Design and Performance Evaluation of Fuzzy Variable Fractional-Order $[PI]^{\lambda}D^{\mu}$ Controller for a Class of First-Order Delay-Time Systems*

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Abstract: This paper presents the design of a fuzzy variable fractional-order PID (FVF[PI]λDμ) controller and its level of performance. The first-order plus delay-time (FOPDT) pressure control fractional-order plant model is identified using an experimental dataset and designed for study the control scheme. The conventional integer-order PID (IOPID) controllers' performance is not satisfactory for the systems with nonlinear characteristics, plant uncertainties, and external load disturbances. Thus, a modified control structure is proposed in which the exponent terms λ and μ of the controller are changed lively using an intelligent fuzzy logic controller (FLC). To investigate the usefulness of the proposed control method, simulation studies are carried out using MATLAB software. Further on, the proposed controller performance for the identified fractional-order plant is compared with that of the IOPID and classical fractional-order PID (FOPIλDμ) controllers. The controllers' performance is demonstrated during the parameter change and under external load disturbance conditions. The results confirm that the FVF[PI]λDμ controller is capable to achieve an improved performance and robustness for the identified FOPDT pressure control systems.

Keywords: Fractional-order controller, Fuzzy logic controller, Pressure control, First-order delay-time system, PID controller.

1. Introduction

It is well known that the PID controller is widely used in most of the industrial control applications. The control law related to the PID controller is simple and effective which makes them easily understand and implement. Therefore, for more than five decades the IOPID controllers have been used for industrial control systems due to their advantages such as simple control structure, and their being easily comprehensible and implementable (Ang et al., 2005). However, the performance IOPID controller is not good enough for complex and nonlinear systems (Jezernik & Rodic, 2009). Hence, the research works are focused to enhance the performance of the IOPID controller. Thus, the modified PID control structure was developed, which is known as the $FOPI^{\lambda}D^{\mu}$ controller. In the conventional IOPID controller, the orders of the integrals and differentiation are restricted to integers, which may limit the controller performance specifically when the system provides plant uncertainty and load disturbances (Monje et al., 2008, Luo et al., 2010).

In recent years, the fractional-order controllers (FOCs) are used in different fields of science and engineering. Research studies are focused on creating a new control structure for FOC and its design methods as an extension of the integer-order control scheme (Podlubny et al., 2002, Erenturk, 2013). The fractional-order control method was introduced by (Podlubny, 1999), in this design the integer-order differentiator and integrator in the conventional IOPID controller has been replaced by fractional-order differentiator (D^{μ}) and integrator (I^{λ}). In $FOPI^{\lambda}D^{\mu}$ controller, the orders of the integrator and differentiator (λ and μ) are real non-integer values. Since they were introduced, fractional-order controllers have been used by many researchers and confirmed their enhanced performance compared to the integer-order controllers (Vahid & Mohammad, 2016, Liu et al., 2018). Recently, the Harmony search optimized fractional-order PID control method suggested by (Vinu, Varghese & Kishore, 2018) and confirmed that its performance is superior to that of the conventional controllers in controlling the speed of a DC motor of fuel cell powered vehicles under dynamics load conditions. In principle, the IOPID controller has three tuning parameters, while the $FOPI^{\lambda}D^{\mu}$ controller has five tuning parameters, which provide greater flexibility in the controller design. Further, FOC-based systems allow an exact description of the real-time system parameters in order to achieve a high degree of control performance. The FOC

* This paper is an extended version of the paper called "Fuzzy fractional-order PID controller for fractional model of pneumatic pressure system", published in Mathematical Problems in Engineering, 2018, Article ID 5478781, DOI: 10.1155/2018/5478781. In the current paper, a novel control method based on a fuzzy variable fractional-order PID (FVF[PI]λDμ) controller is used to enhance the system performance. The simulation studies are carried out in order to demonstrate controllers' performance during the parameter change and under external load disturbance conditions.

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could be implemented in the closed-loop system with an integer-order or fractional-order plant (Chen, 2006). However, it is most commonly used with the integer-order plant if the model of the plant has been already confirmed.

The FOPI$^{\lambda}$D$^{\mu}$ controller has the advantage of providing five parameters for controller tuning. However, the increasing number of tuning parameters will increase the complexity of the system and the computation time. (Khalfa et al., 2008) have studied the FOPI$^{\lambda}$D$^{\mu}$ controller and stated that its design has been relatively difficult due to the higher number of parameters has to be tuned. Thus, a FLC-based control structure is proposed in this paper. (Das et al., 2012, Jianpeng & Lichuan, 2015) have recommended various types of controller tuning techniques to estimate the parameters of FOPI$^{\lambda}$D$^{\mu}$ controller, and a few research studies had demonstrated the use of the Ziegler-Nichols tuning method for FOCs. The use of intelligent controllers such as the FLC with fractional-order controllers becomes one of the core areas of research in control theory. Combining the FLC with fractional-order operators provides additional flexibility in the system design at the same time handling plant uncertainty and disturbance rejection ability of the controller could be improved. (Das et al., 2012) have designed a controller structure that combines the fuzzy logic with fractional-order PID and demonstrated that its performance is superior to that of the IOPID and integer fuzzy PID controller. The recent researches confirmed that the FLC has the ability to determine the appropriate controller parameters by the use of its rule-base inference mechanism. Hence, an attempt is made in this work to use FLC as a part of a new control structure meant to enhance the performance of the system. The FLC changes the exponent terms λ and μ dynamically based on the system dynamics. To investigate the proposed control method, a pressure control plant is chosen and its mathematical model is identified based on an experimental dataset. The IOPID and FOPI$^{\lambda}$D$^{\mu}$ controllers' performance is also studied for the same plant and compared with the FVF[PI]$^{\lambda}$D$^{\mu}$ control scheme.

This paper is organized as follows: In section 2 the fractional-order controller design specifications are discussed in detail. The identification of the fractional plant model is explained in section 3. In section 4 the proposed controller design procedure is discussed. The fuzzy logic controller design is described in section 5. Section 6 includes the simulation results and their related discussion. The findings of the above-mentioned research are provided in section 7 as a conclusion.

2. Fractional-Order Controller Design Specifications

2.1 Fractional-Order Plant

In general, most of the real-world industrial plants are modelled into either first-order or second-order equations depending on the nature of the process. The first-order fractional plant model can be obtained using an experimental data-based S-shaped process reaction curve. The first-order plant model usually has a single integer-order pole which is believed to better characterize the process experimental data. In the case of fractional plant model, the single integer-order pole will be replaced by a non-integer pole and it is expressed in the following form

$$P(s) = \frac{K}{Ts^{\alpha} + 1} e^{-L_d s} \tag{1}$$

where T is time constant, K is the process gain, L_d is considered as delay and α is a positive real number.

2.2 FOPI$^{\lambda}$D$^{\mu}$ Controller

The FOCs are a generalization of conventional integer-order controllers, which are developed using the concept of fractional calculus. Due to an intensified research in the field of fractional calculus, the controllers with fractional integrals and derivatives are widely applied in control applications. The transfer function of the FOPI$^{\lambda}$D$^{\mu}$ controller can be given as

$$C_1(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{2}$$

where K_p, K_i, and K_d are the proportional, integral, and derivative gain parameters, λ is the order of integral part and μ is the order of derivative part. Using the fractional-order plant transfer function (1) and FOC transfer function (2), the transfer function of the open-loop system is expressed as

$$G_1(s) = P(s)C_1(s) \tag{3}$$

In the case of FOC design, the system fulfills different requirements as robustness to load disturbances, plant uncertainties and high-frequency noise are concerned. Thus, the specifications related to gain margin and phase margin, robustness to parameters variation and sensitivity functions are considered in the system design process. (Franklin, Powell & Naeini, 1986) have recommended two important measures namely the phase margin and gain margin to study the system robustness and performance. Based on the fundamental definition of phase margin and gain crossover frequency, the controller must satisfy the following constraints indicated below.

Phase margin constraint:

$$Arg\left|G_1(j\omega)\right|_{\omega=\omega_c} = Arg\left|C_1(j\omega)P(j\omega)\right| = -\pi + \phi_m \quad (4)$$

where P(jω) is the transfer function of the plant, G_1(jω) is the transfer function of the open-loop system, C_1(jω) is the transfer function of the controller, ω is the gain crossover frequency, and ϕ_m is the required phase margin.

Gain cross-over frequency constraint:

$$\left|G_1(j\omega)\right|_{dB} = \left|C_1(j\omega)P(j\omega)\right|_{dB} = 0 \quad (5)$$

Robustness to parameters variation: The robustness to parameter variations is discussed by (Chen & Moore, 2005). For a specific value of ω the phase Bode plot is flat, in which case the system is considered to be more robust to gain changes and the overshoots corresponding to the step-input response. Hence, the controller could satisfy this condition as

$$\left|\frac{d(Arg(G_1(j\omega)))}{d\omega}\right|_{\omega=\omega_c} = 0 \quad (6)$$

Noise rejection: In the case of noise rejection constraint, the sensitive function T can be given as

$$\left|T(j\omega) = \frac{C_1(j\omega)P(j\omega)}{1+C_1(j\omega)P(j\omega)}\right|_{dB} \leq N\,dB,$$

$$\forall\,\omega \geq \omega_t\ rad/s \Rightarrow \left|T(j\omega_t)\right|_{dB} = N\,dB \quad (7)$$

where N is the required noise reduction for frequencies $\omega \geq \omega_t$ rad/s.

Disturbance rejection: The constraint-related output disturbance elimination sensitive function S can be given as

$$\left|S(j\omega) = \frac{1}{1+C_1(j\omega)P(j\omega)}\right|_{dB} \leq M\,dB,$$

$$\forall\,\omega \leq \omega_s\ rad/s \Rightarrow \left|S(j\omega_s)\right|_{dB} = M\,dB \quad (8)$$

where M is the preferred value of the sensitivity function for frequencies $\omega \geq \omega_s$ rad/s.

2.3 Proposed FVF[PI]$^\lambda$D$^\mu$ Controller

There are many different approaches for integer-order controller tuning such as Zeigler-Nichols and Cohen-Coon etc. In the case of FOC, only a limited number of research papers reported the controller tuning methods. (Luo et al., 2010) have recommended a tuning method for FOCs for an integer-order pole with the delay-time system. (Malek et al., 2013) have extended this tuning method for a fractional-order pole with the time-delay system. In most of these methods, the phase and gain margin are considered specifications for the robust controller tuning. The three gain parameters and exponents of the FOPI$^\lambda$D$^\mu$ controller (2) are usually determined based on the controller tuning and in most of the research papers, these parameters are taken as constant while investigating the system performance. As a result, the system performance reduced sometimes specifically during the parameter change and under load disturbance conditions. Besides, combining the proportional and integral gain parameters (K_p and K_i) and having a common exponent λ, will provide the opportunity to update these gain values lively based on system dynamics. Thus, system performance could be improved. Also, by keeping the derivative term with exponent μ in the controller structure, the system's transient response will be better. The parameters λ and μ can be estimated at a particular time based on system dynamics. The controller structure proposed in this paper is given as

$$C_2(s) = \left[K_p + \frac{K_i}{s}\right]^\lambda + K_d s^\mu \quad (9)$$

where λ and μ are positive real numbers and these parameters' ranges are selected as 0 < λ, μ > 2 in this work. By using the proposed controller configuration (9) and fractional-order plant (1), the open-loop system transfer function can be expressed as

$$G_2(s) = P(s)C_2(s) \quad (10)$$

In the frequency domain, the equation (10) can be represented in the following form by replacing s with jω

$$G_2(j\omega) = P(j\omega)C_2(j\omega) \quad (11)$$

where

$$C_2(j\omega)=\left[K_p+K_i(j\omega)^{-1}\right]^{\lambda}+K_d(j\omega)^{\mu}$$
$$=\left(K_p-\frac{K_i j}{\omega}\right)^{\lambda}+K_d\omega^{\mu}\left(\cos\frac{\mu\pi}{2}+j\sin\frac{\mu\pi}{2}\right)$$
$$=\left(K_p^{\lambda}-\frac{\lambda K_i K_p^{\lambda-1}}{\omega}\right)+K_d\omega^{\mu}\left(\cos\frac{\mu\pi}{2}+j\sin\frac{\mu\pi}{2}\right) \tag{12}$$

$$Arg\left[C_2(j\omega)\right]=\tan^{-1}\left(\frac{-\lambda K_i K_p^{\lambda-1}+K_d\omega^{\mu+1}\sin\frac{\mu\pi}{2}}{\omega K_p^{\lambda}+K_d\omega^{\mu+1}\cos\frac{\mu\pi}{2}}\right) \tag{13}$$

and

$$P(j\omega)=\frac{K}{T(j\omega)+1}e^{-L_d(j\omega)} \tag{14}$$

$$Arg\left|P(j\omega)\right|=-\tan^{-1}\left(\frac{T\omega^{\alpha}\sin\frac{\alpha\pi}{2}}{1+T\omega^{\alpha}\cos\frac{\alpha\pi}{2}}\right)-L\omega \tag{15}$$

Open-loop phase at ω frequency can be expressed as

$$Arg\left|G_2(j\omega)\right|=Arg\left|P(j\omega)\right|Arg\left|C_2(j\omega)\right| \tag{16}$$

$$=\tan^{-1}\left[\frac{-\lambda K_i K_p^{\lambda-1}+K_d\omega^{\mu+1}\sin\frac{\mu\pi}{2}}{\omega K_p^{\lambda}+K_d\omega^{\mu+1}\cos\frac{\mu\pi}{2}}\right]-\tan^{-1}\left(\frac{T\omega^{\alpha}\sin\frac{\alpha\pi}{2}}{1+T\omega^{\alpha}\cos\frac{\alpha\pi}{2}}\right)-L\omega \tag{17}$$

$$Arg\left|G_2(j\omega)\right|=\tan^{-1}\left[\frac{-\lambda K_i K_p^{\lambda-1}+K_d\omega^{\mu+1}\sin\frac{\mu\pi}{2}}{\omega K_p^{\lambda}+K_d\omega^{\mu+1}\cos\frac{\mu\pi}{2}}\right]-\tan^{-1}\left(\frac{B}{A}\right)-L\omega \tag{18}$$

where $A=1+T\omega^{\alpha}\cos\frac{\alpha\pi}{2}, B=T\omega^{\alpha}\sin\frac{\alpha\pi}{2}$

According to the phase margin constraint (4), the open-loop phase could satisfy the following expression

$$Arg\left[G_2(j\omega)\right]=\tan^{-1}\left[\frac{-\lambda K_i K_p^{\lambda-1}+K_d\omega^{\mu+1}\sin\frac{\mu\pi}{2}}{\omega K_p^{\lambda}+K_d\omega^{\mu+1}\cos\frac{\mu\pi}{2}}\right]-\tan^{-1}\left(\frac{B}{A}\right)-L\omega=-\pi+\phi_m \tag{19}$$

According to the gain crossover frequency constraint (5), the open-loop gain at the gain crossover frequency satisfies the following equation

$$Arg\left[G_2(j\omega)\right]=\frac{K\sqrt{\left(-\lambda K_i K_p^{\lambda-1}+K_d\omega^{\mu+1}\sin\frac{\mu\pi}{2}\right)^2+\left(\omega K_p^{\lambda}+K_d\omega^{\mu+1}\cos\frac{\mu\pi}{2}\right)^2}}{\sqrt{A^2+B^2}}=1 \tag{20}$$

$$=\lambda^2K_i^2K_p^{2(\lambda-1)}+K_d^{2(\mu+1)}-2K_p^{-1}\lambda K_iC-2\omega^2D+\omega^2K_p^{2\lambda}=E \tag{21}$$

where

$$C=\omega^{\mu+1}K_p^{\lambda}K_d\sin\frac{\mu\pi}{2}, D=\omega^{\mu}K_p^{\lambda}K_d\cos\frac{\mu\pi}{2},$$

$$E=1+T^2\omega^{2\alpha}+2T\omega^{\alpha}\cos\frac{\mu\pi}{2}$$

According to the robustness to parameters variation constraint (6), the open-loop phase is expressed as

$$\left(\frac{d(Arg(G_2(j\omega))}{d\omega}\right)=\frac{\mu C+\lambda K_p^{-1}K_i\left[K_p^{2\lambda}+(\mu+1)D\right]}{\omega^2K_p^{2\lambda}+\omega^{\mu+1}\left(K_d^2\omega^{\mu+1}+2D\right)+\lambda K_p^{-1}K_i(\lambda K_p^{2\lambda-1}K_i-2C)}=F \tag{22}$$

where $F=\dfrac{\alpha T\omega^{\alpha-1}\sin\frac{\alpha\pi}{2}}{1+T^2\omega^{2\alpha}+2T\omega^{\alpha}\cos\frac{\alpha\pi}{2}}-L$

Based on equations 19, 21, and 22 the controller parameters K_p, K_i, K_d, λ, and μ can be found. Alternatively, in the present system design, the estimation of these parameters is carried out using the FOMCON tool of MATLAB.

3. Identification of Plant Model

Generally, the industrial systems are mathematically described through first-order or second-order transfer function in order to study their performance. The pressure control experimental system considered for the present study is illustrated in Figure 1. The prototype system consists of a pneumatic pressure tank which is connected to a pneumatic compressor via an equal percentage control valve at the inlet side. The control diagram of the pressure regulating system is depicted in Figure 2. To determine the mathematical model of this plant, an open-loop system response for the step input has been recorded. The system is assumed to be a single input single output (SISO) one, the integer-order model of the plant can be determined by applying Ziegler-Nichols (ZN) or Cohen-Coon (CC) method presented by (Coughanowr, 1991).

Figure 1. Experimental setup of the pressure control system

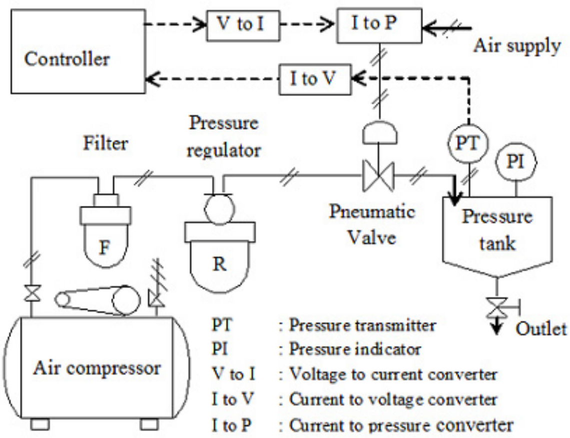

Figure 2. Control diagram of the pressure control system

However, in the case of fractional-order plant model, the MATLAB FOMCON toolbox is used for its identification. (Tepljakov et al., 2011) have presented the procedure to apply FOMCON tool for identifying a continuous-time fractional-order system model using a transient experimental dataset. Besides, the identified model can be analyzed based on the frequency-domain and time-domain categories. To estimate the fractional-order plant parameters K, T, L, and α related to the model (1), the experimental dataset is used in MATLAB FOMCON tool. In this identification process, an error signal is defined and a minimization of this error will be carried out. The error signal estimation is expressed as

$$E_r = \sum_{0}^{T_N} \left| y_E(t) - y_I(t) \right| \tag{23}$$

where $y_E(t)$ is experimental output data for the step input, $y_I(t)$ is the identified model response to the step input and.T_N is the finishing time for simulation and practical results. The experimental dataset used for identification and the identified model validation is shown in Figure 3. As a procedure presented by (Stefan, 2007) for time-domain system identification, the fractional pole polynomial is generated with commensurate order q = 1 and order N = 1. Using the Trust-Region-Reflective algorithm recommended by (Oustaloup et al., 2000) with coefficients limited to [-100; 100] and exponents limited to [0.001; 10] the identified initial fractional-order plant model is given as

$$P(s) = \frac{2.158}{4.80s^{1.26} + s^{3.506e-14}} e^{-1.4s} \tag{24}$$

After normalization of coefficients and exponential terms, the above identified model (24) can be expressed as

$$P(s) = \frac{2.158}{4.809s^{1.25} + 1} e^{-1.4s} \tag{25}$$

The map of poles and zeros related to the identified fractional-order plant is plotted in Figure 4. This figure confirms the stability of the identified system, since there is no pole or zero on the right side positive real axis.

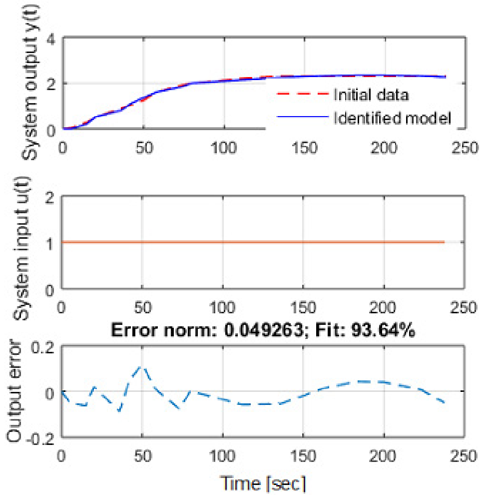

Figure 3. Plant experimental data and response of the identified model

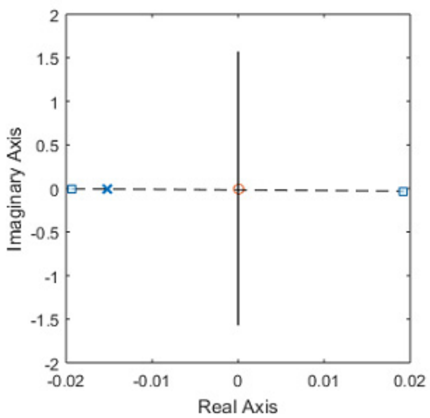

Figure 4. System stability

4. Controller Design Procedure

4.1 Fractional-Order PID Controller

In this section, the design approach for the FOC with regard to the identified fractional-order pressure control plant is introduced. According to the design procedure (Tepljakov et al., 2011), for the case of the fractional-order plant transfer function (25), the following design specifications are considered for the FOPI$^{\lambda}$D$^{\mu}$ controllers

- phase margin $\phi m = 60°$
- gain crossover frequency $\omega = 0.001$ rad/s.
- gain margin =10 dB
- high-frequency noise rejection:

$$\left|T(j\omega)\right|_{dB} \leq -20\,dB, \forall\, \omega \geq \omega_t = 10 \; rad/s$$

- disturbance elimination sensitivity function:

$$\left|S(j\omega)\right|_{dB} \leq -20\,dB, \forall\, \omega \geq \omega_s = 0.001 \; rad/s$$

By applying the integral absolute error (IAE) performance metric and Nelder-Mead optimization algorithm discussed by (Stefan, 2007), the identified FOPI$^{\lambda}$D$^{\mu}$ controller for the pressure control system is expressed as

$$C_2(s) = 0.220 + \frac{0.080}{s^{0.97}} + 0.368 s^{0.89} \tag{26}$$

Based on the fractional integrator approximation suggested by (Oustaloup et al., 1995), the exponent terms λ and μ have been implemented by selecting the frequency range from 0.001;100 rad/s. and the order of approximation is 5 (number of zeros and poles). The system validation is made by an open-loop system Bode plot and is shown in Figure 5. Based on this plot, one can observe that the gain crossover frequency and phase margin constraints are met. Moreover, the phase is flat for the specified gain crossover frequency which indicates that the system is considered to be more robust to parameters change and the overshoots corresponding to the step-input. The magnitude and phase plot of the disturbance elimination sensitivity function S(s) and noise rejection function T(s) for the system is shown in Figures 6 and 7 respectively and satisfying the constraints (7) and (8).

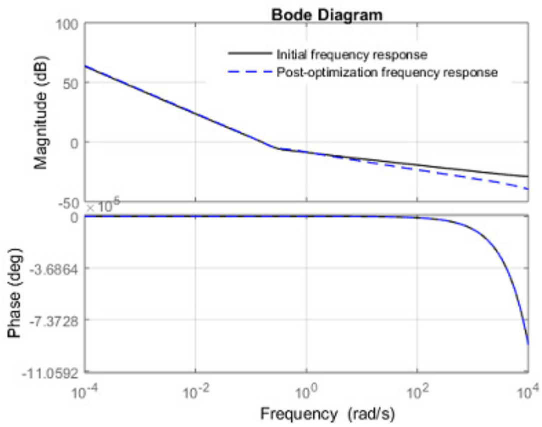

Figure 5. Open-loop system Bode plot for FOPI$^{\lambda}$D$^{\mu}$ controller

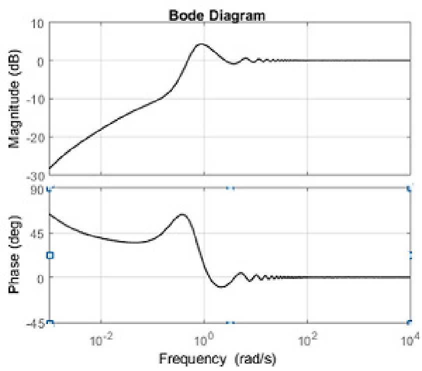

Figure 6. Magnitude and phase angle plot for sensitivity function S(jω)

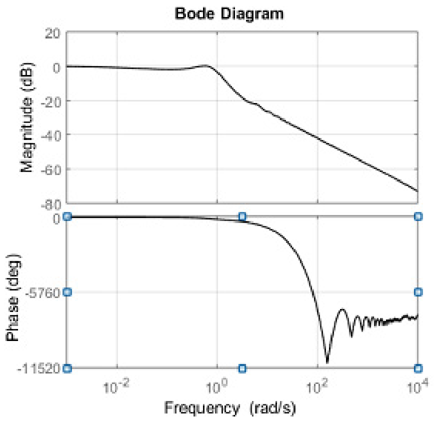

Figure 7. Magnitude and phase angle plot for noise rejection T (jω)

4.2 FVF[PI]$^{\lambda}$D$^{\mu}$ Controller

In the case of FVF[PI]$^{\lambda}$D$^{\mu}$ controller design, the three gain parameters (K_p, K_i and K_d) and two exponent terms (λ and μ) are chosen in the following manner. The same value of FOPI$^{\lambda}$D$^{\mu}$ controller gain parameters is used for FVF[PI]$^{\lambda}$D$^{\mu}$ controller design. But, two exponent terms are initially kept at 1 and subsequently, these parameter values are changed lively by the FLC based on the system dynamics. The FLC determines its input parameters error and change in error based on the system output and generates the outputs λ and μ accordingly. In the design processes, the parameters λ and μ are chosen to obtain a smooth, fast and stable system response when the system is subject to parameter variations and external load disturbances.

5. Fuzzy Logic Controller Design

The structure of the proposed FVF[PI]$^{\lambda}$D$^{\mu}$ control scheme is illustrated in Figure 8. In this control scheme, the FLC changes the exponent terms λ and μ of the controller dynamically based on the system dynamics. In the designed system, only two exponent terms were selected for online modification. (Dingyu et al., 2015) have stated that changing more parameters at the same time might be a difficult task as it increases the system complexity and the computation time. Further on, the changes in the exponential terms themselves will provide sufficient controller output to enhance system performance. In Figure 8, e(t), r(t), u(t) and y(t) are error, reference input, controller output and system output respectively at a particular time. The FLC uses the error e(t) and change in error ce(t) inputs to estimate the process-related dynamic conditions accordingly in order to generate outputs λ and μ by means of the rule-base fuzzy inference mechanism. The fuzzy system inputs are determined based on the following expressions

$$e(t) = r(t) - y(t) \qquad (27)$$

$$ce(t) = e(t) - e(t-1) \qquad (28)$$

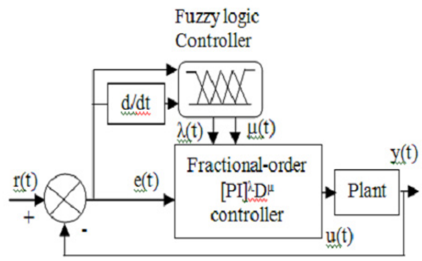

Figure 8. FVF[PI]$^{\lambda}$D$^{\mu}$ control structure

The triangular shape membership functions are used for inputs and outputs and are shown in Figure 9. The range of input parameters is normalized in the range of -1 to 1 and outputs are in the range of 0 to 2. To minimize the computational time, the input ce(t) is designed with three membership functions. The input e(t) and two outputs are designed with five membership functions. The membership function of each input and output is described based on the linguistic variables. The linguistic variables are Zero (ZE), Small (SM), Very Small (VS), Medium (ME), High (HG), Very High (VH), Negative Big (NB), Negative Small (NS), Positive Big (PB), Positive Small (PS), Negative (N), Positive (P) and they are assigned to the inputs and the outputs as illustrated in Figure 9.

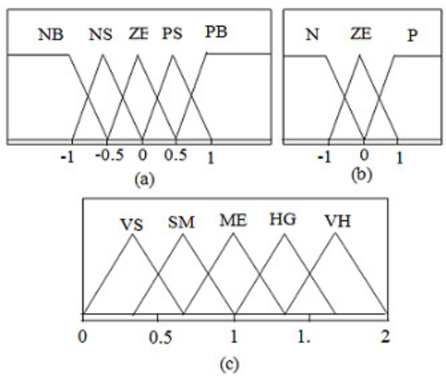

Figure 9. Membership functions of FLC (a) Error e(t), (b) Change in error ce(t), (c) Output (λ and μ)

The fuzzy inference mechanism will map the system's input and output using rule-base and described using linguistic variables. The fuzzy rule-base is designed using the fundamental control theory related to the influence of λ and μ inputs on system output and by using certain theoretical concepts. The rule-base applied for the present FLC design is given in Table 1. To get a precise output, the center of area defuzzification method is used and is given as

$$y_{COA} = \frac{\int_y \mu_A(y) y dx}{\int_y \mu_A(y) dx} \tag{29}$$

where y_{COA} is the crisp output, $\mu_A(y)$ is the aggregated membership function and y is the output variable.

Table 1. Rule-base

λ		e(t)				
		NB	NS	ZE	PS	PB
ce(t)	N	HG	HG	ME	ME	SM
	ZE	SM	ME	ME	HG	VG
	P	VS	VS	SM	ME	ME
μ						
ce(t)	N	SM	SM	VS	VS	ME
	ZE	VH	HG	ME	ME	VS
	P	VH	HG	VS	VS	ME

6. Simulation Results and Discussion

To demonstrate the effectiveness of the designed FVF[PI]$^{\lambda}$D$^{\mu}$ controller, simulation studies are carried out under various conditions. The system response for the step input for three different controllers is shown in Figure 10. The step response of the system obviously shows that the transient performance of the designed FVF[PI]$^{\lambda}$D$^{\mu}$ controller is better than that of the other two types of controllers. The proposed controller features an enhanced output response in terms of settling time and rise time. Due to the lively change of exponent terms λ and μ using FLC, the FVF[PI]$^{\lambda}$D$^{\mu}$ controller can perform better than other controllers in the initial period of the step response. On the other hand, the IOPID and FOPI$^{\lambda}$D$^{\mu}$ controllers can only reach an average performance and it takes more time for them to reach the final steady-state value due to their fixed parameters. Figure 11 illustrates the parameter changes related to the FVF[PI]$^{\lambda}$D$^{\mu}$ controller during the step input response. This figure shows that, because of changes in the exponent value, the proportional plus integral and derivative terms are updated continuously to attain an improved system performance.

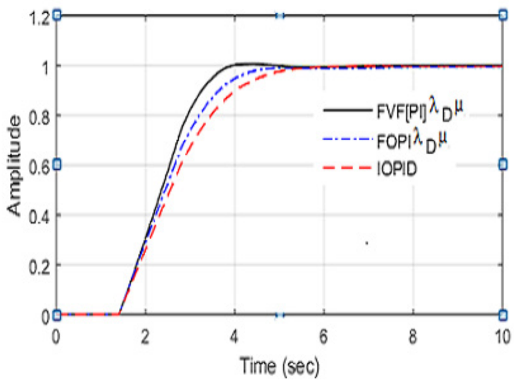

Figure 10. System response for the step input.

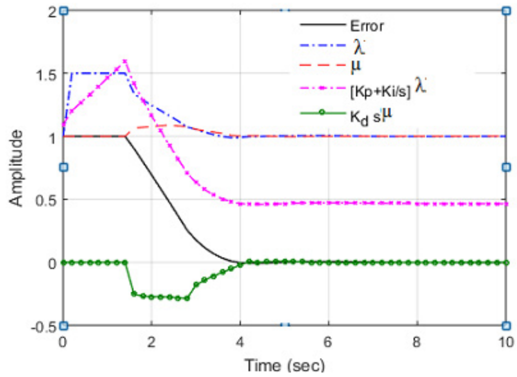

Figure 11. Controller parameters for the step input

To validate the robustness of the designed controller, the controller performance for the external load disturbance and set-point change has been studied. The system response for external load disturbance conditions using three different controllers is depicted in Figure 12. Based on this result, one can understand that the FVF[PI]$^{\lambda}$D$^{\mu}$ controller enables the system to recover quickly from the disturbance it was subject to and its output to reach the original steady-state value within a short time. The system using a FOPI$^{\lambda}$D$^{\mu}$ controller also performs similarly to FVF[PI]$^{\lambda}$D$^{\mu}$ controller. But, the IOPID controller takes little extra time to make the system output reach its original value during the load disturbance.

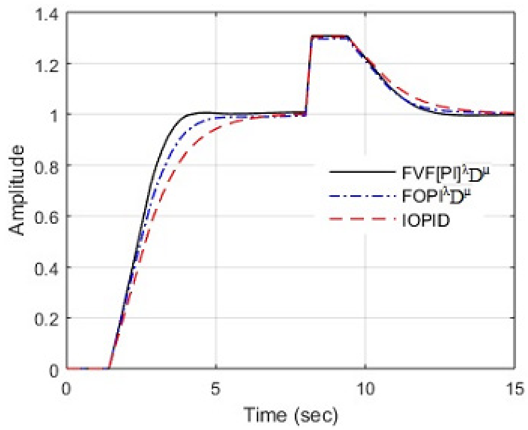

Figure 12. System response for the load disturbance

The controller parameters' modification by FLC during load disturbance is also recorded and is shown in Figure 13. As it is shown in this Figure, the FLC immediately identifies the disturbances to the system and accordingly modifies the proportional plus integral and derivative terms using their exponent appropriately in order to enable the system to recover from the disturbances. The proposed controller performance for the set-point change is illustrated in Figure 14. Based on this response, one can understand that the proposed FVF$[PI]^{\lambda}D^{\mu}$ controller outperforms the other two above-mentioned controllers.

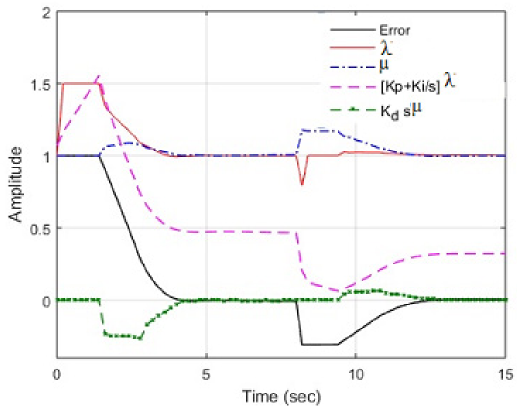

Figure 13. Controller parameters during the load disturbance

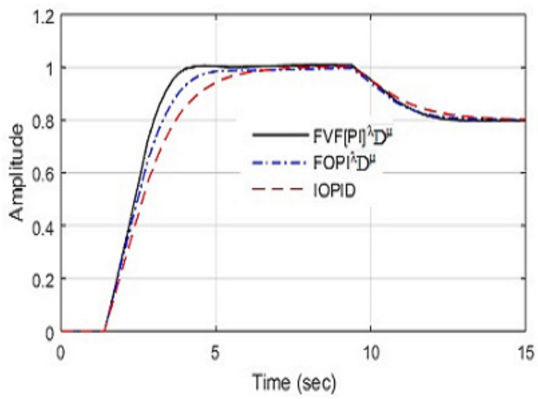

Figure 14. System response for the set-point change

As the changes in the input parameters are immediately observed and the necessary changes in the above-mentioned controller are done by FLC, the FVF$[PI]^{\lambda}D^{\mu}$ exhibits better results than the other controllers. Hence, these results obviously indicate that the lively change of the value of the orders related to the controller parts of the FOC will enrich the robustness of the system.

The system performance during step input is compared and summarized numerically using performance indexes such as integral absolute error (IAE), integral square error (ISE), rise-time and settling time. The comparison of performance indexes for the three above-mentioned controllers is illustrated in Table 2. Based on the numerical comparison, the FVF$[PI]^{\lambda}D^{\mu}$ controller exhibits its strength with a significantly lower settling time and rise-time when compared to other controllers. With regard to error criterion, the FVF$[PI]^{\lambda}D^{\mu}$ controller features lower values for both ISE and IAE when compare to the FOPI$^{\lambda}$D$^{\mu}$ and IOPID controllers.

Table 2. Step response numerical comparisons

Controller type	Performance specifications			
	ISE	IAE	Settling time (sec.)	Rise-time (sec.)
FVF$[PI]^{\lambda}D^{\mu}$	2.03	2.38	3.94	1.8
FOPI$^{\lambda}$D$^{\mu}$	2.15	2.65	5.24	2.4
IOPID	2.28	2.91	6.95	3.2

7. Conclusion

A novel control method based on a FVF$[PI]^{\lambda}D^{\mu}$ controller has been designed and its performance was demonstrated based on simulation studies. A prototype pressure control system has been modelled into a FOPDT system using an experimental dataset. The FLC of the proposed FVF$[PI]^{\lambda}D^{\mu}$ controller structure has been designed to lively change the order value related to integral and derivative parts based on the system dynamics. The system performance was examined through simulation studies under different conditions such as load disturbance, parameter change, etc. The results confirm that the designed controller performs well when the system is subject to external load disturbance and parameter change conditions. Moreover, it has significantly reduced the settling time when compared to classical FOPI$^{\lambda}$D$^{\mu}$ and IOPID controllers. The results obviously prove that the proposed control structure can achieve a high dynamic control performance and exhibit robustness concerning parameter variation and external load disturbances as well.

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