DESIGN AND REAL-TIME IMPLEMENTATION OF A STRUCTURALLY STABLE COMPENSATOR FOR A BIBO UNSTABLE MULTI-MODEL SYSTEM

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ABSTRACT
The paper confronts a real-time robust control of a BIBO unstable multi-model system, of which design drawbacks are not few, with a structurally stable compensator, the implementation of which is enabled by heuristic improvements.
KEYWORDS : robust control, real-time control, microcomputer-based controllers.

1. INTRODUCTION

Although the control strategies meant for multi-model plants include tendencies of parameterizing controllers (with solutions requiring quite sophisticated implementations), the desired specifications can often be obtained using constant robust controllers (successfully implementable on low cost environments), designed for a nominal work point of a plant, using state-space methods [1] or H_∞ approaches [2].

However, real-time robust control of BIBO unstable multi-model systems complicates things because of the quantization and overflow effects, and because of theoretically making it possible that the closed-loop stability for poles "ideally" assigned to a nice stability region be lost [3]. In such cases, the response quality could be significantly improved by heuristic logic.

The paper is a case-study on the observer-based control of a frequently referred example of BIBO unstable multi-model system: a loading bridge [4], [5], [6] with large variations in both physical parameters (cable length and load mass). [7] proved that response specifications could not be received gradually if PID controllers were used and [8] offered the results of a simulation of the observer-based state feedback control. Section 2 describes an experimental model used by laboratories. The following two sections deal with the design of a structurally stable compensator and with its numerical implementation. 4) Section 5 is concerned with deadlocks in real-time operation, while Section 6 provides a "tend-to-be" optimum solution in such an approach. The final Section describes the controller in nuce.

2. EXPERIMENTAL MODEL

The fourth order model of the system concerned:

$$
\begin{cases}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & f_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & f_{43} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ g_2 \\ 0 \\ g_4 \end{bmatrix} u \\
y = \begin{bmatrix} 1010 \end{bmatrix} \begin{bmatrix} x_1 x_2 x_3 x_4 \end{bmatrix}
\end{cases}
$$

was simulated in an adequate scaled form on an analog computer.

The control input **u** is the trolley-driven element and the load position represents the measurable output **y**. The state variables are defined as: x_1- trolley position, $x_2 = x_1$ and x_3- cable angle, $x_4 = x_3$.

The family induced by this multi-model plant is parameterized by the values of the load mass **m** and the cable length **l**, according to:

$$
\begin{cases}
f_{23} = \mathbf{m}/1000 \\
f_{43} = -(\mathbf{m}+1000)/(100\,\mathbf{l}) \\
g_2 = 1/1000 \\
g_4 = -1/(1000\,\mathbf{l})
\end{cases}
$$

The laboratory model implemented on the analog computer allows the simulation of all the parameter variations that can occur in the real operation. The difficulties in designing and implementing the controller arise from the structural particularities of the plant, with two null eigenvalues (which correspond to a Jordan block of dimension 2) and two eigenvalues on the imaginary axis; none of them can be preserved by the closed-loop structure, and thus a greater sensitivity against the plant parameter variations manifests.

A first-order model:

$$U(s) = G_d(s)\, V(s), \quad G_d(s) = \frac{1}{T_d s+1}, \quad T_d = 500 \text{ mseconds}$$

is used to simulate the d.c. electric drive of the plant supplying the system input u, with v denoting the compensator output.

Since the current operation of the plant concerned involves large parameter variations: 500 kg $\leq$ **m** $\leq$ 2,000 kg, 2 m $\leq$ **l** $\leq$ 10 m, the design must by all means finalize a controller rendering stability and the performant characteristics to a parameterized family of BIBO unstable systems.

3. CONTROLLER DESIGN

A discrete-time structurally stable compensator (incorporating a state estimator and an exogenous internal model) [1] has been designed by means of CASAD [9] for a work point representing the average values of the parameters **m** = 1,500 kg and **l** = 10 m. An exogenous step was considered. The design of compensator gets simplified if the state-variable associated with the d.c. electric drive is overlooked in computing the feedback law [10]. Overlooking this state-variable is encouraged by a transient response which takes short time and matches with the described closed-loop dynamics.

The T = 0.5 sec. sampling period used for the discretization of the system resulted from the following considerations:

a) The CPU characteristics used for implementing the real-time application, giving a lower bound to T.

b) Better controllability of the plant is given by $T \leq \Pi/(4\,\omega)$ [4], ω denoting the frequency determined by the two imaginary eigenvalues; for the considered parameter values $\omega = \sqrt{2.5}$. Smaller values for T do not improve notably the controllability.

Although by implementing the input-output equation of the compensator, fewer floating point multiplications than the state- space representations are required, the latter is preferable because an overflow is easier to prevent when changing the set- point.

For the closed-loop specifications: time response about 30 sec. and overshoot about 5% (corresponding in s-domain to two dominant poles $s_{1,2} = -0.141 \pm j\,0.141$ and to other three located in $s_{3,4,5} = -1$), the following matrix has been obtained for the state feedback:

$$H = [20.293196,\ 100.233252,\ 0.536246,\ -0.659463]$$

and

$$H_I = -0.9518406$$

for the state variable introduced by the internal model [1].

The state estimator, of which dynamics corresponds to the assigned eigenvalues $z_i = e^{-2.5} = 0.08208499$, i = 1,2,3,4, has the equation:

$$\hat{x}(k+1) = F\,\hat{x}(k) + L\,y(k) + B\,u(k),$$

where:

$$F = \begin{bmatrix} -4.4291603. & 0.5 & -0.0525122 & 0.0003028 \\ 1.4788354 & 1 & 0.0215311 & 0.0017793 \\ 235.06789 & 0 & 3.0541196 & 0.4495202 \\ -645.42279 & 0 & -7.5780285 & 0.7034407 \end{bmatrix},$$

$$L = \begin{bmatrix} 5.4291603 \\ -1.4788354 \\ -235.06789 \\ 645.42279 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0012117 \\ 0.0046971 \\ -0.1186237 \\ -0.4495202 \end{bmatrix}.$$

The designed compensator was validated by the numerical simulation of the closed-loop for several values of the physical parameters **m** and **l**. The controller output u and the closed-loop system output are shown in Figure 1-a and Figure 1-b respectively, for three different pairs **(m,l)**: (m = 1,500 kg, l = 10m, m = 1,500kg, l = 2m, m = 1,800kg, l = 12m) for a set-point change from 0 to 1 and a total control time of 50 seconds.

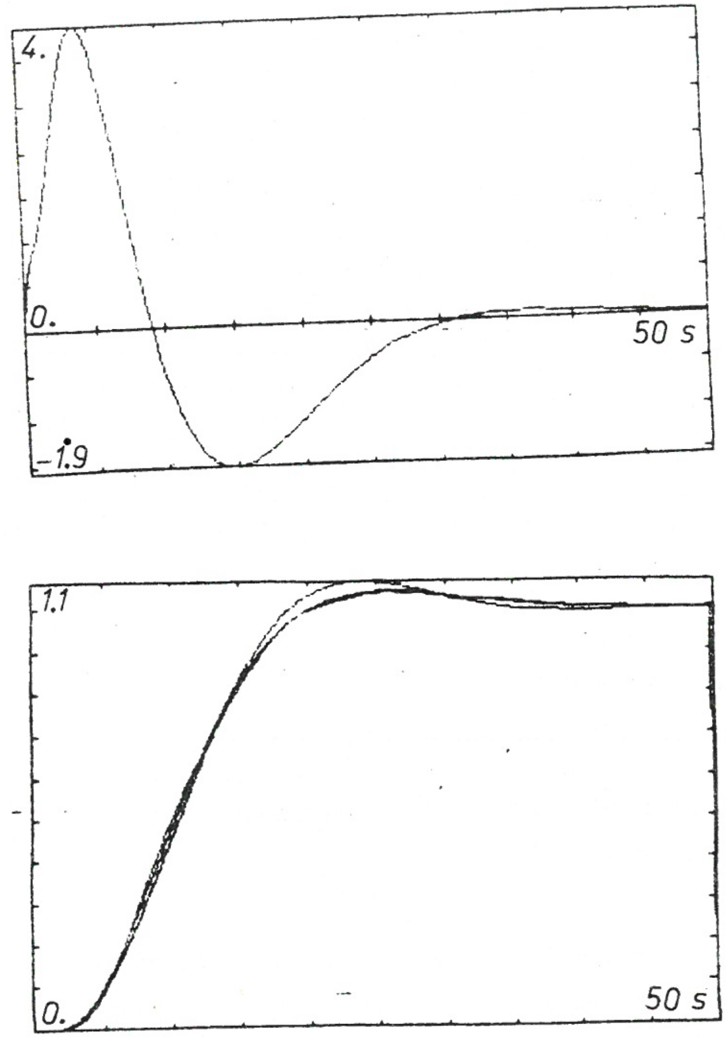


Figure 1. Numerical simulation of the closed-loop system, the structurally stable compensator incorporating a full order estimator and an internal model of the exogenous:

a) command supplied by the compensator v

b) closed-loop output y

Three pairs of values are considered for **m** and **l**

(**m** = 1,500kg, **l** = 10m, **m** = 1,500kg, **l** = 2m, **m** = 1,800kg, **l** = 12m).

A rigorous analysis of the closed-loop robustness was made in order to find that range of the parameters **m** and **l** within which changes of the operating conditions could preserve satisfactory transient response performances. The dominant parcel **D** and the left unbounded region **N** containing the neglectable closed-loop poles in **s-Plane** were mapped into the **(m,l)** - **Plane**, defining the region **R** as shown in Figure 2. The boundaries of **R** were computed using the facilities of symbolic manipulation software.

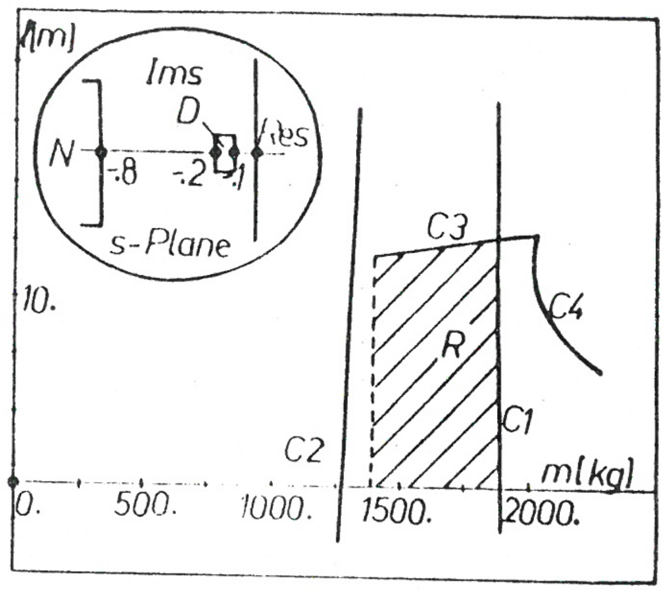


Figure 2. Robustness regions D ∪ N and R in s-Plane and in (m,l)-Plane.

4. REAL-TIME CONTROLLER IMPLEMENTATION

The implementation of the compensator requires the parallel running of the following tasks:

(T1) reads the system output (A/D conversion), computes the command, activates, if necessary, an anti-windup algorithm, sends the command to the actuator (D/A conversion).

(T2) modifies the set-point according to the value read from an input device.

(T3) displays on an output device the values useful for supervising the experimental model.

Task (T1) has been assigned the top priority and is activated at each sampling instant; (T1) interrupts the execution of (T2) or (T3). Task (T3) has the lowest priority.

In order that a synchronized co-operation of the tasks should be achieved, a simple programming technique, based on CP/M facilities, has been devised for overcoming the main drawbacks of the operating system: the single task operation and non- reentrance of libraries. All the service routines corresponding to each interrupt level are separately linked; intertask communication is carried out by mail-box type operations and non-interruptable 16-bit transfers; data are exchanged via a common memory bank.

A comparative study will point to the optimum solution (from the point of view of the execution time and of the machine executable program length) as being Assembler and Fortran codes versus Assembler and Pascal or Assembler and C. A slightly modified code which adequately replaces the calling sequences to BDOS can be used for obtaining a PROM version. The length of the resulting code is of 13 k.

5. SHORTCOMINGS OF THE REAL-TIME IMPLEMENTATION

The command v supplied by the controller to the actuator, the input u and the output y of the system were simultaneously plotted on X-Y recorders using step set-points and producing step disturbances.

Experiments revealed some main drawbacks of the compensator designed according to the previous section: the transient response did not match with the prescribed specifications and oscillations occurred in the steady-state (in contrast with the nice response obtained by numerical simulation). Unlike the case of BIBO stable plants where the controller design taking a balanced state-space representation could diminish the negative effects of the quantization errors, for this case-study no improvement was obtained.

The quantization errors of 5-10 mV must be seen in the context of these significant errors which can be introduced by the term L y(k) and make clear-cut distinction between the real states and the estimated states, especially for the small values of **y**. This phenomenon imposes an upper bound for $||L||$ or, equivalently, a lower one for the sampling period and module of the assigned eigenvalues. No less might have the small comands **u** an undesirable effect when the great values of the state feedback matrix H can lead to large parasitic commands.

On the other hand, if experiments are performed on a laboratory model, connecting the state feedback directly to the physical states (i.e. without state estimator) (Figure 3), one can analyse the major influence of the inherent errors introduced by the estimator when the plant is BIBO unstable, offering a thorough understanding of the part played by the plant stability in observer-based feedback control.

At the same time a comparison between Figure 1-a and Figure 3-a reveals similar wave forms for the signal v, while Figure 3-a let the effects of quantization be known.

6. IMPROVING RESPONSE QUALITY BY HEURISTICS

The oscillations occurring in the steady-state are substantially diminished and the desired transient specifications are obtained quite well (Figure 4) when the compensator computes the command using the first three estimated states only, because the relative errors made in estimating the fourth state prove to be much greater than those in the other states (due to the great value of the fourth element in L (645.42279)), whereas the corresponding gain in the state feedback H is small enough (-0.659463). A faster variation of the command supplied by the compensator (Figure 4-a) is easy to notice in comparison with the compensator without observer (Figure 3-a). Moreover, the errors induced by the estimated state $\hat{x}_4$ (x_4 representing an angular velocity) are greater than those in estimating x_3 which represents an angle.

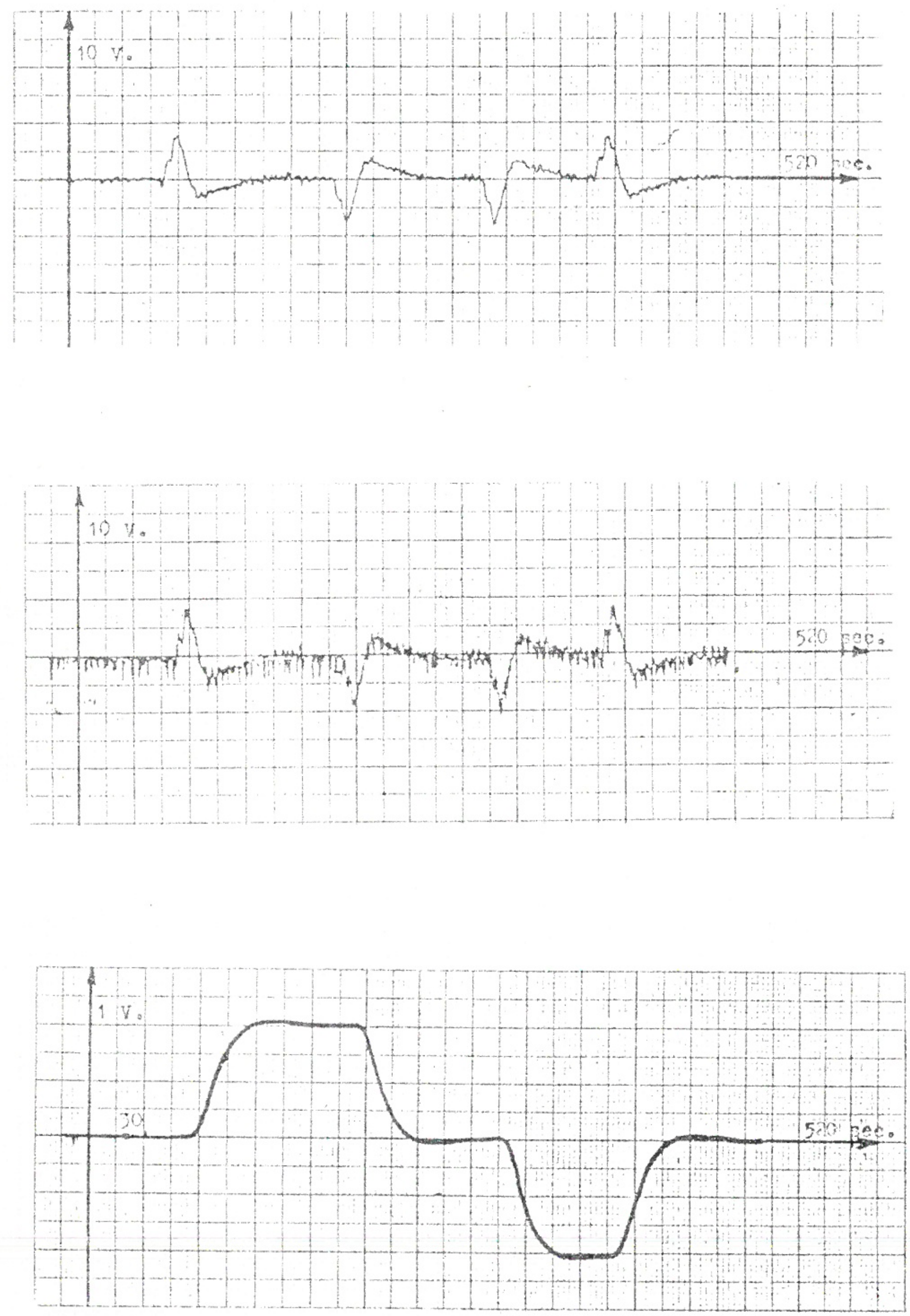


Figure 3. Experiments performed on the laboratory model, the structurally stable compensator utilizing the state feedback:

a) command supplied by the compensator v

b) closed-loop output y

c) plant input u

(**m** = 1,500kg, **l** = 10m).

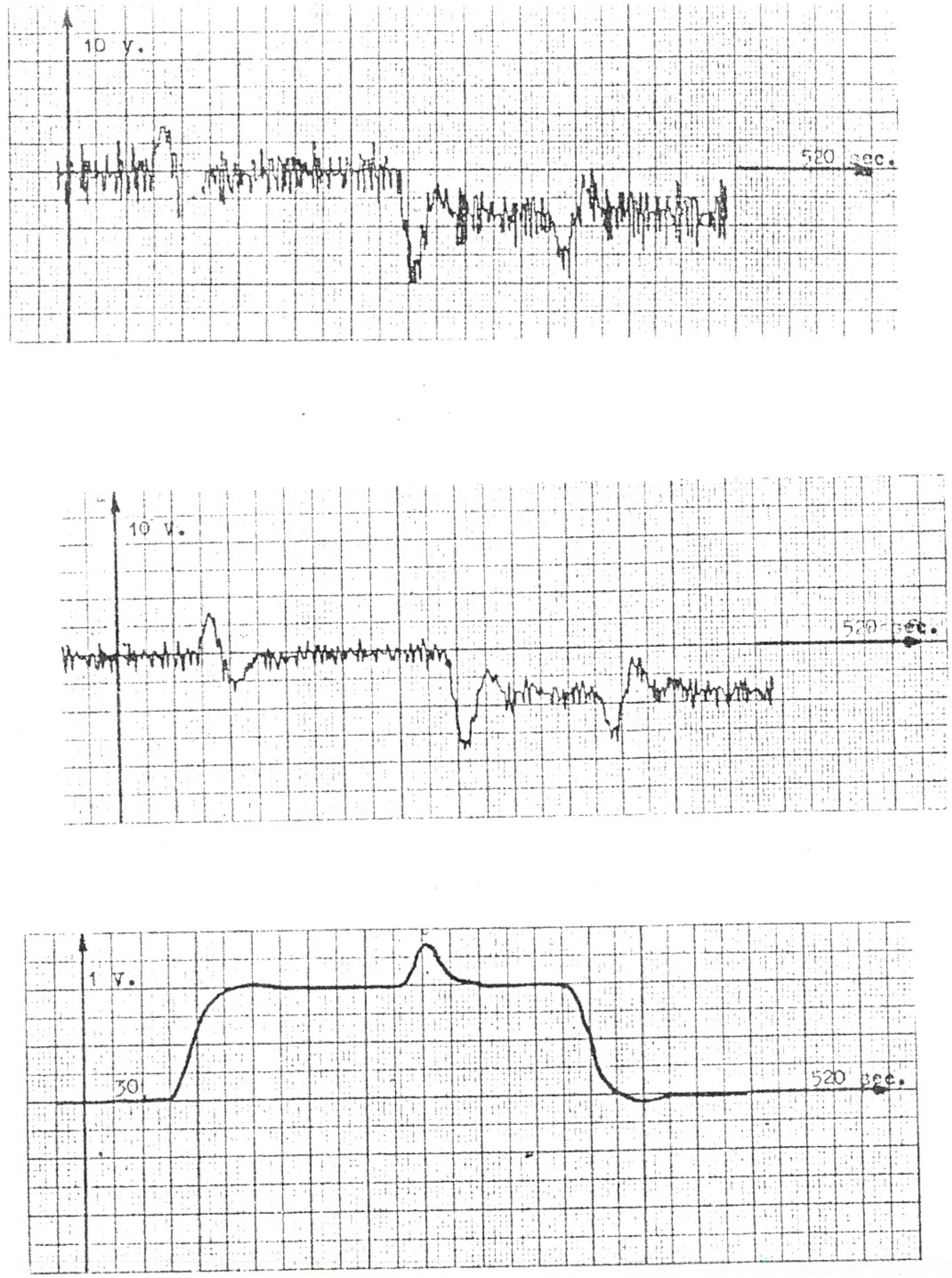


Figure 4. Experiments performed on the laboratory model, the structurally stable compensator utilizing for the proportional feedback only the first three estimated states:

a) command supplied by the compensator v

b) closed-loop output y

c) plant input u

(**m** = 1,500kg, **l** = 10m).

Variations of the physical parameters occur after 165 seconds (**m** = 1,800kg, **l** = 15m).

Various experiments have proved that the step response is not considerably altered when parameters are varying within the previously determined range **R** ; the controller being efficient in stabilizing and regulating the family of BIBO unstable systems. Figure 4 also illustrates the effects of the variation in the physical parameters after about 165 seconds.

7. A SIMPLIFIED SOLUTION FOR CONTROLLER IMPLEMENTATION

Since on a real plant, sensors can be used in order to measure the states x_1 and x_3, a simplified controller (fewer floating point multiplications required and smaller sampling periods allowed) has been implemented to replace the state estimator by numerical differentiators for $\hat{x}_2$ and $\hat{x}_4$.

For the sampling period T = 0.1 sec. and the same pole assignment as in Section 3, the following feedback matrix results:

$$H = [34.519656, 169.151956, -0.55215011, -1.1601539]$$

and

$$H_I = -0.338596$$

for the state variable introduced by the internal model.

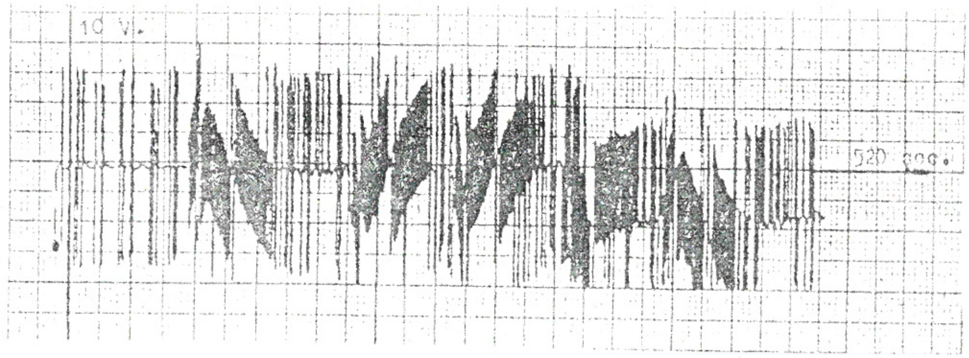
The curves drawn in Figure 5 show the good performances attained in both transient - and steady-state.

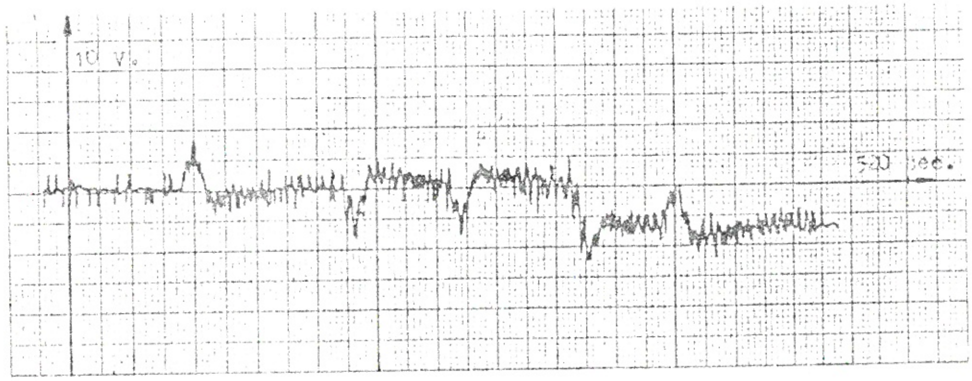
Different experiments proved the preservation of these performances for the multi-model system corresponding to large variations of the physical parameters within the admissible **R** range. Figure 5 also illustrates the effects of a change in **m** and **l** occurred after about 240 seconds (**m** = 1,800kg and **l** = 12m).

Due to differentiators, the command v supplied by the compensator to the actuator varies more quickly than in case of the full order observer, but the system input u has a similar form, because of the filtering effect of $G_d(s)$. By increasing the sampling period, the estimation made by the differentiators turns to be very poor and the closed-loop behaviour turns to be unsatisfactory.

8. CONCLUSIONS

Aspects analysed here offer an overview of the advantages and limitations of the robust numeric control of multi-model systems, highlighting the particular difficulties encountered when the plant is BIBO unstable and the importance of heuristic-type improvements. Even though many problems discussed in the previous sections are not currently considered by design procedures, this contribution proves their role in obtaining the desired closed- loop specifications. It is expected that all these remarks are largely helpful in the design of controllers for industrial plants.





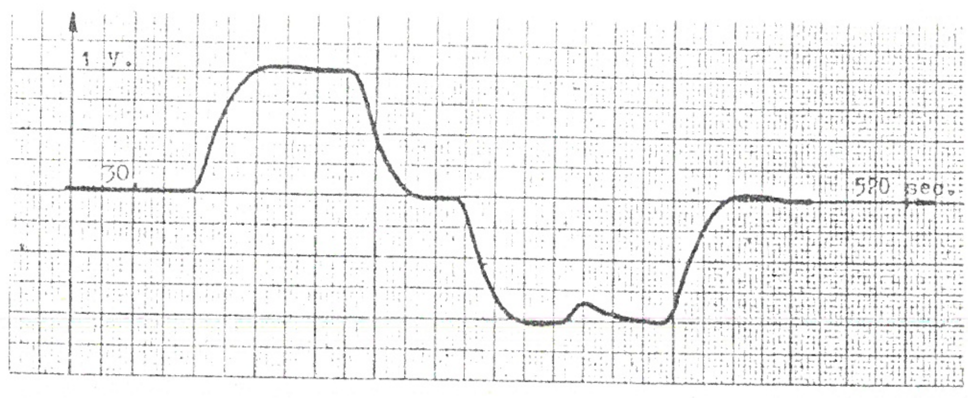


Figure 5. Experiments performed on the laboratory model, the structurally stable compensator utilizing a differentiator-based estimator;

a) command supplied by the compensator v
b) closed-loop output y
c) plant input u
(m = 1,500kg, l = 10m).
Variations of the physical parameters occur after 240 seconds (m = 1,800kg, l = 12m).

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