Efficient Decisions in Ambulance Systems Management

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Abstract: This paper proposes an efficient decision support system for an ambulance dispatching management system. The system is represented in the form of a causal graph, in which each arc represents a causal connection between the medical service providers (ambulances) and the medical service beneficiaries (patients). The generated solution is based on determining the optimal strategies by using specific game theory procedures. The determination of the intervention costs is based on the existing road infrastructure. The novelty of the proposed solution lies both in considering the road infrastructure as a time-varying system, with variable costs and road segments that may become inaccessible, as well as in the possibility of generating a solution based on the current positions of the medical service providers.

Keywords: Causal graph, Management system, Game theory, Emergency services.

1. Introduction

The ambulance system can be considered a supply-distribution system or a transportation system, considering ambulances as providers of medical services and patients as beneficiaries (consumers) of medical services.

The specialized literature presents a multitude of approaches for generating optimal or close to optimal solutions in this field.

The vast majority of them can only be used in ideal cases, where the road infrastructure is stable and the costs are fixed, previously determined. The use of these methods, especially those based on linear programming, in real situations with variable costs and road segments that may become unusable is not possible or, in some cases, it requires considerable computational efforts.

A method for determining a cost-efficient solution in a transportation network prone to critical incidents was proposed in (Bîlbîie, Dimon & Popescu, 2024), by using a modified form of Dijkstra's algorithm, usable even for real-time applications.

Considering that an essential element of the ambulance system is the intervention time, which triggers the need to offer quick solutions, this paper proposes an approach based on simple and efficient algorithms, which can quickly generate a solution and can easily adapt to changes in the system.

Research in the field has experienced an explosive development, especially following the experiences during the Covid-19 pandemic, a timespan in which significant delays were observed in providing medical care, especially during periods of multiple requests.

In (Jankovič et al., 2024) the authors propose an optimized spatial distribution of ambulances using a hierarchical p-median model which, as a result of the experiments carried out, generated an increase in the number of timely interventions by 8.7% for the urban areas, respectively by 10.5% for the rural areas in Slovakia.

A model for optimizing pre-hospital emergency services was developed and presented in (Olave-Rojas & Nickel, 2021). The study was based on hybrid simulation and machine-learning and the model was validated by using data collected from a service coordination center in northern Germany.

Further on, models for optimizing ambulance allocation and determining the locations of parking centers based on genetic algorithms are presented in (McCormack & Coates, 2015) and in (Kochetov & Shamray, 2021). By applying the proposed method to the London ambulance service (the former work) and Vladivostok city EMS service (the latter work), the authors obtained a substantial improvement with regard to the response time.

Jánošíková et al. (2021) proposed a bi-criteria mathematical programming model considering the access of high-priority patients to medical services the accessibility of high-priority patients within a short time limit and the average response time for all patients, to relocate emergency medical stations where ambulances waiting to be dispatched are parked. The methodology was verified using real ambulance trip data from

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Slovakia. The response time was improved by 58 s (as an average) and the number of "on-time" interventions increased by 6% for high-priority calls and 5% for the other calls.

In (Thai & Huh, 2022) the authors propose a cloud-computing and big-data approach to optimizing ambulance services in South Korea. The application, based on a web architecture, optimizes patient transport based on distances, pathologies, and hospitals that can provide specialized treatment.

Another model, based on a combination between the Discrete Event Simulation method and an artificial neural network is presented in (Hosseini Shokouh, Mohammadi & Yaghoubi, 2022). The solution is obtained by solving the mathematical model based on genetic algorithms.

In (Selvan et al., 2025) an AI-based solution is proposed, using a deep neural network for adjusting the ambulance routes.

The present paper considers the modeling of a management system for emergency medicine based on a simple form of a causal graph (Bîlbîie, Dimon & Popescu, 2020), or a directed acyclic graph (DAG) (Dawid, 2024), where the nodes represent the entities of the system (ambulances and patients) and the arcs represent the possibility of each ambulance to offer medical services to some (or all) patients. The optimization of ambulance allocation for patient service is made using procedures specific to the strategic game theory.

This model was applied over a road infrastructure prone to incidents affecting road segments (resulting in an increase or decrease of the travel time) and an optimal solution was generated in real time for ambulance dispatching in an emergency medical system. Both the causal graph theory and game theory procedures offer easy-to-implement and fast enough algorithms to be used in real situations.

The results obtained from the simulation based on the case study presented in Section 4 indicate a possible reduction of the resources used by 11.1% (less personnel and fewer vehicles) without affecting the quality of services (time to arrival) or, if the resources are kept the same, a 16.6% increase in the number of possible assisted patients.

The remainder of this paper is organised as follows.

Section 2 presents the causal graph associated with the management system for emergency medicine, the method of determining the initial costs and the method of constructing the game matrix. Section 3 describes the main methods employed for generating an optimal solution by using procedures specific to game theory. Further on, Section 4 presents a case study related to the simulator developed for implementing the proposed mechanisms, and the results obtained based on the simulation carried out. Finally, Section 5 outlines the main conclusions resulting from this study.

2. Causal Graph Associated With the Management System for Emergency Medicine

The architecture of the proposed system can be represented in the form of a causal graph (Figure 1), with two categories of elements: service providers (ambulances) and service beneficiaries (patients seeking medical assistance).

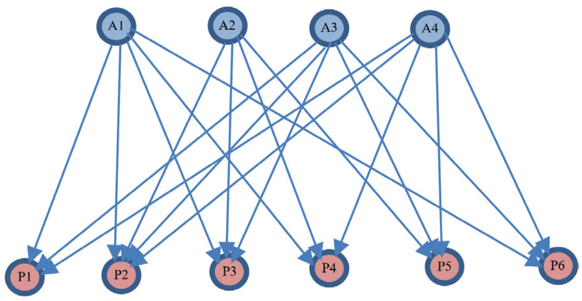

Figure 1. The causal graph associated with the transportation system

The causal relationships between the variables of the management system can be described by using a matrix-vector representation.

The matrix M expresses the causal dependence between the vectors a and p:

$$p = M * a \tag{1}$$

where:

$$\begin{aligned} a^T &= \left[a_1,\ a_2,\ \ldots,\ a_p\right] \\ p^T &= \left[p_1,\ p_2,\ \ldots,\ p_m\right] \end{aligned} \tag{2}$$

represent the provider nodes (ambulances) that will meet the desired requirements of the beneficiary nodes (patients).

The elements of matrix M are:

$$m_{ij} = \begin{cases} 1, \text{ if path } a_j \rightarrow p_i \text{ exists} \\ 0, \text{ if not} \end{cases} \quad (3)$$

Given a matrix M and a vector p already known, vector a can be calculated according to equation 4. The configuration of the vector a provides the algebraic solution of the causal graph.

To determine the suppliers, the matrix representation is used:

$$a = M^t * p \quad (4)$$

where M^t is the transpose of the matrix M.

Solving the graph in a causal direction starts from the known state regarding the availability of providers, given by the known vector a (service providers), to estimate the components of the patient vector p, related to the patients who require medical assistance.

The anti-causal direction of operation starts from the patient vector p which is related to the patient requirements for locating the appropriate service providers denoted by a.

Since the management system reacts to the emergence of requests for medical assistance on the patient side, the anti-causal approach is more appropriate.

When new requests appear, the patients who made the requests are activated, and by solving the graph in the anti-causal direction, all the free providers who can offer the requested services are determined.

In Figure 2 one such situation is illustrated, indicating the service providers denoted by a, for a specific intervention request (patients p_1 to p_6):

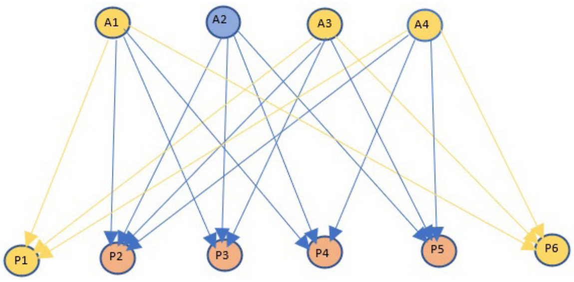

Figure 2. Graph solution for a specific intervention request

For this situation, the value of the patient vector is:

$$p = [1\ 0\ 0\ 0\ 0\ 1] \quad (5)$$

and the ambulances able to provide services are determined by using equation (4), which for the matrix associated with the graph in Figure 2, gives:

$$a = [1\ 0\ 1\ 1] \quad (6)$$

2.1. Determining the Costs of the Intervention

The road network can be represented as an undirected graph, where nodes represent intersections and arcs represent connections between adjacent nodes. Each connection has an associated initial cost, determined by previous measurements, a cost that changes over time depending on the real situation at the time of the analysis.

For each of the connections determined when solving the causal graph, the image of the road network is constructed and a minimum cost path for the intervention is generated using the Dijkstra's algorithm (Dijkstra, 1959). For the system shown in Figure 1, the corresponding minimum cost paths were determined for each possible combination $a_i \rightarrow p_j$, for which m_{ij} =1.

The algorithm that determines the estimated cost for the proposed route from the source node to the destination node follows the steps detailed below:

2.1.1 The Initial Step

- It starts from the source node, the initial minimum cost is zero for the source node and it has a large value (considered "infinite") for the other nodes;
- All the neighbors of the source node are analyzed, and for each of them the cost of transportation from the source node to the analyzed node is memorized;
- The source node is removed from the list of nodes to be analyzed.

2.1.2 The Intermediary Steps

The following operations are executed successively until the analyzed node is the destination node:

- The node with the lowest cost is chosen from the list of remaining nodes to be analysed.

It is obvious that as this node features the lowest cost, the route already determined from the source node to this node is the optimal route; no other route will be able to generate a lower cost;

- The current node is removed from the list of remaining nodes to be analyzed;
- For each neighbor node related to the current node that is still in the list, the minimum cost is established as the sum of the transportation cost to the removed node and the transportation cost from the current node to the neighbor node;
- If the new minimum cost determined is lower than the previous minimum cost, then the new minimum cost is retained, and the current node is retained as the previous node;
- If the new minimum cost is higher than the previous one, it means that another minimum route was previously determined, and the next node is analysed;

2.1.3 The Final Step

- If the destination node is reached, the algorithm stops;
- If all the nodes were reached according to the previous step and the destination node was not yet analyzed, it means that there is no path between the source node and the destination node.

When a significant event that changes the previously estimated costs appears, a new route is determined using the same algorithm by considering the current point of departure for the ambulance, updating the costs for the road segments involved (or eliminating the respective segments or portions from the network if necessary) and generating a new route to the destination. The new obtained cost is considered when proposing a new optimal solution, following the analysis of the entire system.

3. Choosing the Optimal Solution Through Specific Game Theory Procedures

All the costs obtained previously are centralized in the form of a matrix, C, in which the rows represent the active ambulances and the columns represent the active patients. Each element c_{ij} of the matrix represents the cost of providing the service for patient “j” by ambulance “i”. If there is no possible route between an ambulance and a patient, the travel cost is indicated with the value -1 so as not to be considered in the optimization process.

Based on the causal graph represented in Figure 1, the following cost matrix is constructed (Table 1):

Table 1. Network-associated cost matrix

C	P1	P2	P3	P4	P5	P6
A1	23	37	49	15	-1	21
A2	-1	22	11	39	17	-1
A3	31	44	28	-1	22	41
A4	17	24	-1	39	16	33

The matrix thus obtained can be interpreted as a matrix corresponding to a strategic game, in which the elements involved represent the “losses” of the service provider. Thus, the procedures specific to game theory can be used for choosing the optimal strategy either by employing a complementary form of these procedures based on inverting the minimum and maximum conditions, or by transforming the matrix C into a matrix of “gains” and then applying the procedures in their classical form.

Considering the value of an intervention to be, for example, 1000 price units (p.u.), the “gains” matrix is constructed by attributing to each element of the matrix C the value $1000 - c_{ij}$ (Table 2).

Table 2. Game Matrix

G	P1	P2	P3	P4	P5	P6
A1	977	963	951	985	-1	979
A2	-1	978	989	961	983	-1
A3	969	956	972	-1	978	959
A4	983	976	-1	961	984	967

Each value in this matrix is variable over time, being affected by the state of the road infrastructure. The reconfiguration of the matrix is done quickly, and the generation of a new solution in real time is possible thanks to the procedures specific to game theory, which are easy to implement and extremely fast, even for complex cases.

It is possible that after reconfiguring the game matrix, solutions will be generated that differ greatly from the initial ones, having as a practical representation the redirection of ambulances to other patients than those to which they were initially assigned.

Ambulance redistribution can also occur in cases in which a new patient is registered in the system, when the nearest ambulance dispatch center can no longer provide services (no ambulance is available). In this case, the game matrix is expanded with another column, the new costs for each ambulance-patient pair are determined and a new solution is provided.

To generate the optimal solution, the max-min, Hurwicz, Laplace and Savage game theory procedures were selected. They were selected both in order to determine the computational time for applying each procedure and to analyze the ability of each procedure to generate an optimal solution.

3.1. The Max-min Procedure

This procedure assumes that player A adopts the strategy:

$$\max_{1\le i\le n}\left\{\min_{1\le i\le m} c_{ij}\right\} \quad (7)$$

Player A will determine the minimum gains for each strategy considering all of player P's strategies, then he will select the strategy that ensures a maximum gain.

3.2. The Hurwicz Procedure

According to this procedure, player A will choose a "coefficient of pessimism", q, which will be used for computing, for each of his strategies, an average value:

$$Avg_i = \frac{q\min\{c_{ij}\}+(1-q)\max\{c_{ij}\}}{2}, \quad (8)$$
$$j=1,2,\dots,m, 0\le q\le 1$$

Player A will then choose the strategy that ensures maximum gain based on the obtained values:

$$\max\{Avg_i\} \quad (9)$$

3.3. The Laplace Procedure

Based on the Laplace procedure, player A will compute an average value for each of his strategies. Average values are calculated based on the following formula:

$$\bar{c}_i = \frac{1}{m}\sum_{j=1}^{m} c_{ij}, i=1,2,\dots,n \quad (10)$$

Finally, the strategy that ensures

$$\max \bar{c}_i, i=1,2,\dots,n \quad (11)$$

is chosen.

3.4. The Savage Procedure

This procedure works well under conditions of total uncertainty, which means nothing is known about the opposing player regarding the strategy he will adopt at a given moment.

Player A will construct a "matrix of erroneous decisions" based on the initial game matrix, then he will use the max-min procedure on the new game matrix.

The matrix of erroneous decisions is constructed in columns as follows (for column k):

$$\begin{cases} R_{1k} = \max\{c_{ik}\} - c_{1k} \\ R_{2k} = \max\{c_{ik}\} - c_{2k} \\ \quad\vdots \\ R_{nk} = \max\{c_{ik}\} - c_{nk} \end{cases} \quad (12)$$

Thus, each element of the new matrix is obtained by subtracting the corresponding element of the game matrix from the element with the maximum value of the respective column.

After obtaining the new matrix, player A will choose the strategy that ensures:

$$\min_{1\le i\le n}\{\max_{1\le j\le m} R_{ij}\} \quad (13)$$

Considering the values corresponding to the total number of ambulances and all possible locations of patients in real situations, which will generate a relatively big game matrix, all these procedures are applied to a game matrix that contains only active ambulances and active patients.

4. Case Study

As a case study, one chose the activity of a private company that serves patients located within a geographical area that offers, both due to its relief and the sufficiently branched and diversified road network, very good conditions for testing the proposed optimization mechanisms.

The administrative area for ambulance intervention is marked with a dotted line in Figure 3, and the three ambulance parking points are marked in brown (X, Y and Z).

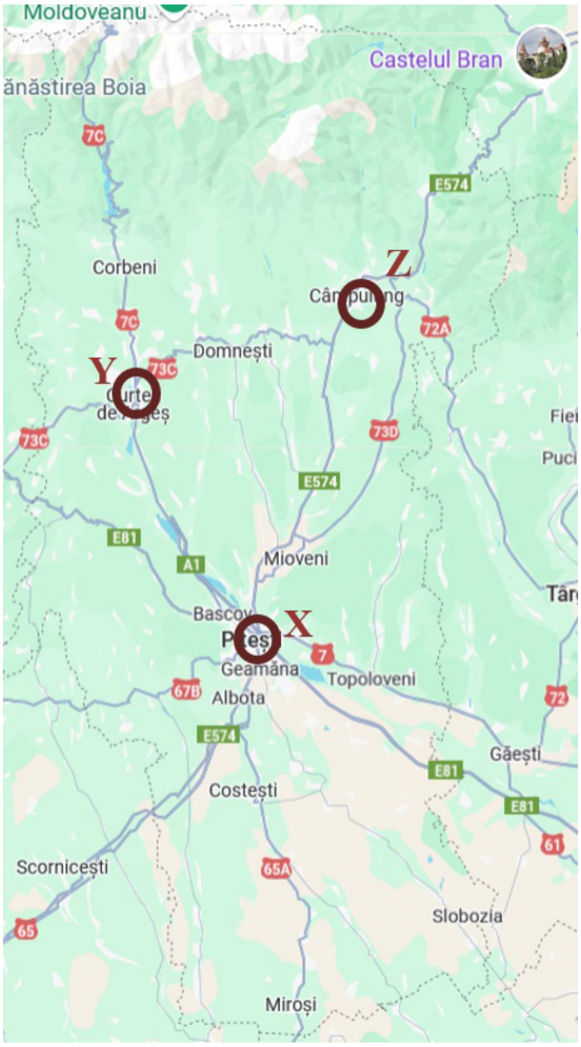

Figure 3. Administrative map of the analyzed area

There are several ambulances at each base station, and these will be treated as independent entities.

The case study was limited to the analysis of situations in which it is not necessary to transport the patients to the hospital or to other institutions, and the ambulances have identical equipment, sufficient for a day with up to 20 successive interventions. The staffing of the ambulances is also identical, so that each ambulance can provide services to any patient, in case of need.

The case study starts from an initial situation, with no patients requiring an intervention and with all ambulances at the parking points.

During the simulation, patients are randomly generated in various locations, using the "simple random sampling" method, which assumes equal probabilities of requesting an intervention for each location. In this way, the generation of events in diversified locations is ensured and the response of the proposed system can be studied more easily.

After generating a solution, critical events are randomly inserted that require recalculating the minimum cost paths, rebuilding the game matrix, and determining a new solution.

Although for some places the optimal route seems easy to choose, for most of them there are alternative routes which, depending on the weather and traffic conditions, may (or may not) represent better solutions (Figure 4).

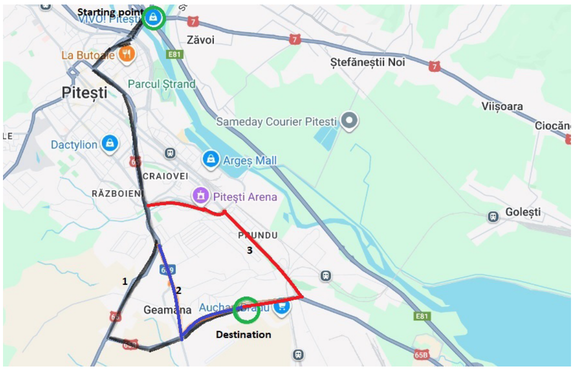

Figure 4. Road infrastructure with alternative routes

For all these points, one starts from the ideal situation in which the costs are known and, depending on the events occurring on the chosen route, the new costs are determined using Dijkstra's algorithm.

The game matrix is constantly updated, both based on the previously collected data and on the recent data obtained with regard to traffic conditions.

Thus, whenever a significant event occurs that determines a change in the intervention costs, the game matrix is reconstructed and the optimal strategies are reevaluated.

Based on the specific road infrastructure, a game matrix was built as illustrated in Table 3, which takes into account the initial costs, corresponding to the stable (ideal) situation.

Table 3. Complete game matrix (excerpt)

G	P_1	P_2	P_3	P_4	P_5	...	P_{102}
A_1	1000	938	958	984	991	...	975
A_2	1000	938	958	984	991	...	975
...	...	...	...	...	...	...	...
A_{10}	938	1000	949	954	936	...	958
...	...	...	...	...	...	...	...
A_{18}	958	949	1000	950	952	...	941

Each row corresponds to an ambulance, and each column corresponds to a possible intervention point (the location of a patient – 102 administrative-territorial units).

From this matrix, depending on the patients' requests, a sub-matrix is extracted that includes only the columns specific to active patients. For this new matrix, the classical procedures specific to game theory (the Max-min, Laplace, Hurwicz, and Savage procedures) are applied and a solution is generated.

The dimensions of the matrix are variable, the rows corresponding to ambulances involved in an intervention were eliminated for the duration of the intervention, and the columns correspond only to patients who require transport for the medical intervention.

4.1 Simulator Description

The analyzed company operates under the following conditions:

- it has three locations where ambulances are stationed (X, Y and Z), which feature a number of 9, 5 and 4 ambulances, respectively;
- each ambulance carries out an average number of 12 interventions per day, with a minimum of 9 and a maximum of 15 interventions;
- the treatment applied to a patient after moving to the intervention point takes on average 35 minutes, with a minimum time of 15 minutes and a maximum time of 60 minutes.

Starting from the data collected from the analyzed company, the simulator runs in three stages:

- Stage 1: Establishing the baseline. This involves randomly generating a number of patients per day similarly to a real scenario and determining the average ambulance travel time to the patient (the average cost);
- Stage 2: A gradual reduction of the number of ambulances and the analysis of its impact on the average travel time;
- Stage 3: A gradual increase in the number of patients per day and the analysis of its impact on the average travel time.

The initial data is taken from a database that includes the average (standard) travel times from the three ambulance parking points to each of the locations of potential patients (102 administrative-territorial units).

To simulate the traffic conditions (better or worse) and the occurrence of critical events that affect the preset routes and require the generation of alternative routes, the standard travel times to the patients are adjusted randomly based on percentages between -10% and +25%. When the average travel time for a road segment is modified, that value will be used for all the subsequent trips on that segment.

All the results, both the partial and final ones, are saved in a database to allow their easy verification.

The method used for generating random events is "simple random sampling" which ensures a uniform distribution of the generated values between the minimum and maximum limits.

4.2. Implementation Considerations

For a better integration with the software used by the analyzed company, the simulator was implemented under Windows OS, using the C# programming language.

The game matrix G (Table 3) was constructed based on the existing road infrastructure, determining for each element the optimal travel route by using Dijkstra's algorithm presented in subsection 2.1. The game matrix is rebuilt whenever an incident occurs on the road infrastructure and the travel times change.

When a new intervention request is generated by a patient, a new, smaller matrix is generated, containing only the active patients (as columns) and the available ambulances (as rows). In this situation, the active patients are all the patients waiting for ambulance arrival and the available ambulances are considered all ambulances which are not involved in providing treatment to a patient.

It is possible, in this way, to change the destination of an ambulance sent to a patient, if necessary, in order to obtain a better total time of intervention.

For the new matrix all four game theory procedures presented in section 3 (the max-min, Hurwicz, Laplace and Savage procedures) are applied and the best result is chosen.

To that, all the inputs and outputs of the simulator are in the form of text files, for easier human verification.

4.3. Simulation Results

In the first stage, based on the data collected from the company providing ambulance services, an average travel time to the patient of 32.3 minutes was obtained, a value which represents a 12.7% reduction of the real travel time of 37 minutes (according to the information obtained).

The randomly generated data on the road segment costs and patient distribution was saved in an external database.

For the second stage the same data was used, reducing the number of available ambulances, but keeping the other conditions identical.

To avoid particular configurations resulting from random patient generation, the simulation was run for 100 (simulated) days, with an average of 216 patients per day (18 ambulances with 12 interventions each).

It is noted that, thanks to the efficient ambulance dispatching algorithms for intervention purposes, the average travel time was reduced by 4.7 minutes in comparison with to the real-life scenario.

The centralized results obtained in stage 2 are included below (Table 4):

Table 4. The impact of the number of ambulances on the travel time

No. of ambulances	Average travel time	Blockage duration
9, 5, 4	32.3 min	0.5 min
8, 5, 4	34.1 min	0.5 min
9, 4, 4	32.9 min	0.5 min
9, 5, 3	33.3 min	0.6 min
8, 4, 4	36.2 min	1.2 min
8, 5, 3	38.3 min	2 min
9, 4, 3	35.6 min	0.9 min
7, 5, 4	39.1 min	2 min
9, 3, 4	34.7 min	0.8 min
9, 5, 2	37.4 min	1.4 min
8, 4, 3	43 min	4.4 min

By reducing the number of ambulances by one, regardless of the location from which an ambulance is removed, the algorithms used still allow for the provision of medical assistance at a level specific to real-life scenarios.

The simulation shows that it is even possible to eliminate two ambulances (11.1% less personnel and fewer vehicles) if 9 ambulances are kept in the headquarter X, and the response times change very little (35.6 minutes for the combination 9, 4, 3 and 34.7 minutes for the combination 9, 3, 4, respectively).

For these two variants, the average duration of the blockage (the period of time during which all ambulances are busy and no ambulance can be sent to the patient) is under one minute, an extremely short time interval.

However, if three ambulances are eliminated, one from each location, the time required to travel to a patient's address increases substantially, reaching an average of 43 minutes, with an average blockage period of 4.4 minutes.

For the third stage, both the number of ambulances and the travel times were taken over from the first stage, in order to determine the influence of the number of patients on the travel time. If in the first and second stages the number of patients was 216 (ensuring an average of 12 patients for the 18 available ambulances), in the third stage the number of patients was gradually increased (adding 18 patients at each step, which resulted in an average of one more patient for each ambulance). The centralized results are presented in Table 5.

Table 5. The impact of the number of patients on the travel time

No. of patients	Average travel time	Blockage duration
216	32.3 min	0.5 min
234	33.7 min	0.4 min
252	36.7 min	1.2 min
270	46.9 min	6.2 min
288	69.4 min	24.3 min

The results obtained from the simulation show that the initial number of ambulances can successfully cope with a larger influx of patients, with each ambulance being able to provide assistance to 14 patients daily (a 16.6% increase for the initial number), that is 252 patients on average per day, without affecting the intervention times (a travel time of 36.7 minutes to the patient and 1.2 minutes of congestion).

However, for a larger number of patients per day, a larger number of ambulances is required, as starting with 270 patients the average blockage duration reaches 6.2 minutes, and the average travel time increases to 46.9 minutes.

5. Conclusions

The proposed method allows the generation of rapid solutions for an ambulance services management system based on a road infrastructure with variable costs over time and with a variable number of customers.

Based on the simulation carried out, a very detailed description of the road infrastructure has a major importance with regard to the additional costs arising from an incident. The more detailed the network, the closer the total costs are to the ideal ones.

The new solutions generated following the occurrence of critical incidents are quite close in value to the initial ones (differences below 5%), and their generation is extremely fast, in an interval of less than 50 milliseconds.

When a new patient appears, thanks to the use of algorithms that do not require complex calculations, the new solutions are generated just as fast.

The results obtained from the simulation show that, as a rule, several game theory procedures generate an optimal solution. Due to the simplicity of the algorithms used, in real applications it is possible to determine a solution through each procedure and then choose the most efficient one.

The proposed management system can be used both for optimizing the number of vehicles needed to ensure timely interventions and for estimating the number of patients that can be allocated a certain number of ambulances.

The simulation results also demonstrate the validity of the proposed model and of the employed methods, leading to an increase of 16.6% in the number of patients that can be assisted by the analyzed company or in a decrease of 11.1% of the resources used (ambulances and personnel) if the number of patients remains the same.

In conclusion, the proposed system meets the requirements of an ambulance dispatching management system, providing real-time solutions that can be used as a basis for human decision-making.

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