A PROOF THEORY FOR GENERAL UNIFICATION

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The monograph presents an approach to general E-unification and higher-order unification based on applying the non-deterministic transformations for systems of equations as originated from Herbrand and developed, in the case of standard first-order unification, by Martelli and Montanari. The mathematical formalism which is used permits an effective separation of the logical instruments from the specification of the procedural information and provides rigorous means for analysing the properties of these more complex types of unification problems.

In both cases, the basic set of transformation rules for standard unification is extended by capturing the essential scheme which makes terms identical in these two generalizations of unification, i.e. the least congruence induced by the set of equations for E- unification, and modulo the conversion rules of the typed lambda- calculus for the higher-order unification.

Apart from being a unitary and clear presentation of the various developments in the field of unification, the book contains for the first time a proof of the completeness of a general E- unification method.

The plan of this monograph is as follows.

In Chapter 1 the author introduces a short historical comment regarding the evolution of the unification theory and the connections with its frequent fields of applications which are automated reasoning and logic programming.

Chapter 2 presents an overview of the method of transformations for unifying systems of equations, which resembles to the Gaussian elimination in linear algebra, and provides a formal support for analysing the unification problems in various settings.

Chapter 3 represents a comprehensive introduction to the basic definitions and results concerning the properties of term algebra, substitutions, unification of systems of equations using the Herbrand-Martelli-Montanari transformation methods, equational logic, term rewriting and completion of equational theories.

Chapter 4 presents the main results related to E-unification and a detailed proof of the "narrowing" method which is now the most general form used for investigating this type of unification.

In Chapter 5 the soundness and completeness of the set of transformations BT is proved for arbitrary E-theories using the "equational proof trees" as a new representation for proofs of E- unifiability.

Chapter 6 presents an improved set of transformations T with its corresponding soundness and completeness results. This chapter also includes a comparison with other methods applied to more general forms of E-unification and the study of an open

In Chapter 7 the Herbrand-Martelli-Montanari method is extended for coping with the higher-order unification. The material of this chapter consists of an overview of the basic notions regarding the typed lambda-calculus, the conversion rules, higher-order substitutions and the presentation of two sets of transformations HT and PT with their soundness and completeness properties.

Chapter 8 summarizes the principal results obtained in this book.

As the monograph covers the actual topics of Unification Theory and contains all the necessary material in a very well-organized and well-presented form, it can be recommended as a valuable source of documentation to those interested in automated reasoning, equational logic, Knuth-Bendix completion procedures, first-order and higher-order logic programming, program synthesis, transformations and development, etc.

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