Parameterization of Feedback Controllers: A Polynomial Approach

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Abstract: This paper gives a new parameterization of all stabilizing or pole-placement controllers in unitary feedback systems. A polynomial matrix fraction description of the system under consideration is taken as starting point, and the coefficients of certain polynomial matrices are given the role of parameters. To make sure that the resulting controller is proper, some constraints are imposed on the degrees of the elements of the polynomial matrices involved. These features make the proposed parameterizations very simple, easy to handle, and suited to multi-goal controller design. Based on such a parameterization, an algorithm is proposed to design the controller for robust asymptotic tracking and disturbance rejection. Finally, some numerical examples are presented.

Keywords: Pole-assignment; stabilization; controller parameterization; feedback control; robust asymptotic tracking and disturbance rejection.

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1. Introduction

A fundamental and implicit requirement in any control system design is to achieve closed-loop stability. Additional requirements such as decoupling, (asymptotic) disturbance rejection, robustness, etc. are often to be met with. A convenient way to cope with this multi-goal design specification, is the controller parameterization method, according to which all stabilizing or pole-assigning controllers are first parameterized, while the final controller is selected within the parameterized collection by fixing the "free" parameters so as to properly achieve additional design goals.

Since the publication of the celebrated paper by Youla, Jabr and Bongiorno [1], many papers have appeared which introduce various kinds of parameterization for pole assigning or stabilizing controllers, see e.g. [2]-[9].

As it is well-known, in systematic (computer-aided) control system design the properness of the parameterized family of controllers is an important issue [2]. When the considered system is represented by the polynomial matrix fraction description (MFD), ensuring properness of the resulting controllers is all but a trivial issue. Only for single-input single-output plants, have G. Celentano and G. De Maria [6] recently proposed a new parameterization of stabilizing controllers which actually ensures properness of the resulting controllers.

This paper gives new parameterizations of proper pole assigning or stabilizing controllers in unitary feedback systems. The new parameterization involves polynomial matrices only, and ensures properness of the resulting controllers. Furthermore, since the free parameters are simply coefficients of certain matrices, the computations entailed by the subsequent controller design are very straightforward.

In what follows, we use the following notations: ∂A denotes the degree of polynomial A, or the maximum degree of the elements of polynomial vector A; $\partial c_i B$ ($\partial r_i B$) denotes the *i*th column (row) degree of the polynomial matrix B. For any rational matrix G,Π [G] and SP[G] denote the polynomial part and the strictly proper part of G, respectively; i.e $G = \Pi[G] + SP[G]$, where SP[G] is strictly proper, and $\Pi[G]$ is a polynomial matrix. Furthermore, unless otherwise specified, all matrices in this paper are polynomial matrices. Arguments of polynomial or rational matrices are dropped for the sake of simplicity.

2. Parameterization of Pole-assignment Controllers

Consider the unitary feedback control system shown in Figure 1, where P is the (rxm) matrix transfer function of the plant, while C is the matrix transfer function of the controller to be designed. Let (D_l, N_l) and (D_r, N_r) be irreducible left and right MFD of P, respectively. D_l is row reduced with row degrees n_i, $i = 1, 2, \ldots, r$; D_r is column reduced with column degrees k_i, $i = 1, 2, \ldots, m$. Let k be the maximum, with respect to i, of k_i; furthermore let (X_l, Y_l) and (X_r, Y_r) be irreducible left and right MFD of C, respectively.

For the sake of simplicity, we assume that P is strictly proper. The results presented here can easily be extended to the case when P is proper, but not strictly proper. As it is well-known [13], the pole assignment problem, or more precisely (in the multivariable case) the denominator matrix assignment problem, is equivalent to solving the Diophantine equation:

$$D_l X_r + N_l Y_r = F \tag{2-1}$$

subject to the constraint that $C := Y_r X_r^{-1}$ is proper, where F is the closed-loop denominator matrix (characteristically polynomial).

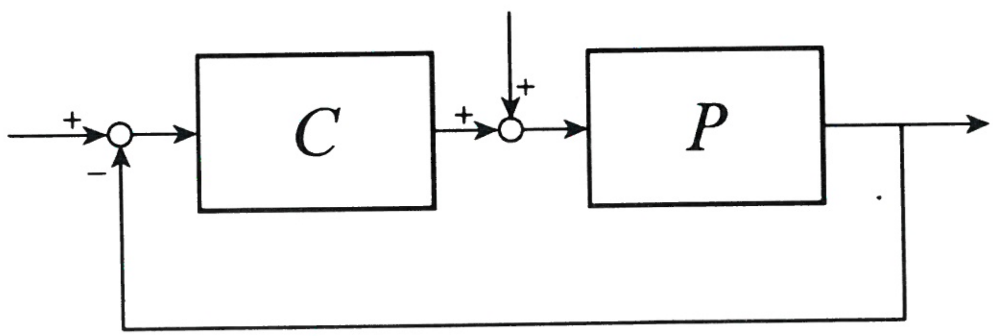

Figure 1. Unitary Feedback System

First, multi-input single-output (MISO) systems are dealt with. In this case, F is the closed-loop characteristic polynomial, and the considered pole-assignment problem admits a solution [13] if (D_l, N_l) are coprime, and $\partial F \geq \partial D_l + k - 1$ The following theorem provides a parameterization of all proper controllers that solve the pole-assignment problem.

Theorem 1

Suppose that the MISO plant P of the unitary feedback system shown in Figure 1 is strictly proper, and

$$\partial F \geq \partial D_l + k - 1 \tag{2-2}$$

where F is the desired closed loop characteristic polynomial. Then, the family of all proper pole-assignment controllers can be parameterized as follows:

$$C := Y_r X_r^{-1} \tag{2-3-a}$$

$$X_r = X_0 - N_r T \tag{2-3-b}$$

$$Y_r = Y_0 + D_r T \tag{2-3-c}$$

where (X_0, Y_0) is any solution of the Diophantine equation (2-1) such that $D_r^{-1} Y_0$ is strictly proper, while T is a parameter polynomial vector subject to the following structural constraints:

$$\partial T_i \leq \partial F - \partial D_l - k_i \quad , \quad i = 1, 2, \ldots, m, \tag{2-4}$$

where T_i is the i-th element of T, and $\partial T_i = -1$ implies $T_i = 0$.

Proof. First, we prove that C, as given by (2-3-a), is a proper pole-assignment controller if inequality (2-4) holds. As a matter of fact, one may check by direct substitution that X_r and Y_r, as given by (2-3), solve the Diophantine equation (2-1), for all T. Since $D_r^{-1} Y_0$ is strictly proper, we have:

$$\partial Y_0 < \max_i \partial r_i D_r = \max_i \partial c_i D_r = k \tag{2-5}$$

Thanks to the predictable degree property [12], and in view of D_r being column-reduced, one has:

$$\partial(D_r T) = \max_{i, T_i \neq 0} (\partial T_i + k_i) \tag{2-6}$$

From (2-5) and (2-6), it follows that

$$\partial Y_r \leq \max_i (\partial T_i + k_i) \quad ; \tag{2-7}$$

and in view of (2-4)

$$\partial Y_r \leq \partial F - \partial D_l \quad . \tag{2-8}$$

Since the plant P is strictly proper (by assumption), one has

$$\partial(N_l Y_r) < \partial F \tag{2-9}$$

whereby, in view of eq. (2-1),

$$\partial F = \partial D_l + \partial X_r \tag{2-10}$$

Inequality (2-8) and eq. (2-10) imply that C is proper. Since (2-3-b) and (2-3-c) are the general solution of Diophantine equation (2-1), then [14] for any pole assignment controller C, be it proper or not, there exists a T such that C can be written in the form of (2-3). To conclude our proof we need to show that C, as given by (2-3-a), is proper only if (2-4) holds. This part of the proof will be by contradiction. Suppose that, for some j,

$$\partial T_j > \partial F - \partial D_l - k_j \quad , \tag{2-11}$$

and note that, in view again of the predictable degree property [12],

$$\partial(D_r T) > \partial F - \partial D_l \quad . \tag{2-12}$$

From (2-2) and (2-12) it follows that

$$\partial(D_r T) > k - 1 \quad ; \tag{2-13}$$

in turn, this and (2-5) imply

$$\partial Y_r > \partial F - \partial D_l \quad . \tag{2-14}$$

Since, by assumption, P is strictly proper, i.e. $\partial D_l > N_l$, (2-14) clearly implies, in view of eq. (2-1), that C is not proper. Hence, the properness of C contradicts (2-11).

Q.E.D.

Now, the multi-input and multi-output (MIMO) case will be discussed. Precisely, we deal with the closed-loop denominator matrix assignment problem defined in [13]; matrix F of eq. (2-1) is the desired closed-loop denominator matrix.

When F is a matrix, eq. (2-1) has a solution such that $Y_r X_r^{-1}$ is proper [11] if (D_l, N_l) are left coprime, and F is a row-column-reduced polynomial matrix, with row powers n_i column powers b_i, i = 1, 2, ... , r, and

$$\partial b_i \geq k - 1 \quad . \tag{2-15}$$

The following theorem provides a convenient parameterization of all proper controllers solving the closed-loop denominator-assignment problem.

Theorem 2

Suppose P is strictly proper. For any arbitrary row-column-reduced denominator matrix F, with row powers n_i, column powers b_i, i = 1, 2, ... , r, and $\partial b_i \geq k - 1$, the proper solutions to the denominator-assignment problem can be parameterized as follows

$$C := Y_r X_r^{-1} \tag{2-16-a}$$

$$X_r = X_0 - N_r T \tag{2-16-b}$$

$$Y_r = Y_0 + D_r T \tag{2-16-c}$$

where (X_0, Y_0) is any solution of the Diophantine equation (2-1) such that $D_r^{-1} Y_0$ is strictly proper, while T is a parameter polynomial matrix subject to the following structural constraints:

$$\partial T_{ij} = \begin{cases} b_j - k_i \ , & \text{if } b_j \geq k_i \ , \\ 0 & , \text{otherwise} \ , \end{cases} \tag{2-17}$$

where T_{ij} is the (i,j)-th element of T, i=1, 2, ... , m, j=1, 2, ... , r.

Proof. First, we prove that C, as given by (2-16-a), is a proper denominator-assignment controller if condition (2-17) holds. As a matter of fact, one may check by direct substitution that X_r and Y_r, as given by (2-3), solve the Diophantine equation (2-1), for all T. Since $D_r^{-1} Y_0$ is strictly proper, then

$$\partial c_i Y_0 \le \max_h \partial c_h Y_0 = \max_h \partial r_h Y_0 < \max_h \partial c_h D_k = $$
$$= \max_h \partial r_h D_r = k \quad ;$$

namely:

$$\partial c_i Y_0 \le k - 1 \le b_i \tag{2-18}$$

Furthermore, since D_r is column-reduced, eq. (2-17) implies that

$$\partial c_i (D_r T) = \max_{j,\ T_{ji} \ne 0} (\partial T_{ji} + k_j) \le b_i \quad ,$$
$$i=1, 2, \dots, r \quad , \tag{2-19}$$

hence

$$\partial c_i Y_r \le b_i \quad , \quad i=1, 2, \dots, r. \tag{2-20}$$

Let:

$$H := \mathrm{diag}\{s^{n_1}, s^{n_2}, \dots, s^{n_r}\} \quad ,$$
$$H_c := \mathrm{diag}\{s^{b_1}, s^{b_2}, \dots, s^{b_r}\} \quad .$$

In view of inequality (2-20), $Y_r H_c^{-1}$ is proper. Since F is row-column-reduced with row-power n_i and column power b_i, it can be given the form:

$$F = H F\_ H_c \quad , \tag{2-21}$$

where $F\_$ is biproper. Similarly, D_l can be expressed as $D_l = H\ D\_$, with $D\_$ biproper. The assumed strict properness of P implies the strict properness of $H^{-1}N_l$. Now let:

$$X_r = X\_ H_c \tag{2-22}$$

and note that biproperness of $X\_$ would imply properness of C. As a matter of fact, should $X\_$ be biproper, then X_r would be column-reduced with column degrees b_i, i=1, 2, ... , r, hence the properness of C would follow from inequality (2-20). We conclude, then, the first part of the proof by showing that $X\_$ is in fact biproper.

From (2-1) and (2-21), by letting:

$$M := F\_ - (H^{-1} N_l)(Y_r H_c^{-1})$$

and recalling that $D\_$ is biproper by definition, one has:

$$X\_ = D\_^{-1} M \quad . \tag{2-23}$$

Since $H^{-1}N_l$ is strictly proper and $Y_r H_c^{-1}$ is proper, matrix M is biproper. This and biproperness of $D\_$ imply biproperness of $X\_$.

To conclude our proof (second part) we need only to show that C, as given by (2-16-a), is proper only if eq.(2-17) holds. This part of the proof will be by contradiction. Suppose that, for some (i, j),

$$\partial T_{ij} > b_j - k_i \quad ; \tag{2-24}$$

then,

$$\partial c_j (D_r T) = \max_{i,\ T_{ij} \ne 0} \left(\partial T_{ij} + k_i\right) > b_j \,. \tag{2-25}$$

Since $D_r Y_0^{-1}$ is strictly proper, it follows that

$$\partial c_j Y_r > b_j$$

namely, C is not proper. Hence, the properness of C contradicts (2-24).

Q.E.D.

Remark 1. By minor changes, Theorem 1 can be extended to the case in which P is proper, but not strictly proper.

Remark 2. It should be apparent that the parameters, namely the coefficients appearing in the elements of T, carry in a quite direct way into the coefficients appearing in the elements of X_r and Y_r , thus making such a parameterization particularly convenient. The number of free parameters is limited by the order of the controller, only.

3. Parameterization of Stabilizing Controllers

By considering again the unitary feedback system shown in Figure 1, we say that a controller is stabilizing if the overall system is closed-loop (asymptotically) stable. Referring to the Diophantine equation (2-1), the only constraint to take into consideration in dealing with stabilizing controllers is det(F) being Hurwitz. Thus, as compared with the case of pole- or denominator-assignment, there is much more freedom here in fixing the coefficients of the polynomial entries of F.

By taking advantage of the parameterization of pole-assignment controllers dealt with in the preceding section, the parameterization of stabilizing controllers can now be tackled.

Since $(D_l,\ N_l)$ are left coprime, there exist polynomial matrices (X,Y) satisfying the Bezout equation:

$$D_l X + N_l Y = I \quad . \tag{3-1}$$

Write, then, the rational matrix $D_r^{-1}Y$ as:

$$D_r^{-1}Y = \Pi[D_r^{-1}Y] + SP[D_r^{-1}Y] := Q + R \quad , \quad (3\text{-}2)$$

and note that $D_r R$ is a polynomial matrix. Similarly, write the rational matrix RF as:

$$RF = \Pi[RF] + SP[RF] \quad (3\text{-}3)$$

and note that $D_r(SP[RF])$ is a polynomial matrix as well.

Lemma 1

A particular solution to the Diophantine equation (2-1) can be expressed as follows:

$$X_{r0} = X F + N_r (Q F + \Pi[R F]) \quad (3\text{-}4\text{-a})$$

$$Y_{r0} = D_r SP[R F] \quad (3\text{-}4\text{-b})$$

where $D_r^{-1}Y_{r0}$ is strictly proper.

Proof. Checking that $D_r^{-1}Y_{r0}$ is strictly proper is straightforward. By substituting X_{r0} and Y_{r0} into the left hand side of eq. (2-1), one gets:

$$D_l X F + D_l N_r (Q F + \Pi[R F]) + N_l D_r SP[R F] =$$

$$= D_l X F + N_l D_r (Q F + \Pi[R F] + SP[R F]) =$$

$$= D_l X F + N_l D_r (Q + R) F = D_l X F + N_l Y F = F.$$

Q.E.D.

In the light of Theorem 2 and Lemma 1, the following Theorem 3 gives a parameterization of all proper stabilizing controllers for a unitary feedback control system.

Theorem 3

If P is strictly proper, all proper stabilizing controllers for the unitary feedback control system of Figure 1 can be parameterized as follows:

$$C := Y_r X_r^{-1} \quad (3\text{-}5\text{-a})$$

$$X_r = X F + N_r (Q F + \Pi[R F]) - N_r T \quad (3\text{-}5\text{-b})$$

$$Y_r = D_r SP[R F] + D_r T \quad (3\text{-}5\text{-c})$$

where X_r is column reduced with column degrees b_i, i=1, 2, ... , r. The closed-loop denominator matrix F is any arbitrary row-column-reduced matrix, with row powers n_i and column powers b_i, i=1, 2, ... , r, such that det(F) is Hurwitz. T is any arbitrary (parameter) polynomial matrix such that:

$$\partial T_{ij} = \begin{cases} b_j - k_i \ , & \text{if } b_j \geq k_i \ , \\ 0 & , \text{ otherwise} \ , \end{cases}$$

where T_{ij} is the (i,j)-th element of T, i=1, 2, ... , m, j=1, 2, ... , r.

Proof. Immediate from Theorem 2 and Lemma 1.

Remark 3. The parameterization given in Theorem 3 is a linear one, i.e. the coefficients in anyone of the elements of X_r or Y_r are linear functions of the parameters, namely of the coefficients appearing in the elements of F or T.

Remark 4. The constraint that det(F) be Hurwitz can obviously be given the form of a set of (highly non-linear) Routh-Hurwitz inequalities. A priori fixing some of the parameters with the specific purpose of reducing the stiffness (nonlinearity) of these inequalities may prove to be, on occasion, an expedient trick.

Remark 5. In the MIMO case, pole-assignment amounts to nothing but asking that det(F) be a prescribed polynomial. This is generally different from closed-loop denominator assignment [13], where all entries of F have to take on a prescribed (polynomial) value. If, in Theorem 3, the condition that det(F) be Hurwitz is substituted by det(F) be equal to a given polynomial, a parameterization is obtained of all proper pole-assignment controllers in MIMO unitary feedback systems.

Remark 6. The previously discussed constraints on F do not imply at all that the coefficients corresponding to the highest power of s, in all entries of F, must be different from zero.

4. Robust Controller Design

As an illustration of the way the results obtained in the preceding section can conveniently be used in unitary feedback control systems design, the problem of control design for robust asymptotic tracking and disturbance rejection is here considered in some detail.

Denote by $f(s)$ the polynomial made up of the modes of all reference and disturbance signals. If all such modes (the zeroes of $f(s)$) lay within the closed right half plane, then robust asymptotic tracking and disturbance rejection is achieved if [11]:

i) $m \geq r$ (there are at least as many control as controlled variables),

ii) no zero of $f(s)$ is a transmission zero of the plant P.

iii) $f(s)$ divides every element of X_r, i.e. X_r incorporates an internal model of $f(s)$.

iv) the closed-loop system is internally stable.

General systematic methods for the design of controllers that achieve robust asymptotic tracking and disturbance rejection invariantly result in a high order controller. When Theorem 2 is used to design a controller that solves the closed-loop denominator-assignment problem, it is a simple matter to incorporate in X_r an internal model of $f(s)$, (for a discussion of the robust asymptotic tracking and disturbance rejection problem within a somewhat more general parameter tuning approach, see also [15, Sect.2]). Suppose, for the sake of simplicity, that $f(s)$ has n simple zeroes $s_1, s_2, \dots, s_n$; then, selecting T such that

$$X_0(s_i) = N_r(s_i)\, T(s_i) \quad , \quad i = 1, 2, \dots, n \quad , \qquad (4\text{-}1)$$

is sufficient in fact to ensure that also condition (iii) above is met with. In conclusion, we may set up the following algorithm for the design of denominator-assignment controllers endowed with the robust asymptotic tracking and disturbance rejection property.

Algorithm

Step 1. Solve the Diophantine equation (2-1) for X_0 and Y_0, and express X_r and Y_r in the form (2-16-b,c).

Step 2. Solve eq.(4-1) subject to constraints (2-17); then, determine T.

Step 3. Substitute T in eq. (2-16), and get the controller.

In order to obtain a robust controller of minimum order, a conceivable line of attack is to start with an F of order as low as possible, then to check whether equation (4-1) admits a solution (Step 2). In case it does not, increase the order of F, and go back to Step 1; otherwise, proceed to Step 3.

When no specific requirements exist on the closed-loop denominator matrix, whence Theorem 3 can be used to design just a stabilizing controller, the order of the resulting controller may decrease further. Assuming again that $f(s)$ has n simple zeroes, $s_1, s_2, \dots, s_n$, let X_r and Y_r be in the form of eqs. (3-4). In order for matrix X_r to incorporate an internal model of $f(s)$ it is sufficient that the parameters of F and T are given a value such that

$$X(s_i)\, F(s_i) + N_r(s_i)\, (Q(s_i)\, F(s_i) + \Pi[R(s_i)\, F(s_i)]) = \\ = N_r(s_i)\, T(s_i) \qquad (4\text{-}2)$$

for $i=1, 2, \dots, n$. It is straightforward to modify the algorithm above accordingly, so as to make it suitable for the design of stabilizing controllers endowed with the robust asymptotic tracking and disturbance rejection property.

5. Numerical Examples

To illustrate some of the potential advantages entailed by the proposed parameterizations two numerical examples will now be discussed in detail.

Example 1

Consider the plant

$$P = \frac{s+5}{(s+1)^2}$$

and suppose that one has to design a controller C such as to produce a fast and smooth response of the controlled variable to a step variation of the reference signal, with robust asymptotic tracking. As it is well-known, C must incorporate an integrator.

Let F be given the form: $F = s^3 + a_2 s^2 + a_1 s + a_0$; the corresponding (asymptotic) stability region is specified by the following (Routh-Hurwitz) inequalities:

$a_i > 0 \quad , \quad i = 0, 1, 2 \quad ,$

$a_1 a_2 - a_0 > 0 \quad .$

From the Bezout identity:

$(s+1)\, x(s) + (s+5)\, y(s) = 1$

one gets

$x(s) = 0.1875\, s + 1\ , \ y(s) = -\,0.1875\, s - 0.4375\ ,$

and

$$Q(s) = -0.1875\ ,\ R(s) = \frac{-0.0625s+0.1875}{(s+1)^2},$$

$$\Pi[R\,F] = 0.0625\ [-s + (5-a_2)\,s + (5\,a_2 - a_1 - 9)]\ ,$$

$$SP[RF]=\frac{-0.0625[(5a_0-a_1-9a_2+13)s+(3a_0+a_1-5a_2+9)]}{(s+1)^2}.$$

By substituting the above expressions in eqs. (3-5) one gets:

$$X_r = s + 0.0625\ (a_0 - 5\,a_1 + 25\,a_2 - 45)$$

$$Y_r = 0.0625\ [(5\,a_0 - a_1 - 9\,a_2 + 13)\,s + + (3\,a_0 + a_1 - 5\,a_2 + 9)]$$

In order for X_r to incorporate a zero at the origin, we have to set:

$$a_0 - 5\,a_1 + 25\,a_2 - 45 = 0\ ;$$

hence, we are left with two degrees of freedom that can be used to move to a desirable region the zeroes of F, namely the poles of the closed-loop system. In any case, the corresponding controller will be a simple PI. For instance, $a_2 = 14$ and $a_1 = 145$ yield $a_0 = 420$ and

$$p_1 = -4\quad ,\quad p_{2,3} = -5 \pm j\,8.944\ .$$

As for the controller, one gets:

$$C := Y_r X_r^{-1} = \frac{12s+84}{s}\ .$$

Example 2

Consider now a strictly proper plant with 2 inputs and 2 outputs:

$$P = \begin{vmatrix} \frac{4s+1}{s^2-s} & \frac{1}{s} \\ \frac{-3s-2}{s^2-s} & \frac{-2}{s} \end{vmatrix}$$

its polynomial matrix fraction description is as follows:

$$D_l = \begin{vmatrix} s & s \\ 2 & s+1 \end{vmatrix} \qquad N_l = \begin{vmatrix} 1 & -1 \\ -3 & -2 \end{vmatrix}$$

$$D_r = \begin{vmatrix} -0.2s+0.2 & -0.4s+0.4 \\ 0.8s+0.2 & 0.6s+0.4 \end{vmatrix} N_r = \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}.$$

Suppose F is given the form:

$$F = \begin{vmatrix} a_1s+a_0 & b_1s+b_0 \\ c_1s+c_0 & d_1s+d_0 \end{vmatrix}.$$

It is then easy to ascertain that det(F) is Hurwitz if and only if $(a_1\,d_1 - b_1\,c_1)$, $(a_0\,d_0 - b_0\,c_0)$ and $(a_0\,d_1 + +a_1\,d_0 - b_1\,c_0 - b_0\,c_1)$ have all the same sign, and none of them is zero.

Resorting to Theorem 3, a stabilizing controller can be parameterized as follows:

$$X_r = \begin{vmatrix} a_1-c_1 & b_1-d_1 \\ c_1 & d_1 \end{vmatrix}$$

$$Y_r = \begin{vmatrix} 2a_0+2a_1-c_0-c_1 & 2b_0+2b_1-d_0-d_1 \\ -3a_0+2a_1-c_0-c_1 & -3b_0+2b_1-d_0-d_1 \end{vmatrix}.$$

If we select: $a_0 = a_1 = d_1 = 1, b_0 = 3, b_1 = c_1 = 0, c_0 = -1$, and $d_0 = 10$, then

$$C := Y_r X_r^{-1} = \begin{vmatrix} 1 & 0 \\ 0 & -4 \end{vmatrix}$$

is a nondynamic totally decentralized stabilizing controller. In fact, the determinant of the corresponding closed-loop denominator matrix

$$F = \begin{vmatrix} s+1 & 3 \\ -1 & s+10 \end{vmatrix}$$

zeroes at $p_1 = -6.8467$, $p_2 = -15.1533$ (closed-loop poles).

6. Conclusions

Referring to systems for which a polynomial matrix fraction description is available, this paper presents new parameterizations of *proper* pole-assigning or merely stabilizing controllers. These parameterizations have linear form and involve polynomial matrices only, so as to be pretty easy to handle.

Although the present paper deals with continuous-time systems only, the extension to discrete-time systems is almost straightforward, the major difference being the way of computing the stability region (Jury inequalities instead of Routh-Hurwitz inequalities).

ACKNOWLEDGMENT

This work has been partially supported by MURST (Italian Ministry for University and Scientific Research), while the author was visiting with the Department of Electronics and Information, at the Politecnico di Milano, Italy.

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