords: Large Scale Systems, decentralized control, structural control, overlapping subsystems.

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1. INTRODUCTION

A Large Scale System (LSS) is generally composed of a set of units or subsystems interlinked by several connections to form an interconnected system. Examples of such LSS are power systems, digital communication networks, economic systems and urban traffic networks. In such complex and large scale systems,to solve the control problems for the whole system is very costly or even impossible. It is often unrealistic in a LSS for every actuator to know the output of all sensors in the system due to insufficient information exchange between subsystems and thus, the controllers use only partial information about the overall system. This is the concept of LSS decentralized control [1]-[4].

It is well- known that in the presence of structurally constrained feedback patterns the fundamental concepts of controllability and observability are extended to the concepts of structural controllability and structural observability which are of major practical interest in the study of LSS. The concepts of structural controllability and observability were introduced by Lin [5]. His results enable structural controllability (and by duality for structural observability) of single-variable systems in a graph-theoretical approach. These results have been extended to multi-variable systems by Shield and Pearson [6] in a purely algebraic approach.

The pole placement problem in systems with decentralized feedback control has been considered by several authors [7]-[9]. In [8], a necessary and sufficient condition for the existence of a solution under decentralized feedback control for a LSS is given in terms of the fixed modes of a system. The question of when a decentralized local static feedback control law is to make the closed loop system observable and controllable from a single station is answered in [9] with graphical methods. In [10], a matrix rank test characterization of the fixed modes of a LSS under decentralized control has been proposed. The same results are considered in [11] from an algebraic point of view.

In this paper the problem of structural controllability for a class of two level hierarchical systems at the supervising level is considered in Part 3. In Part 4, an interconnection pattern where an optimum solution is identical with a completely decentralized one, is investigated.

2. PRELIMINARIES

A structural model and a control algorithm have been proposed in [12], when exploring the inherent structural properties of two level hierarchical systems. A decentralized large scale system consists of N linear subsystems

S_1, S_2, S_N interconnected with a common linear subsystem S_o as shown in Figure 1.

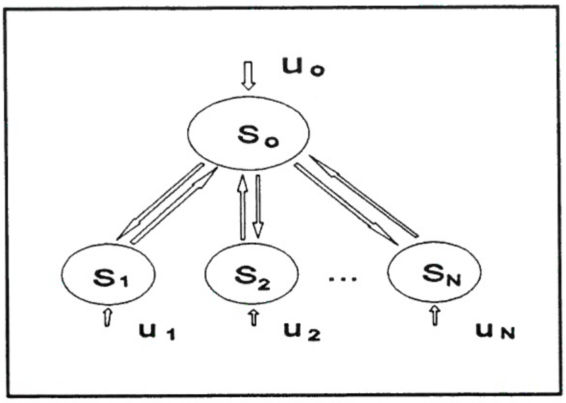

Figure 1. A Two- Level Structural System

Since in the state space description, as to be shown, the matrix describing the dynamics of this system consists of block elements depicting an arrow structure, this system is called a Block Arrow Structure (BAS) decentralized large scale system. The regulation of the above system can be obtained by using large scale techniques [1]-[3]. The approach in [12] makes direct use of the interconnections of the system to obtain an overlapping, partially decentralized controller. A modified BAS method is given in [13]. It scores better than the previous one. In this paper we study some more theoretical aspects of the structural properties of a BAS system with N subsystems S_i and a coordinator S_o.

The objective is to find the controllers u_i for each subsystem, in a time interval $[t_o, \infty]$ in order to

1) stabilize the overall interconnected system
2) meet some predefined criteria.

Following the formulation in [13], we get the mathematical model below :

$$S_i: \dot{x}_i(t) = a_i x_i(t) + b_i u_i(t) + a_{io} x_o(t), \quad x_i(t_0) = x_{i0} \tag{1a}$$

$$y_i(t) = c_i x_i(t) \tag{1b}$$

$$S_o: \dot{x}_o(t) = a_o x_o(t) + b_o u_o(t) + \sum_{i=1}^{N} a_{oi} x_i(t), \quad x_o(t_0) = x_{o0} \tag{1c}$$

$$y_o(t) = c_o x_o(t) \tag{1d}$$

where $x_i(t) \in R^{n_i}$, $x_o(t) \in R^{n_o}$, $u_i(t) \in R^{r_i}$, $u_o(t) \in R^{r_o}$ and

$y_i(t) \in R^{k_i}$, $y_o(t) \in R^{k_o}$ are the state, control and output vectors for subsystems S_i and S_o respectively.

When a criterion is wanted,based on which different stabilizing controllers of the system (1) are to be compared, the next associated quadratic cost functional is used

$$J\,(\,x_0,\ t_0,\ \infty,\ u(t)\,) = \int_{t_0}^{\infty} (\,x'(t)\ Q\ x(t)\ +\ u'(t)\ R\ u(t)\,)\ dt \tag{2}$$

where $Q = block\ diag.[q_1, q_2, ..., q_N, q_o] \in R^{nxn}$ $R = block\ diag.[r_1, r_2, ..., r_N, r_o] \in R^{rxr}$ are constant, symmetric, block diagonal positive semidefinite and positive definite weighting matrices respectively. In that case the above quadratic cost functional can be written

$$J(x_0,\ t_0,\infty,\ u(t))=\sum_{i=1}^{N+1} J_i\ (\ x_{i0},\ t_0,\ \infty,\ u_i(t)) \tag{3}$$

where

$$J_i(x_{i0},\ t_0,\ \infty,\ u_i(t)\ )=\int_{t_0}^{\infty} (x_i'(t)\ Q_i\ x_i(t)+u_i'(t)\ R_i\ u_i(t))dt \tag{4}$$

The matrices $a_i \in R^{n_i x n_i}$, $a_o \in R^{n_o x n_o}$, , $b_i \in R^{n_i x r_i}$, $b_o \in R^{n_o x r_o}$ and $c_i \in R^{k_i x n_i}$, $c_o \in R^{k_o x n_o}$ describe the dynamics, control and output distribution for S_i and S_o respectively. The interconnections (or information transfer from) between S_o and S_i and S_i and S_o are represented by the matrices $a_{io} \in R^{n_i x n_o}$ and $a_{oi} \in R^{n_o x n_i}$ respectively.

As can be seen from (1), these state equations are representative of a class of two level hierarchical and/or decentralized systems where N+1 subsystems are interconnected in a way such that N from the subsystems is at the lower hierarchical level while the subsystem left is at the upper hierarchical level and acts as a coordinator for the system. The transition matrix A for the overall system has the following form, which depicts an arrow constructed from the block matrices a_i, a_{io}, a_{oi}, ... a_o , hence the name Block Arrow Structure (BAS),

$$A = \begin{bmatrix} a_1 & & & & a_{10} \\ & a_2 & & \underline{0} & a_{20} \\ & & \ddots & & \vdots \\ & \underline{0} & & a_N & a_{N0} \\ a_{01} & a_{02} & \cdots & a_{0N} & a_0 \end{bmatrix} \in R^{nxn} \tag{5}$$

while the input and output matrices are respectively

$$B = block\ diag.\ [\ b_1,\ b_2,\ ...,\ b_N,\ b_o\ ] \in R^{nxr} \quad C = block\ diag.\ [\ c_1,\ c_2,\ ...,\ c_N,\ c_o\ ] \in R^{kxn} \tag{6}$$

Another useful representation of the overall system (1), which indicates the partitioning of the system into N+1 subsystems is the following:

$$\dot{x}(t) = A\ x(t) + \sum_{i=1}^{N+1} B_i u_i(t) \tag{7a}$$

$$y_i(t) = C_i\ x(t) \tag{7b}$$

where

$$B_i = \begin{bmatrix} \underline{0} \\ \vdots \\ b_i \\ \vdots \\ \underline{0} \end{bmatrix} \in R^{nxr_i}\ ,\ C_i = [\underline{0}\ \cdots\ c_i\ \cdots\ \underline{0}] \in R^{k_i xn} \quad \forall\ i=1,2,...N+1. \tag{8}$$

The problem of eigenvalue placement of such a BAS structured system is of great importance, since such types of systems are common in practice where,due to physical or cost restrictions, N subsystems can only exchange information through a supervisory subsystem. If there are no interconnections between subsystems

$$(a_{io} = \underline{0} \wedge a_{oi} = \underline{0}) \ \forall \ i = 1,2,\ldots N. \tag{9}$$

the problem of eigenvalue placement for the system concerned is broken into N subproblems of lower dimensions, to be solved independently from one another.

If only some of the interconnection matrices are identical with a zero matrix of the same order

$$(a_{io} = \underline{0} \vee a_{oi} = \underline{0}) \ for \ i = i_1, i_2, \ldots i_s \ where \ i_1, i_2, \ldots i_s \in \{1,2,\ldots N\}. \tag{10}$$

it is obvious that the eigenvalues of the overall system are the eigenvalues of the subsystems for which (10) holds together with the eigenvalues of a lower order BAS subsystem consisting of the rest of the subsystems.

To prevent such trivial cases as the above discussed ones, it is assumed that

$$(a_{io} \neq \underline{0} \wedge a_{oi} \neq \underline{0}) \ \forall \ i = 1,2,\ldots N \tag{11}$$

It is also useful to develop the next

Notation.

For N-channel systems, $\mathbf{N}$ denotes the set { 1, 2, ... N } and $\mathcal{L}$ is a non empty subset of $\mathbf{N}$ with elements i_1, i_2, ... i_s ordered so that $i_1 < i_2 < \ldots < i_s$. $B_{\mathcal{L}}$ and $C_{\mathcal{L}}$ are defined

$$B_{\mathcal{L}} = [B_{i_1} \ B_{i_2} \ \ldots \ B_{i_s}] \tag{12}$$

and

$$C_{\mathcal{L}} = [C_{i_1} \ C_{i_2} \ \ldots \ C_{i_s}] \tag{13}$$

$\wp(\mathbf{N})$ is a power set of $\mathbf{N}$, which is the set of all subsets of $\mathbf{N}$, $\mathbf{N} - \mathcal{L} = \{x \mid x \in \mathbf{N} \ \wedge \ x \notin \mathcal{L}\}$, $\rho(A)$ is the rank of matrix A and $\sigma(A)$ is the spectrum of eigenvalues of A.

When the system consists of N+1 subsystems $\mathbf{N+1}$ denotes the set { 1, 2, ... N, N+1}.

3. The Single Channel Controllability Problem in a BAS Structured System

In this paper we consider the problem of determining the conditions under which a BAS system of form (7) can be made observable and controllable from the input and output variables of one channel by static feedback applied to the other channels. These conditions are characteristic of the given system and are valid in case that the controlling channel is the coordinator S_o, or even one of the subsystems S_i. The first case, of course, is the most interesting one.

The general problem of single channel controllability of a system consisting of N interconnected subsystems was first considered in [9] using a geometric approach. The same problem has been considered in [10],[11] from an algebraic point of view.

It is assumed that all the subsystems are "locally" controllable and observable, so a complete decentralized solution is always possible. It is also assumed that the overall system is completely observable and controllable from a fictitious measurement and control station, or $(A,[B_1,B_2,\ldots,B_N,B_o])$ is fully controllable and $(A,[C_1,C_2,\ldots,C_N,C_o])$ is completely observable. But the system (7) is neither completely observable nor controllable from any one of the N+1 subsystems.

We shall begin by trying to find out the conditions under which the system, after closing the feedback loops $u_i = K_i x_i$ in the lower level subsystems, can be made completely controllable from $u_o(t)$. In this case the system is

$$\dot{x}(t) = \left(A + \sum_{i=1}^{N} B_i K_i \right) x(t) + B_o u_o(t) \tag{14}$$

In the general case, the system can become observable and controllable from every subsystem, irrespective of its belonging to the upper or lower hierarchical level [10],[11]. Next theorem gives us a necessary and sufficient condition for controlling a BAS system from a single station.

THEOREM 1

Given the BAS system (7), there exist feedback matrices $K_i \in R^{n_i x n_i}$, $i = 1,2,...N+1$ making the system completely observable and controllable from a single control station if and only if for all $\mathcal{L} \in \wp(N+1)$ we have

$$C_{N+1-\mathcal{L}} (\lambda I - A)^{-1} B_{\mathcal{L}} \neq 0 \tag{15}$$

and

$$\rho \begin{bmatrix} \lambda I - A & B_{\mathcal{L}} \\ C_{N+1-\mathcal{L}} & \underline{0} \end{bmatrix} \geq n \quad \forall \lambda \in \sigma(A) \tag{16}$$

Proof

In [10], an algebraic characterization of fixed modes for a general decentralized controlled system which consisted of N subsystems (or control stations), is made. Using these results, it is derived that a necessary and sufficient condition for

$$\lambda I - A - \sum_{i=1}^{N} B_i K_i C_i \tag{17}$$

to have rank < n- α for all K_i, some fixed complex λ and a non-negative α is that for some partition $\mathcal{L} \in \wp(N)$ there holds

$$\rho \begin{bmatrix} \lambda I - A & B_{\mathcal{L}} \\ C_{N+1-\mathcal{L}} & \underline{0} \end{bmatrix} < n - \alpha \tag{18}$$

That means that if such a partition exists, the controllability of the interconnected system from the N+1 th station gets loose. In [11], a more general case where an interconnected system can be made controllable from every station is considered.

Applying the above to a BAS system shows that when (15) and (16) hold the system can be made controllable from every subsystem when a local feedback control is applied to N other subsystems.

The above theorem provides a necessary and sufficient condition for $\lambda_o \in \sigma(A)$ as to be a decentralized fixed mode of the system if there exists at least one complementary subsystem such that

$$\rho \begin{bmatrix} \lambda_o I - A & B_{\mathcal{L}} \\ C_{N-\mathcal{L}} & \underline{0} \end{bmatrix} < n \tag{19}$$

(There exists $2^{(N+1)}$-2 complementary subsystems for the system (1)).

This result let us get an insight into the reason why fixed modes occur. λ_o is a decentralized fixed mode of system (7) if there exists a disjoint partition of the system in two aggregate stations a and b such that λ_o is simultaneously uncontrollable by one of these stations and unobservable from the other one.

Example 1

We have the following BAS system which consists of three interconnected subsystems, and where the overall system matrices A,B,C are:

$$A = \left[\begin{array}{cc|ccc|cc} 6 & 0 & & & & 0 & 0 \\ 10 & 20 & & \underline{0} & & 0 & 1 \\ \hline & & 10 & 0 & 8 & 0 & -1 \\ & \underline{0} & 2 & -10 & 2 & 0 & 0 \\ & & 0 & 0 & -2.5 & 1 & 0 \\ \hline -1 & 0 & 0 & 0 & 0 & 0 & 0.3 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0.2 \end{array}\right]$$

$$B = \left[\begin{array}{c|c|cc} 2 & & & \\ 0 & & \underline{0} & \\ \hline & 0 & & \\ \underline{0} & 0 & \underline{0} & \\ & 1 & & \\ \hline \underline{0} & \underline{0} & 0.1 & 0 \\ & & -1 & 0 \end{array}\right] \qquad C = \left[\begin{array}{cc|ccc|cc} 0 & 1 & & & & & \\ 0 & 0 & & \underline{0} & & \underline{0} & \\ \hline & & 1 & 0 & 0 & & \\ & \underline{0} & 0 & 1 & 0 & \underline{0} & \\ & & 0 & 0 & 1 & & \\ \hline & \underline{0} & & \underline{0} & & 1 & 0 \\ & & & & & 0 & 1 \end{array}\right]$$

From these matrices one can easily derive the matrices corresponding to a specific subsystem. For example, the matrices of subsystem 1 are:

$$a_1 = \begin{bmatrix} 6 & 0 \\ 10 & 20 \end{bmatrix} \quad b_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad c_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad a_{10} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad a_{01} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_1 = [\,2\ 0\ 0\ 0\ 0\ 0\ 0\,]' \qquad C_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Subsystems are all controllable and observable when they get disconnected from each other $\mathrm{rank}(\mathrm{ctrb}(a_i,b_i)) = n_i$ and $\mathrm{rank}(\mathrm{obsv}(a_i,c_i)) = n_i$ for i=1,2,o where $\mathrm{ctrb}(a_i,b_i))$ and $\mathrm{obsv}(a_i,c_i)$ are the controllability and observability matrices of the i system respectively. When the interconnections are taken into account the system is neither controllable nor observable from a single station, or rank(ctrb(A,B_i)) < n or rank(obsv(A,C_i)) < n for i=1,2,o e.g. rank(ctrb(A,B_0)) = 6 and rank(obsv(A,C_0)) = 5.

The system is globally controllable and observable, rank(obsv(A,[C_1;C_2;C_0])) = 7 and rank(ctrb(A,[B_1;B_2;B_0])) = 7.

After checking the conditions of theorem 1 for the $2^{(2+1)}-2 = 6$ and for the seven eigenvalues of matrix A, (6*7=42 matrix ranking tests) we conclude that the system can be made controllable from a single station. Indeed, the gains k1 k2 of the linear quadratic regulator problem for subsystems with q_i and r_i identity weighting matrices of appropriate dimensions, at the lower hierarchical level make the system controllable from the co-ordinator. The new transition matrix is AA and

rank(ctrb(AA,B_0)) = 7, rank(obsv(AA,C_0)) = 7

The response of a state to a step input is shown in Figure 2. Dashed line indicates the optimum response of state x_4 which belongs to the second subsystem in a step input applied to u_1 which is an input of the first subsystem. Continuous line indicates the response of the same state when the system is controlled from the coordinator while the static closed loop feedback gains for the subsystems 1 and 2 are correspondingly

$$k_1 = -[\ 26.1921 \quad 52.7873\ ]$$

$$k_2 = -[\ 31.9926 \quad 0.0031 \quad 20.2847\ ]\ .$$

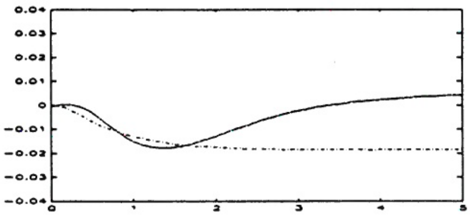

Figure 2. Step Response of State x_4

It can be shown that there are cases where the interconnections play no role in system's performance (so they can be omitted), while in other cases they play a significant role not only in system's performance but also in system's stability.

4. The Neutral Interconnections BAS Information Pattern

It is well- understood that when (9) holds, each subsystem can built its own local observer and controller and this is the optimal solution for the overall system. Beyond this trivial case, there exists a whole class of interconnection patterns which does not move the eigenvalues of the system from this optimal completely decentralized location, as the next theorem shows.The study of such interconnection patterns is of great importance, because in that case the solution of the overall system can still be broken into N+1 lower order problems.

THEOREM 2

Given the N+1 completely observable and controllable systems (c_i, a_i, b_i) $i=1,2,...N$, (c_o, a_o, b_o), with a separable quadratic cost criterion (3), the class of BAS interconnection patterns

$$a_{io} = H_i P_o, \quad a_{oi} = -H_i^T P_i, \quad i=1,2,...N \tag{20}$$

where P_i (P_o) is the solution of the Riccati equation for system S_i (S_o) and H_i arbitrary, does not change the solution of system (1) against the optimum decentralized one.

Proof

When there are no interconnections between the subsystems, the solution of the overall problem is given by:

$$P A + A^T P - P B R^{-1} B^T P + Q = \underline{0} \tag{21}$$

where $A = block\ diag.\ [\ a_1, a_2, \dots, a_N, a_o\ ]$, $B = block\ diag.\ [\ b_1, b_2, \dots, b_N, b_o\ ]$,

$Q = block\ diag.\ [\ q_1, q_2, \dots, q_N, q_o\ ]$, $R = block\ diag.\ [\ r_1, r_2, \dots, r_N, r_o\ ]$.

The solution of (21) results in a diagonal matrix $P = block\ diag.\ [\ P_1, P_2, \dots, P_N, P_o\ ]$,

where every P_i satisfies one of the N+1 Riccati equations of type (21) corresponding to each of the subsystems.

When the interconnections are taken into account, the solution of the problem is produced by an equation of type (21), but this time the matrix A is of the BAS form

$$A = \begin{bmatrix} a_1 & & & & a_{10} \\ & a_2 & & \underline{0} & a_{20} \\ & & \ddots & & \vdots \\ & \underline{0} & & a_N & a_{N0} \\ a_{01} & a_{02} & \cdots & a_{0N} & a_0 \end{bmatrix} \in R^{nxn} \tag{22}$$

The structural perturbation of adding interconnections a_{io}, a_{oi} makes the new system matrix

$$A + \tilde{A} = \begin{bmatrix} a_1 & & & & \\ & a_2 & \underline{0} & & \\ & & \ddots & & \\ & \underline{0} & & a_N & \\ & & & & a_0 \end{bmatrix} + \begin{bmatrix} & & & & a_{10} \\ & & & & a_{20} \\ & & \underline{0} & & \vdots \\ & & & & a_{N0} \\ a_{01} & a_{02} & \cdots & a_{0N} & \end{bmatrix} \in R^{nxn} \tag{23}.$$

Generally, this will result in the solution of (21) changing to $P + \tilde{P}$ so that the new Riccati equation for the system is

$$(P+\tilde{P})\ (A+\tilde{A}) + (A+\tilde{A})^T\ (P+\tilde{P}) - (P+\tilde{P})\ B\ R^{-1}\ B^T\ (P+\tilde{P}) + Q = \underline{0} \tag{24}.$$

We claim that for the interconnection pattern (20), $\tilde{P} = \underline{0}$. In order to let this happen, it is necessary that $P \tilde{A} + \tilde{A} P = 0$, or

$$\begin{bmatrix} P_1 & & & & \\ & P_2 & \underline{0} & & \\ & & \ddots & & \\ & \underline{0} & & P_N & \\ & & & & P_0 \end{bmatrix} \begin{bmatrix} & & & & a_{10} \\ & & & & a_{20} \\ & & \underline{0} & & \vdots \\ & & & & a_{N0} \\ a_{01} & a_{02} & \cdots & a_{0N} & \end{bmatrix} + \begin{bmatrix} & & & & a_{01}^T \\ & & & & a_{02}^T \\ & & \underline{0} & & \vdots \\ & & & & a_{0N}^T \\ a_{10}^T & a_{20}^T & \cdots & a_{N0}^T & \end{bmatrix} \begin{bmatrix} P_1 & & & & \\ & P_2 & \underline{0} & & \\ & & \ddots & & \\ & \underline{0} & & P_N & \\ & & & & P_0 \end{bmatrix} = \underline{0} \tag{25}$$

which is true when

$$P_i\, a_{i\bullet} + a_{\bullet i}^T\, P_\bullet = \underline{0} \qquad \forall\ i = 1,2,\ldots N \tag{26}$$

and this is the case, easily observable, when the interconnection pattern is as given in (20) ■.

Example 2

Once again consider the three subsystems of example 1, but this time interconnect them with a pattern as given in Theorem 2 with P_i the solution of the "local" Riccati equations and $H_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $H_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$. The new transition matrix A is:

$$A = \left[\begin{array}{cc|ccc|cc}
6 & 0 & & & & 0 & 0 \\
10 & 20 & & \underline{0} & & 0.4991 & 1.4159 \\
\hline
 & & 10 & 0 & 8 & -0.4991 & -1.4159 \\
 & \underline{0} & 2 & -10 & 2 & 0 & 0 \\
 & & 0 & 0 & -2.5 & 4.5759 & 0.4991 \\
\hline
0 & 0 & -31.992 & -0.0031 & -20.284 & 0 & 0..3 \\
-26.393 & -69.637 & 51.126 & 0.0003 & 31.9926 & -1 & 0..2
\end{array}\right]$$

The matrix B is as before, while the weighting matrices are $Q = I_7$ and $R = I_4$.
The solution of the Riccati equation for the overall problem is:

$$P = \left[\begin{array}{cc|ccc|cc}
13.0960 & 26.3937 & & & & & \\
26.3937 & 69.6375 & & \underline{0} & & \underline{0} & \\
\hline
 & & 51.1261 & 0.0003 & 31.9926 & & \\
 & \underline{0} & 0.0003 & 0.0500 & 0.0031 & \underline{0} & \\
 & 31.9926 & 0.0031 & 20.2847 & & & \\
\hline
\underline{0} & \underline{0} & & & & 4.5759 & 0.4991 \\
 & & & & & 0.4991 & 1.4159
\end{array}\right]$$

and the feedback gain $K = -R^{-1} B^T P$ is

$$K = \left[\begin{array}{cc|ccc|cc}
26.1921 & 52.7873 & & \underline{0} & & \underline{0} & \\
\hline
 & & 31.9926 & 0.0031 & 20.2847 & \underline{0} & \\
\hline
 & \underline{0} & & \underline{0} & & -0.0415 & -1.366 \\
 & & & & & 0 & 0
\end{array}\right]$$

The above two block diagonal matrices consisted of blocks which are solutions to the complete decoupled subsystems. In this way the seventh order overall problem has been broken into three lower order problems, two of the second order and one of the third order. Even if the Theorem 2 type of patterns are unlikely to meet with in practice, this in no way reduces the significance of the result.

5. CONCLUSIONS

This paper is an analysis of the inherent structural properties of the two- level hierarchical controlling scheme proposed in [12]-[13]. Although previous work related about some appealing characteristics of the BAS regulator such as the possibility of parallel implementation [12]-[13], the structural flexibility of other more generalized structural approaches [14] or the simulation of a given system and the comparison of the results with other approaches, say the two time scale approach, little was done as to some theoretical structural analysis.

In this work, some criteria about when a BAS two- level hierarchical system can be made observable and controllable from a single station irrespective of this station being at the lower hierarchical level or the coordinator, have been given. It is possible, therefore, that a control centre for the overall system is set up in one of the subsystems (mainly in the coordinator), while static closed loop feedback control is exercised on the other subsystems.

An interconnection pattern of great theoretical importance, where the optimum quadratic local control coincides with the optimum quadratic global control, has been investigated. In that case, local controllers of much lower order than the order of the overall system provide the best solution. It is reasonable to expect that the decentralized solution for a BAS system deviates between acceptable margins, even though the interconnection pattern slightly differs from that in (20), and so, can also be used in such cases.

REFERENCES

[1] MESAROVIC, M.D.,MACKO, D. and TAKAHARA, Y. **Theory of Hierarchical Multilevel Systems**, ACADEMIC PRESS, New York,1970.

[2] SINGH, M.G., **Dynamical Hierarchical Control**, NORTH-HOLLAND, Rev. Ed., The Netherlands ,1980.

[3] JAMSHIDI, M., **Large Scale Systems Modelling and Control**, NORTH-HOLLAND, U.S.A.,1983.

[4] HODZIC, M. and SILJAK, D.D., **Estimation and Control of Large Sparse Systems**, AUTOMATICA, Vol. 29,1985, pp 277-292.

[5] LIN, C.T., **Structural Controllability**, IEEE TRANS. AUT. CONT. Vol. AC-19, No. 3., 1974, pp 201-208.

[6] SHIELDS, R.W. and PEARSON, J.B., **Structural Controllability of Multi-input Linear Systems**, IEEE TRANS. AUT. CONT. Vol. AC- 21, No. 2., 1976, pp 203-212.

[7] AOKI, M., **On Feedback Stabilizability of Decentralized Dynamic Systems**, AUTOMATICA, Vol. 8, No.2, 1972, pp 163-173.

[8] WANG, S.H. and DAVISON, E.J., **On the Stabilization of Decentralized Control Systems**. TRANS. AUT. CONT., AC-18, 1973, pp. 473-478.

[9] CORFMAT, J.P. and MORSE, A.S., **Decentralized Control of Linear Multivariable Systems**, AUTOMATICA, Vol. 12, No. 5, 1976, pp 479-495.

[10] ANDERSON, B.D.O. and CLEMENTS, D.J., **Algebraic Characterization of Fixed Modes in Decentralized Control**, AUTOMATICA, 17, 1981, pp. 703-712.

[11] LEE, J.S. and WANG, P.K.C., **Feedback Control of Linear Decentralized Control Systems: An Algebraic Approach**, INT. J. CONTROL, Vol. 52, No.6, 1990.

[12] GROUMPOS, P.P., **Structural Dynamic Hierarchical Stabilization and Control of L S S in C.Leondes (Ed.) Decentralized / Distributed Control and Dynamic Systems**, CONTROL AND DYNAMIC SYSTEMS, 22 , ACADEMIC PRESS, New York, pp. 117-163.

[13] LEROS, A.P. and GROUMPOS, P.P., **Time Invariant (BAS) Decentralized Large Scale Linear Regulator Problem**, INT.J.CONTROL, Vol. 46, No. 1, 1987, pp. 129-152.

[14] SUNDARESHAN, M.K., **Generation of Multilevel Control and Estimation Schemes for Large-Scale Systems: A Perturbational Approach**, IEEE TRANS. SMC-7, March 1977, pp. 144-152.

[15] PAGALOS, A.V. and GROUMPOS, P.P., **The Block Arrow Structure (BAS) Regulator versus the Two Time Scale Regulator. A comparison Study**, European Control Conference ECC '93, Groningen, The Netherlands, 1993.