

# A Hierarchical Approach for Effective Loading in Flexible Manufacturing Systems

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**Abstract:** A hierarchical approach is presented to deal with the problem of batching with tool configuration, and routing problems in flexible manufacturing systems. The objectives considered here are balancing workload and minimizing the total number of cutting tools. In the top level of the approach, an appropriate tool allocation strategy is determined by structuring the relationship between parts and tools. Based on the selected tool allocation strategy, batching, tool configuration, and routing are carried out at the middle level and the bottom level of the hierarchical approach in consideration of the limited capacity tool magazine and the available machining time of each machine. A different type of heuristics is proposed for each tool allocation strategy to obtain an acceptable solution in a reasonable computation time. A numerical example is given to demonstrate the effectiveness of the proposed approach.

**Keywords:** flexible manufacturing system, group technology, machine-loading, heuristics

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## 1. Introduction

To date, various types of flexible manufacturing systems (FMSs) have been developed and implemented to cope with the low-volume large-variety production requirement.

Kusiak [7] classified FMSs into five classes based on the number of numerically controlled (NC) machines and their arrangement. A typical FMS can be expressed as the one which consists of a group of processing stations comprising numerically controlled machines, machining centres with automatic tool interchange capabilities, and robots, linked together with automatic material handling system and automatic storage system that together operate under the control of a central computer [9]. In order for an FMS to achieve the efficiency of high-volume mass production while retaining the flexibility of low-volume job shop production, careful system setup should be done on the operational problems such as planning, grouping, and scheduling [7].

This paper focuses on the loading problem in an FMS. The FMS loading problem is concerned with the allocation of operations and tools to machines subject to the resource and technological constraints of the system. To date, a relatively large number of research works have been reported on the loading problem. Stecke [10] described the following six alternative objectives of loading in FMS and formulated the loading problem for each objective as the nonlinear 0-1 mixed integer program: (1) balancing the machine processing times, (2) minimizing the number of movements, (3) balancing the workload per machine for a system of groups of pooled machines of equal sizes, (4) unbalancing the workload per machine for a system of groups of pooled machines of unequal sizes, (5) filling the tool magazines as densely as possible, and (6)

maximizing the sum of operation priorities. Kusiak [7] formulated the loading problem as 0-1 integer programming problem by simplifying the magazine tool capacity constraint where the total machining cost had been chosen as the objective function. Co et al [5] considered the loading problem in conjunction with batching and tool configuration problems in an FMS and formulated them as a mixed integer programming (MIP) problem.

Although these research works well provide considerable insights into the machine loading problem in an FMS, more realistic factors such as the number of identical tools required based on its tool life and the tool sharing among parts cannot be handled in most cases. In this paper, a hierarchical approach to solving the machine loading problem in an FMS is proposed where the above two factors can also be counted to deal with such realistic problems. The approach comprises three hierarchical levels. At the top level, tool allocation strategy is determined based on group technology (GT) concept. According to the tool allocation strategy determined at the top level, batching, tool configuration and routing are carried out at the middle level and the bottom level in consideration of the available machining time and the limited capacity tool magazine of each machine. The effectiveness of the proposed approach is also shown through a numerical example.

## 2. Problem Formulation

With reference to the loading objectives, one of the following three measures were considered in most of the research works, i.e. balancing workload, balancing machine processing time, and minimizing production cost [4]. Beside these loading objectives, total number of cutting tools to be used will also have a significant impact on the performance of an FMS. Chen et al [3] discussed the problems on the tool management for achieving the high system performance and minimum cost of tool inventory. Ayres [2] indicated that tooling accounts for 25%-30% of the fixed and variable costs of production in an automated machining environment. Reduction of the total number of tools not only can serve to reduce the tooling cost but also can bring high productivity of an FMS through the saving of the setup time for reconfiguration of the tool magazine. From this viewpoint, in this study, minimization of the total number of cutting tools is chosen as one of the loading objectives.

On the other hand, tool allocation strategy can be expected to have a strong effect on both flexibility and performance of an FMS. Gyampah et al [1] examined the four types of tool allocation strategies in association with the part selection rules. In this paper, the following three types of tool allocation strategies are considered: (1) allocate cutting tools to machines so that a part can fully be processed on a single machine (TAS1), (2) allocate cutting tools to machines so that each machining operation can be performed on a specific machine, where there are no pooling machines (TAS2), and (3) allocate cutting tools to machines in such a way that there exist some pooling machines having the same tool sets for performing some identical machining operations (TAS3). In TAS1, batch formation can be performed independently on each machine because there is no interaction among machines. On the contrary, in TAS2 and TAS3, batch formation on each machine should be synchronized with other machines, i.e. tool reconfiguration of the tool magazine should be done simultaneously for all the machines because each part in a batch must visit several machines to complete its machining processes.

Based on those tool allocation strategies, machine loading problem can be formulated as follows. Notation used in the formulations is summarized in Table 1.

Table 1. Notation

$N_p$	number of kinds of parts to be produced
$N_m$	number of machines
$N_c$	number of kind of cutting tools
$n_i$	production volume of part $P_i$
$t_{rk}$	processing time of operation $O_r$ of part $P_i$ with tool $C_k$
$\gamma_{ik}$	load rate of tool $C_k$ for machining part $P_i$
$T_{rk}$	tool life of $C_k$ for the machining operation $O_r$
$L_j$	available machining time of machine $M_j$
$Q_j$	capacity of tool magazine attached to machine $M_j$
$N_{sj}$	number of batches to be formed on machine $M_j$
$X_{iju}$	take 1 if $P_i$ is processed in $u$ -th batch on $M_j$ , otherwise 0
$X_{irju}$	take 1 if $O_r$ of $P_i$ is processed in $u$ -th batch on $M_j$ , otherwise 0
$Y_{iu}$	take 1 if part $P_i$ is machined in $u$ -th batch, otherwise 0

- $y_{rju}$  take 1 if  $O_r$  is performed in  $u$ -th batch on  $M_j$ , otherwise 0.
- $N_{oi}$  number of machining operations required for part  $P_i$
- $N_o$  number of different types of machining operations
- $L$  makespan for machining all the parts to be produced
- $E_j$  expected load rate of machine  $M_j$
- $\Delta E_j$  allowance on the load rate of machine  $M_j$

(a) For tool allocation strategy TAS1, the objective functions can be given as follows:

$$\text{Minimize } V = \sum_{j=1}^{N_m} \sum_{u=1}^{N_{sj}} \sum_{k=1}^{N_c} \left[ \sum_{i=1}^{N_p} \gamma_{ik} \cdot x_{iju} \right]^+ \quad (1)$$

$$\text{Minimize } E = \sum_{j=1}^{N_m} |UR_j - E_j| \quad (2)$$

Here,  $[t]^+$  means the smallest integer greater than or equal to  $t$ . In the objective function  $V$ ,  $\gamma_{ik}$  is a load rate of tool  $ck$  for machining part  $P_i$  which is defined by eq. (3).

$$\gamma_{ik} = n_i \sum_{r=1}^{N_o} t_{irk} / T_{rk} \quad i=1, \dots, N_p; k=1, \dots, N_c \quad (3)$$

$UR_j$  in the objective function  $E$  means a load rate of machine  $M_j$  and is defined by eq. (4).

$$UR_j = \sum_{u=1}^{N_{sj}} \sum_{i=1}^{N_p} \sum_{r=1}^{N_o} \sum_{k=1}^{N_c} n_i t_{irk} x_{iju} / L_j \quad (4)$$

Constraint on the available machining time can be written as in eq. (5).

$$\sum_{u=1}^{N_{sj}} \sum_{i=1}^{N_p} \sum_{r=1}^{N_o} \sum_{k=1}^{N_c} n_i t_{irk} x_{iju} \leq L_j \quad j=1, \dots, N_m \quad (5)$$

Eq. (6) shows the constraint on the limited capacity of tool magazine.

$$\sum_{k=1}^{N_c} \left[ \sum_{i=1}^{N_p} \gamma_{ik} x_{iju} \right]^+ \leq Q_j \quad j=1, \dots, N_m; u=1, \dots, N_{sj} \quad (6)$$

Decision variable  $x_{iju}$  should satisfy the following constraints.

$$x_{iju} = 0 \text{ or } 1 \quad i=1, \dots, N_p; j=1, \dots, N_m; u=1, \dots, N_{sj} \quad (7)$$

$$\sum_{j=1}^{N_m} \sum_{u=1}^{N_{sj}} x_{iju} = 1 \quad i=1, \dots, N_p \quad (8)$$

The constraint of eq. (8) ensures that each part is fully processed within only one batch on a single machine.

(b) In case of tool allocation strategy TAS2, the objective functions to be minimized can be represented as follows:

$$\text{Minimize } V = \sum_{u=1}^{N_b} \sum_{j=1}^{N_m} \sum_{k=1}^{N_c} \left[ \sum_{i=1}^{N_p} \sum_{r=1}^{N_o} n_i t_{irk} x_{irju} / T_{rk} \right]^+ \quad (9)$$

$$\text{Minimize } L = \sum_{u=1}^{N_b} H_u \quad (10)$$

where  $N_b$  means a minimum number of batches to be formed in the planning horizon.

Constraints on the production time on each batch can be stated as in eq. (11).

$$\sum_{i=1}^{N_p} \sum_{r=1}^{N_o} \sum_{k=1}^{N_c} n_i t_{irk} x_{irju} \leq H_u \quad j=1, \dots, N_m; u=1, \dots, N_b \quad (11)$$

In eq.(11), a time length of the production window for the  $u$ -th batch,  $H_u$ , should be determined to be a minimum value through balancing the workload among machines. The constraints on the limited capacity of each tool magazine for particular batch are given in eq.(12).

$$\sum_{k=1}^{N_c} \left[ \sum_{i=1}^{N_p} \sum_{r=1}^{N_o} n_i t_{irk} x_{irju} / T_{rk} \right]^+ \leq Q_j \quad j=1, \dots, N_m; u=1, \dots, N_b \quad (12)$$

Decision variables  $x_{irju}$ ,  $y_{rju}$  and  $y_{iu}$  should satisfy the following constraints.

$$x_{irju} = 0 \text{ or } 1 \quad i=1, \dots, N_p; r=1, \dots, N_o; j=1, \dots, N_m; u=1, \dots, N_b \quad (13)$$

$$y_{rju} = 0 \text{ or } 1 \quad r=1, \dots, N_o; j=1, \dots, N_m; u=1, \dots, N_b \quad (14)$$

$$y_{iu} = 0 \text{ or } 1 \quad i=1, \dots, N_o; u=1, \dots, N_b \quad (15)$$

$$\sum_{j=1}^{N_m} y_{iu} \leq 1 \quad i=1, \dots, N_o; u=1, \dots, N_b \quad (16)$$

$$\sum_{i=1}^{N_p} x_{irju} = y_{rju} N_{rp} \quad r=1, \dots, N_o; j=1, \dots, N_m; u=1, \dots, N_b \quad (17)$$

$$\sum_{r=1}^{N_o} \sum_{j=1}^{N_m} x_{irju} = y_{iu} N_{oi} \quad i=1, \dots, N_p; u=1, \dots, N_b \quad (18)$$

$$\sum_{u=1}^{N_b} y_{iu} = 1 \quad i=1, \dots, N_p \quad (19)$$

Eq.(16) states that the operation  $O_r$  can be assigned only one machine within a batch. Eq.(17) makes sure that each machining operation can be assigned to only one machine over all the batches, where  $N_{rp}$  means the number of kinds of different parts which have the machining operation  $O_r$ . Eq.(18) ensures that all the required operations of any part assigned to  $u$ -th batch will be performed within that batch.

(c) With tool allocation strategy TAS3, objective function and constraints are the same as those in the case of TAS2 except that the constraints on the operation assignment to machine represented by eq.(16) and eq.(17) can be omitted.

The FMS machine loading problem formulated above as a non-linear integer program belongs to the NP hard problems and the optimal solution can hardly be obtained in an acceptable computation time when a problem size becomes larger. In order to overcome this difficulty, a heuristic approach based on GT concept is proposed. The next section provides the specification of the proposed approach.

### 3. A GT Based Hierarchical Approach

As mentioned above, machine-loading problem in an FMS includes batching, tool configuration, and routing problems. In order to solve those problems simultaneously, a GT based hierarchical approach is proposed. This approach consists of the following three levels as shown in Figure 1. The top level of the hierarchy is the structuring level in which an appropriate tool allocation strategy is determined based on the production information of parts to be produced in the given planning horizon. According to the tool allocation strategy determined at the structuring level, batching, tool configuration, and routing are carried out at the middle level and the bottom level of the hierarchical approach. Specification at each level of the hierarchy is given in the following sections.

#### 3.1 Determination of the Tool Allocation Strategy

In the structuring level, selecting the tool allocation strategy out of the three types of TASs is based on GT concept. The procedure for TAS selection can be specified by the following steps.

(step 1) Arrange the given production information into the part-tool incident matrix. Each element  $\gamma_{ik}$  of the part-tool matrix is a load rate of cutting tool  $C_k$  in the machining process of part  $P_i$  which is defined in eq. (3).

(step 2) Rearrange the part-tool incident matrix into the structured part-tool matrix in such a way that the correlation among parts and cutting tools is maximized. The quantification theory III

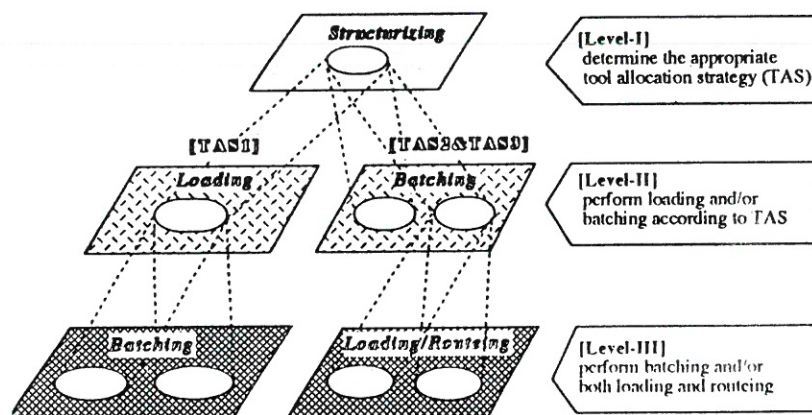


Figure 1. Hierarchical Loading Approach Based on GT

on the correlation analysis can be used for this purpose [8].

The fundamental idea of this method can be stated as follows. First, assign unknown parameters  $u_i$  ( $i=1, \dots, N_p$ ) and  $v_j$  ( $j=1, \dots, N_c$ ) to parts and tools respectively. Next, determine the value of each parameter  $u_i$  so that any two parameters may have closer values to each other as the corresponding two parts are more similar in the use of cutting tools. In the same way, determine the value of each parameter  $v_j$  so that any two parameters may have closer values to each other as the corresponding two cutting tools are used for more similar parts. In this sense, these parameters  $u_i$  and  $v_j$  can be referred to as similarity indices for parts and tools respectively. Here the maximum value of the correlation coefficient  $\rho$  is equal to the maximum eigenvalue except 1 of the following characteristic equation:

$$\sum_{j=1}^{N_c} c_{kj} z_j = \rho^2 Z_k \quad k=1, \dots, N_c \quad (20)$$

where coefficient  $c_{kj}$  is the constant given by the part-tool incident matrix. The similarity indices  $u_i$  and  $v_j$  can be determined according to the eigen vector  $Z=[z_1, \dots, z_i, \dots, z_{N_c}]$  corresponding to the maximum eigen value except for  $\rho=1$  [8]. Finally, obtain the structured part-tool matrix by rearranging parts and cutting tools in decreasing order of the similarity indices  $u_i$  and  $v_j$  respectively.

Here it should be noted that a high value of  $\rho$  near 1 indicates a strong correlation between parts and cutting tools, which will make it easy to construct explicit part-tool groups having less interactions with one another. In this case, it can be recommended to select the tool allocation strategy TAS1 in which all the machining processes of parts within a part-tool group can fully be processed on the assigned machine. For example, consider the part-tool incident matrix shown in Figure 2 (a) where symbol \* is used in place of the load rate  $\gamma_{ik}$ . By applying the quantification theory III to the incident matrix, the structured part-tool matrix shown in Figure 2 (b) is

obtained where the maximum eigenvalue  $\rho$  except 1 is 0.97.

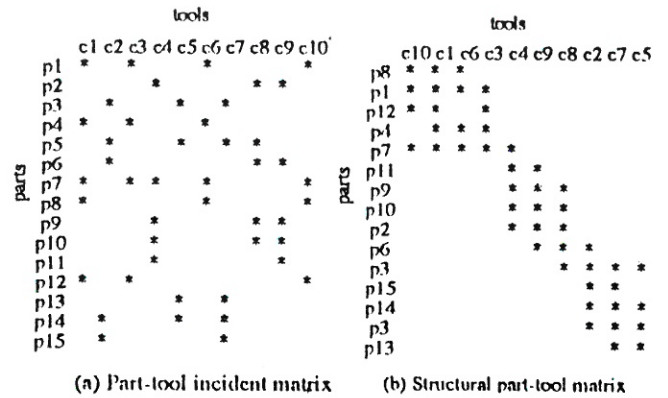


Figure 2. Construction of the Structured Part-tool Matrix

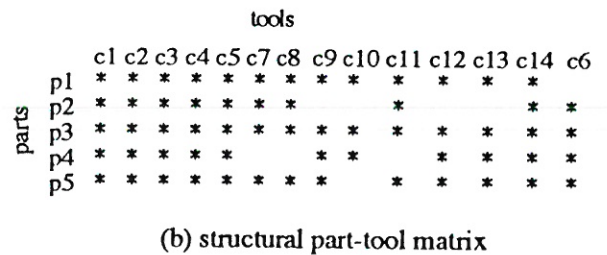
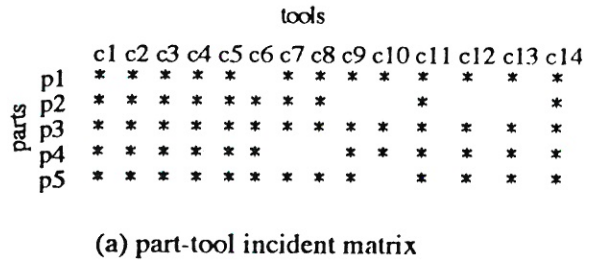
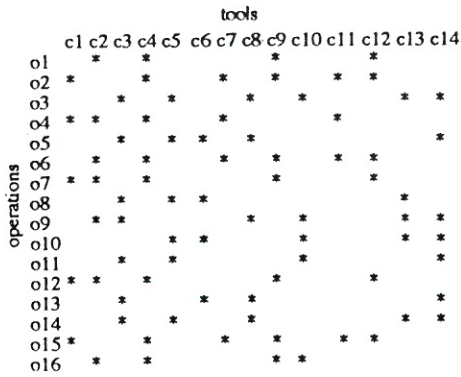
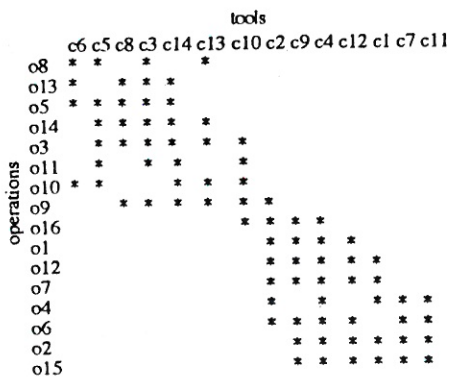


Figure 3. Structured Part-tool Incident Matrix with Small  $\rho$

On the other hand, if the maximum eigenvalue  $\rho$  except 1 has a low value, distinct part-tool groups



(a) operation-tool incident matrix



(b) structured operation-tool matrix

**Figure 4. Construction of the Structured Operation-tool Relation Matrix**

cannot be constructed due to the poor relationship among parts and tools. In this case, the relationship between operations and tools can be explored instead of considering the relationship between parts and tools. For instance, consider the part-tool incident matrix as shown in Figure 3 (a). Figure 3 (b) shows the resultant structural part-tool matrix where the maximum eigenvalue  $\rho$  is 0.132. As shown in Figure 3(b), there can be seen no distinct part-tool structure. Figure 4(a) shows the operation-tool incident matrix obtained from the same production information as the one for the part-tool incident matrix of Figure 3(a). Figure 4(b) shows the structured operation-tool matrix where the maximum eigenvalue except 1 is 0.951.

From the above example, it is found to be reasonable to consider TAS2 or TAS3 as the tool allocation strategy in case the correlation among parts and tools is low. In this case, proceed to step 3 to select the

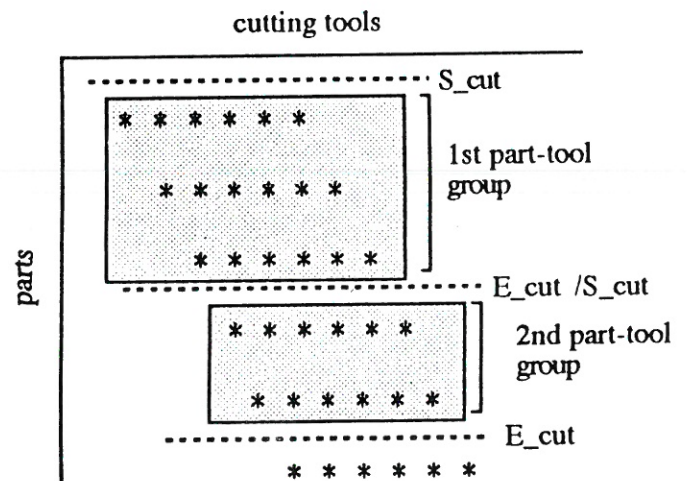
appropriate tool allocation strategy from those two strategies, TAS2 and TAS3.

(step 3) In both TAS2 and TAS3, parts are grouped first into batches by taking into account the magazine capacity of all machines. Then machining operations of each part in a batch are distributed among machines, so that each part has to visit several machines to complete its all machining processes. Here, the selection of either TAS2 or TAS3 depends on the objective to be selected as the primary one. If the minimization of the total number of tools is treated as the primary objective, TAS2 will be selected as the tool allocation strategy. On the contrary, if balancing workload among machines is selected as the primary objective, TAS3 will be chosen as the tool allocation strategy.

Following the above procedure, once the appropriate tool allocation strategy is determined for the given production requirement, batching, loading, and routing are carried out at the middle level and the bottom level of the hierarchical approach. The concrete process at these two levels strongly depends on the tool allocation strategy adopted. In the following sub-sections, the detailed loading procedure is presented for each tool allocation strategy, TAS1, TAS2 and TAS3.

### 3.2. Heuristic Loading Procedure for TAS1

For tool allocation strategy TAS1, the assignment of both parts and cutting tools to machines can be performed so that each part should fully be processed



**Figure 5. Generation of Part-tool Groups**

for its all machining processes on a single machine. To achieve this loading task, a GT based heuristic approach has been developed by Kato et al [6]. In this heuristic approach, both formation of part-tool groups and their assignment to machines are carried out simultaneously at the loading level (the middle level in Figure 1) and then formation of batches on each machine is performed at the batching level (the bottom level in Figure 1). The detailed procedures of the heuristic approach is given below.

### 3.2.1. Loading Procedure (Level-II)

In the tool allocation strategy TAS1, the objective of balancing workload among machines is treated as a constraint shown in eq. (21) where  $E_j$  and  $\Delta E_j$  are the expected load rate and its allowance on machine  $M_j$  respectively.

$$|UR_j - E_j| \leq \Delta E_j \quad j=1, \dots, N_m \quad (21)$$

The allowance of the load rate  $\Delta E_j$  can also be adjusted in an interactive fashion to obtain a feasible solution.

At the loading level, formation of part-tool groups is carried out by applying two types of horizontal cut,  $S\_cut$  and  $E\_cut$ , to the structured part-tool matrix. To make it easy to understand this procedure, consider the structured part-tool matrix as shown in Figure 5. At the beginning of the procedure, both  $S\_cut$  and  $E\_cut$  are set at the first row of the structural part-tool matrix to form a provisional part-tool group containing only a kind of part. Then a total machining time of this block is calculated to check on each machine whether the constraint of eq. (21) is satisfied or not.

If there exists no machine for satisfying this constraint of the provisional part-tool group,  $E\_cut$  is shifted downward by one row in the structural matrix and then the same procedure is repeated until the machine of satisfying eq.(21) can be found. Once the part-tool group is constructed,  $S\_cut$  is reset at the row immediately after the  $E\_cut$ .  $E\_cut$  is then reset at the same row as the renewed  $S\_cut$ . Then the same procedure is repeated until  $E\_cut$  is set at the last row of the structural part-tool matrix. In this way, the formation of mutually independent part-tool groups and their assignment to the machines can be carried out sequentially in consideration of the desired machine load rate and/or load balance among machines.

In this loading stage, there exist several machines to be loaded for the the same part-tool group. Moreover,

different  $E\_cuts$  may be possible for the same machine under the constraint of the desired machine load rate. This combinatorial nature makes the loading problem more complex as the system becomes larger. The branch and bound method can be used for such combinatorial problem to search solutions effectively. This method comprises the following two operations. (1) Select a node with the least lower bound  $V_w$  of the total number of required tools, and then form new child nodes by constructing the part-tool groups remaining to be selected in the part-tool structured matrix (branching operation). (2) Bound all the nodes having a greater value of  $V_w$  than a minimum value of the feasible solutions already found at the preceding node (bounding operation). The lower bound  $V_w$  of the total number of required tools can be given by eq.(22).

$$V_w = \sum_{j=1}^{N_m} \sum_{u=1}^{N_{sj}} R_{qju} + \sum_{k=1}^{N_c} \left[ \sum_{i \in R_m} \gamma_{ik} \right]^+ \quad (22)$$

Here,  $R_{qju}$  is a total number of tools to be reloaded at the  $u$ -th batch on machine  $M_j$  by taking account of the sharing tools among successive batches, which is determined at the batching level of the hierarchical approach, and  $R_m$  is a set of parts remaining to be assigned to the focussing node.

### 3.2.2. Batching Procedure (Level-iii)

At the batching level, forming part groups to be processed without tool reloading under the constraint of limited capacity tool magazine is performed. There exist the following two problems to be solved at this level: (1) determine the minimum number of batches. (2) select the parts to be processed in each batch and form tool groups to be used. The minimum number of batches can be determined in a way similar to that of forming part-tool groups. At this step,  $E\_cut$  is set for each batch so as to contain the parts as much as possible under the given magazine capacity constraint. When  $E\_cut$  reaches the bottom row of the part-tool group matrix, the number of  $E\_cuts$  gives the minimum number of batches to be constructed.

While the formation of each batch can easily be performed in the above manner, the content of the batch is not necessarily optimal from the viewpoint of the minimization of tools. Hence there is a need to reorganize those batches. Reorganization of the batches can be performed effectively by the branch

and backtrack method [6]. The fundamental idea of this method can be stated as follows. Reorganization of the  $i$ -th batch is performed by shifting its  $E\_cut$  upward by one row in the part-tool group matrix assigned to the machine under consideration. This operation causes the reorganization of  $(i+1)$ th batch because  $S\_cut$  of the  $(i+1)$ th batch is shifted automatically upward by one row of the part-tool group matrix. Then the number of required tools for the  $(i+1)$ th batch is calculated by eq.(23) and is checked to see if the constraint of eq.(6) is satisfied.

$$NT_{ju} = \sum_{k=1}^{Nc} [ \sum_{i \in Sju} \gamma_{ik} ]^+ \quad j=1, \dots, Nm; u=1, \dots, Nsj \quad (23)$$

If the constraint of eq.(6) is satisfied for the reorganized  $(i+1)$ th batch, then  $(i+2)$ th batch is tried to be reorganized by shifting its  $E\_cut$  upward by one row in the part-tool group matrix and the same procedure is repeated. On the contrary, if eq.(6) cannot be satisfied for that  $(i+1)$ th batch, reorganization of the  $(i+1)$ th batch is retried by shifting its  $E\_cut$  upward by one row and the same procedure is repeated. Backtracking from the  $i$ -th batch to the  $(i-1)$ th batch is provoked when one of the following three cases occurs (a)  $i$ -th batch is the last batch, i.e.  $Nsj$ -th batch (b)  $i$ -th batch has no part within itself, (c) no more shift of  $E\_cut$  can be done because of violating the constraint of eq. (6) for  $(i+1)$ th batch. At the beginning of the procedure, the  $(Nsj-1)$ th batch is set as a batch to be reorganized. In this way, reorganization of the batches can easily be performed by taking the magazine capacity constraint into account.

The number of tools required on each machine can also be reduced by taking the sharing of tools among successive batches into account. The following equations should be satisfied to reduce tool  $c_j$  by one.

$$\Delta c_{i+n-1,j} \geq 1 \quad (24)$$

$$\Delta c_{i+k-1,j} = [\gamma c_{i+k-1,j}]^+ - (\gamma c_{i+k-1,j} - \Delta c_{i+k-2,j}) \quad k=1, \dots, n \quad (25)$$

$$\Delta c_{0,j} = 0 \quad (26)$$

$$\gamma c_{k,j} = \sum_{i \in Sju} n_i t_{ij} / T_{ik} \quad (27)$$

Eq.(24) indicates that the total number of tool  $c_j$  can be reduced by one at  $(i+n-1)$ th batch only if the remaining tool life of tool  $C_j$  in  $(i+n-1)$ th batch,  $\Delta c_{i+n-1,j}$  is greater than or equal to 1.  $\Delta c_{i+n-1,j}$  can be obtained by sharing tool  $c_j$  among successive  $n$  batches as specified in eq.(25).  $\gamma c_{k,j}$  as defined by eq.(27) means a total load rate of tool  $c_j$  in  $k$ -th batch. It should be noted in applying tool sharing strategy for saving the required tools that the number of tools to be loaded in the tool magazine increases by one on each batch from  $(i+1)$ th batch to the  $(i+n-2)$ th batch. This fact should be taken into account when checking the constraint eq. (6) of the limited magazine capacity.

### 3.3. Heuristic Loading Procedure for TAS2/TAS3

In tool allocation strategies TAS2 and TAS3, the determination of the number of batches and their contents are first carried out and then the assignment of operations in each batch is performed in consideration of the workload balance among machines and the limited capacity tool magazines.

#### 3.3.1. Batching Procedure (Level-II)

Selection of parts to form a batch from the given production requirements is a large sized combinatorial problem especially for a low-volume large-variety production area. In this stage, several heuristic rules can be employed as the effective tools for batch formation. For example, Amoako-Gyampah et al [1] considered the following three types of part selection rules: (i) assign higher priorities to parts that require the largest number of tools (LNT rule), (ii) assign higher priorities to parts with the smallest number of tool requirements (SNT rule), and (iii) assign higher priorities to parts with the earliest due date (EDD rule).

All the parts included in a batch should be processed completely without reloading tools in the tool magazines. Hence the minimum number of batches to be formed is determined from the viewpoint of tool magazine capacity as in eq.(28).

$$Nb_{min} = \sum_{k=1}^{Nc} [ \sum_{i=1}^{Np} \gamma_{ik} ]^+ / \sum_{j=1}^{Nm} Q_j \quad (28)$$

The upper bound of the number of parts to be included in a batch is determined by eq. (29).



$$\sum_{k=1}^{N_c} \left[ \sum_{i \in R_p} \gamma_{ik} \right]^+ \leq \sum_{j=1}^{N_m} Q_j \quad (29)$$

where  $R_p$  means a set of parts to be selected for constructing a batch.

### 3.3.2. Loading Procedure (Level-III)

The assignment of both operations and cutting tools to machines for each batch constructed at the batching level is carried out in the following heuristic procedure comprising three steps in consideration of the limited capacity of tool magazines and the workload balance among machines.

(step 1) Construct the operation-tool incident matrix for a generated batch in level-II and the structure incident matrix in the same way as the one at the structuring level except that each element of the incident matrix is a machining load rate of the cutting tool  $ck$  for the operation  $O_r$  which can be calculated by eq.(30) and eq.(31) for TAS2 and TAS3 respectively.

$$q_{rk} = \sum_{i \in R_p} n_i t_{irk} / T_{rk} \quad r=1, \dots, N_o; k=1, \dots, N_c \quad (30)$$

$$q_{irk} = n_i t_{irk} / T_{rk} \quad i=1, \dots, N_p; r=1, \dots, N_o; k=1, \dots, N_c \quad (31)$$

(step 2) Form operation-tool groups to be assigned to machines by applying the same horizontal cuts as the ones at the loading level for TAS1 to the structured operation-tool matrix by taking account of the tool magazine capacity and the workload balance among machines. A branch and bound method can be used for the effective search of the optimal solution. Here the objective for TAS2 is to minimize the total number of cutting tools and the workload balance among machines is treated as one of the constraints to be satisfied. On the other hand, for TAS3, the objective is to minimize the workload unbalance among machines and the total number of cutting tools required is treated as a constraint.

(step 3) If any feasible solution cannot be obtained for the batch under consideration because of the violation of the limited capacity of tool magazine, eliminate the part of the batch which was selected last in the batching procedure of level-II. Then return to step 1 and repeat the

same procedure until a feasible solution can be obtained.

## 4. Numerical Example

To demonstrate the effectiveness of the proposed approach, let us consider the hypothetical manufacturing data shown in Table 2. There are 32 types of parts to be produced and five machines to be used. Each part is machined by using some of 45 types of cutting tools in its machining process. Figure 6 shows the part-tool structured matrix obtained at the structuring level, where the maximum eigenvalue except 1 is 0.980. From this structural part-tool matrix, it seems rational to select TAS1 as the tool allocation strategy for the given production requirements. Figure 7 shows the branch and bound tree generated at the loading level for TAS1. The desired machine load rate and its allowable deviation for each machine are also given in Table 2. Table 3 shows the resultant batches constructed at the batching level for TAS1. In Table 3, the number of tools required is presented for each batch along with the number of tools saved by sharing tools among batches (bracketed value). Here, note that there exist several alternatives for constructing batches on machine M1, M3, and M5. In this case, a solution can be given by selecting an appropriate alternative through interactive manner in consideration of such criteria as the total load rate of the machine. Table 4 shows the result obtained by combining solution 5 of node 4, solution 1 of node 7, solution 3 of node 21, solution 1 of node 34, and solution 2 of node 35 in Table 3.

Table 2. Example Data

(a) Information on the parts to be produced

Part	Workload	Part	Workload	Part	Workload
p1	17.41	p12	10.29	p23	18.71
p2	23.38	p13	17.78	p24	10.82
p3	16.17	p14	10.76	p25	18.24
p4	23.86	p15	17.19	p26	15.28
p5	12.63	p16	16.17	p27	16.00
p6	14.50	p17	16.00	p28	18.75
p7	14.06	p18	15.10	p29	20.31
p8	21.35	p19	18.23	p30	16.84
p9	16.37	p20	26.24	p31	23.86
p10	15.53	p21	21.12	p32	18.24
p11	16.00	p22	19.77		

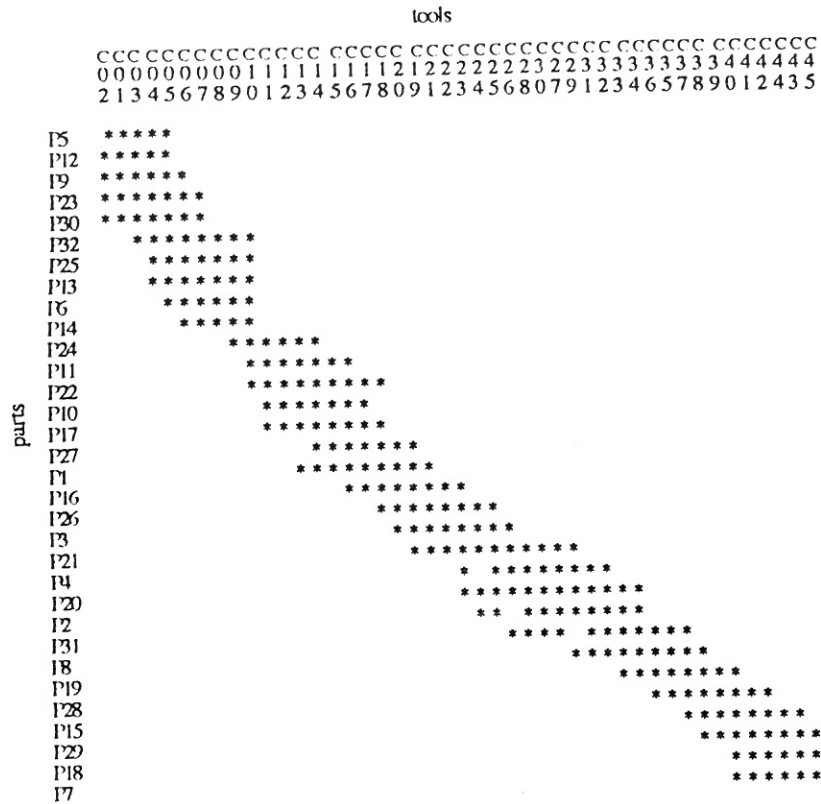


Figure 6. Structured Part-tool Matrix of the Example

Table 3. Alternatives to Batch Formation

Node	batch1	batch2	batch3	T.N.T.
SOL.1	p1-p5	p6-p8	p9-p10	37
SOL.2	p1-p5	p6-p7	p8-p10	37
SOL.3	p1-p5	p6	p7-p10	37
SOL.4	p1-p4	p5-p6	p7-p10	37
SOL.5	p1-p3	p4-p6	p7-p10	37
Node	batch1	batch2	batch3	T.N.T.
SOL.1	p11-p13	p14-p15	p16-p17	31
Node	batch1	batch2	batch3	T.N.T.
SOL.1	p18-p20	p21-p22	p23	33
SOL.2	p18-p20	p21	p22-p23	33
SOL.3	p18-p19	p20-p21	p22-p23	33
SOL.4	p18	p19-p21	p22-p23	33
Node	batch1	batch2	batch3	T.N.T.
SOL.1	p24	p25	p26	18
Node	batch1	batch2	batch3	T.N.T.
SOL.1	p27-p28	p29-p31	p32	26
SOL.2	p27-p28	p29-p30	p31-p32	26
SOL.3	p27-p28	p29	p30-p32	26
SOL.4	p27	p28-p29	p30-p32	26

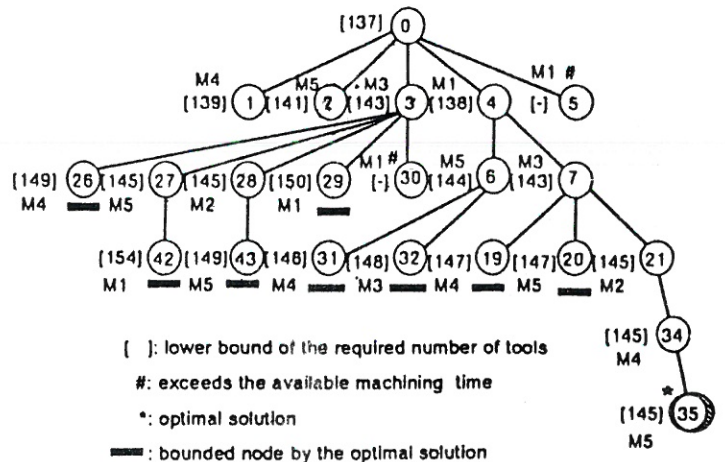


Figure 7. Branch and Bound Tree

**Table 2. Example Data**

(b) Information on the machines to be used

Machine	A.M.T.	Magazine	$E_j$	$\Delta E_j$
M1	180	18	90%	$\pm 5\%$
M2	150	20	80%	$\pm 3\%$
M3	140	14	80%	$\pm 7\%$
M4	100	14	70%	$\pm 5\%$
M5	120	16	80%	$\pm 7\%$

A.M.T.=available machining time  
 $E_j$ =desired load rate of machine  $M_j$   
 $\Delta E_j$ = allowable deviation from  $E_j$

**Table 4. Machine Loading Result**

Machine	Assigned parts	Load (L.R.)	T.N.T.
M1	p5, p6, p9 p12, p13, p14 p23, p25, p30 p32	154.36 (85.76 %)	37
M2	p3, p4, p16 p20, p21, p26	118.84 (79.23 %)	33
M3	p1, p10, p11 p17, p22, p24 p27	111.53 (79.66 %)	31
M4	p2, p8, p31	68.59 (68.59 %)	18
M5	p7, p15, p18 p19, p28, p29	103.64 (86.37 %)	26

L.R. = Machine load rate

T.N.T.= Total number of required tools

### 5. Conclusions

In this paper, a GT based hierarchical approach was proposed to deal with the machine loading problem including batching, tool configuration, and routing problems in an FMS. Three types of tool allocation strategies were presented so as to obtain the feasible solutions in consideration of the workload balance among machines, the available machining time, and the limited capacity of tool magazine. To validate the effectiveness of the proposed approach, a numerical example consisting of 5 machines to be used and 32 parts

to be produced by using some of the 45 types of cutting tools was considered. Future research should examine the effect of the loading strategy on the scheduling results especially for the strategies TAS2 and TAS3.

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