Stabilization and Voltage Regulation of the Buck DC-DC Converter Using Model Predictive of Laguerre Functions

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Abstract: This paper proposes a solution to stability and voltage regulation of switched mode buck DC-DC converter using a model predictive controller (MPC) of Laguerre functions. The MPC is used to compute the optimal control actions subject to constraints. To have a low computation burden and to avoid ill-conditioning, particularly for large prediction horizon, an exponentially weighted Laguerre based model predictive control (LMPC) is used. In order to validate the effectiveness of the proposed scheme, the performance of the proposed controller is compared with a linear quadratic regulator and classical linear state space MPC in MATLAB. Obtained results of simulation show that optimal voltage regulation has been achieved.

Keywords: Model predictive control, Optimal control, Ill-conditioning, Prediction horizon, Exponentially weighted, Laguerre, Linear quadratic regulator.

1. Introduction

The technological advance in the field of power electronics has increased the use of DC–DC power converters in a wide range of applications. Thus, their major role is to bring the voltage to the proper level, providing a regulated output voltage based on a supply voltage that can vary. Nevertheless, the control problems that are related to such converters still pose theoretical challenges. Usually, the control design for these switched systems is based on a continuous or discrete state-space averaged model [1]. For most of DC–DC power converters, the averaged model has to be locally linearized around a specific operating point because of the product between the state vector and the control input. A classic approach is to derive linear control laws based on an averaged linearized model: proportional–integral (PI) controllers [2], [3] state-feedback and Linear Quadratic Regulators (LQR) [4].

While PID controllers have wide application in many control problems, and often perform well without any modification or only with coarse tuning, they can operate badly in some applications, and do not in general perform optimally [5]. LQR is a relatively modern control technique that is effective but restricted to applications related with linear system models. In addition, the solution to the LQR problem, i.e., infinite prediction horizon, is available only in the unconstrained case, whereas MPC employs a finite prediction horizon to make the control problem tractable numerically.

An established solution to constraint handling in process control and in control of other relatively slow systems is MPC [6]. Just as LQR, MPC employs the model of the system to make predictions on future behavior of the system and optimize the control action accordingly. The reason for the slow speed of MPC its huge computational burden. To overcome this drawback, MPC with orthonormal basis function called Laguerre function [7] was proposed. The proposed MPC lowers computational burden significantly which makes it more suitable for real time implementation. In addition, an exponential data weighting is used to minimize numerical issue in MPC with large prediction horizon [8].

In this paper, the buck converter dynamics is reformulated to address the nonlinearity and is described in state space averaging (SSA) model. MPC employing [6], [7] is proposed to stabilize and regulate the output voltage of the buck DC-DC converter with systems constraints. The constraints are introduced from the converter circuitry. In particular, the control variable (duty cycle) is limited between zero and one. Additional constraints are imposed for safety measure such as the limit on inductor current. To prove the effectiveness of the suggested method, time-based simulations are carried out and compared with Optimal Discrete Linear Regulator (DLQR) and with the more common state space approach presented in [15]. The obtained results proved that the Laguerre based MPC (LMPC) is able to control successfully the buck converter in the transient and steady state.

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2. Modeling the buck converter

2.1 Continuous Time Model

The circuit topology of the converter is shown in Figure 1 where R_o is the output resistance which, R_c is the equivalent series resistance of the capacitor, R_L is the internal resistance of the inductor, L and C represent the inductance and the capacitance of the low-pass filter of the converter, respectively, and V_s is the input voltage. The semiconductor switches S_1 and S_2, are operated by a pulse sequence with constants switching frequency f_s (with period T_s). The duty cycle d is defined as $d=t_{on}/T_s$ where t_{on} represents the interval within the switching period during which the S_1 switch is closed.

We consider the state vector

$$x_m(t) = \left[I_L(t) \quad V_o(t) \right]^T \tag{1}$$

The set of continuous-time state space equations describing the converter's behavior of the two configurations in CCM (Continuous Conduction Mode) are:

$$\dot{x}_m(t) = \begin{cases} A_{on}x(t) + B_{on}V_s, kT_s \le t < (k+d(k))T_s \\ A_{off}x(t), (k+d(k))T_s \le t < (k+1)T_s \end{cases} \tag{2}$$

$$y(t) = Cx(t) \tag{3}$$

where the matrices A_{on}, B_{on} and C are given by

$$A_{on} = A_{off} = \begin{bmatrix} -\frac{R_l}{L} & -\frac{1}{L} \\ \frac{R_o}{C(R_o+R_c)}\left(1 - R_cR_l\frac{C}{L}\right) & -\frac{1}{C(R_o+R_c)}\left(1+\frac{R_cR_oC}{L}\right) \end{bmatrix},$$

$$B_{on} = \begin{bmatrix} \frac{1}{L} \\ \frac{R_cR_o}{(R_c+R_o)L} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

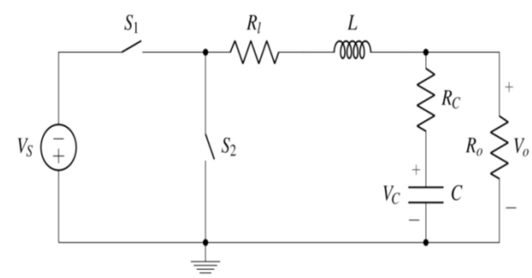


Figure 1. Buck Converter Topology

The continuous-time averaged model is

$$\begin{aligned} \dot{x}_m(t) &= \underbrace{\left[A_{on}d + A_{off}(1-d) \right]}_{A_m} x_m(t) + \underbrace{B_{on}}_{B_m} dV_s \\ &= A_m x(t) + B_m dV_s \end{aligned} \tag{4}$$

$$y_m(t) = \underbrace{[Cd + C(1-d)]}_{C_m} x_m(t) = C_m x_m(t) \tag{5}$$

In the following section, we derive a model to serve as prediction model for the optimal control problem formulation. For this, we reformulate the converter model.

2.2 Reformulated Continuous Time Model

We will be motivated to remove V_s from the model equations by using it to scale the physical quantities (states and output voltage reference) used in the model. Hence, we introduce the state $x'_m(t) = \frac{x_m(t)}{V_s}$ to scale (4) and (5). This yields the reformulated state space equations

$$\dot{x}'_m(t) = A_m x'(t) + B_m u(t) \tag{6}$$

$$y'_m(t) = C_m x'_m(t) \tag{7}$$

where the matrices A_m, B_m and C_m are as in (4) and (5). Hereafter, we use control the notation for input signal, u(t) and duty cycle, d(t) interchangeably as required.

Since in real time operation, the input voltage is either piecewise constant or varies only slowly compared to the fast switching frequency, the normalized converter can serve as a sufficiently accurate prediction model. Since normalizing makes the prediction model equations not influenced by (the time-varying) input voltage V_s, the matrices, A_m, B_m *and* C_m in (4) and (5) are time-invariant. Thus, the only time-varying model parameters are the scaled output voltage reference $v_{o,ref}$ and normalized current limit $(i_{L,max}, i_{L,min})$.

2.3 Discretization of the continuous time model

This paper is based on discrete model based MPC and hence (7) is discretized at sampling period of T_s using a zero-order hold technique, yielding the discrete state-space model

$$x'_m(k+1) = A_d x'_m(k) + B_d u(k) \tag{8}$$

$$y'_m(k) = C_d x'_m(k) \tag{9}$$

where $A_d = e^{A_m T_s}$, $B_d = \int_0^{T_s} e^{A_m T_s} B_m d\tau$, $C_d = C_m$

Let $\Delta x_m^{'}(k) = x_m^{'}(k) - x_m^{'}(k-1)$ and $\Delta u(k) = u(k) - u(k-1)$ denote the incremental state and input vectors, respectively, computed from the corresponding vectors in (8).

The state dynamics in the incremental model

$$\Delta x_m^{'}(k+1) = A_d \Delta x_m^{'}(k) + B_d \Delta u(k) \tag{10}$$

In a similar manner the output incremental dynamics are given by

$$\begin{aligned} y_m^{'}(k+1) = y_m^{'}(k) + C_d A_d \Delta x_m^{'}(k) \\ + C_d B_d \Delta u(k) \end{aligned} \tag{11}$$

By choosing a new state vector $x'(k) = \left[\Delta x_m^{'}(k) \quad y_m^{'}(k)\right]^T$ the augmented state-space model is obtained by combining (9) with (10).

$$x'(k+1) = Ax'(k) + B\Delta u(k) \tag{12}$$

$$y'(k) = Cx'(k) \tag{13}$$

where $A = \begin{bmatrix} A_d & 0_1 \\ C_d A_d & I \end{bmatrix}, B = \begin{bmatrix} B_d \\ C_d B_d \end{bmatrix}, C = \begin{bmatrix} 0_2 & I \end{bmatrix}$

and the 0_1, 0_2 and I, respectively, are the zero and identity matrices of compatible dimensions.

3. Model predictive control design

To get high control performance while respecting state and input constraints we have used a model predictive control (MPC) approach. MPC provides a systematic way of handling constraint optimal control problems. In this and following sections the performance of a DC-DC controller designed using Laguerre-based MPC (LMPC) is compared with that designed using the more common linear state-space method (SS-MPC) as suggested in [15].

3.1 Linear State-Space MPC (SS-MPC)

Linear MPC, as unlike the usual forms of MPC, is attractive because the plant is modelled using a linear state space and plant constraints are modelled using linear equalities and inequalities. Using the augmented state-space model (13), the future state variables are calculated sequentially leading to the following equations:

$$\begin{aligned} &x(k+1|k) = Ax(k) + B\Delta u(k) \\ &x(k+2|k) = A^2 x(k) + AB\Delta u(k) + B\Delta u(k+1) \\ &\vdots \\ &x(k+Np|k) = A^{N_p} x(k) + A^{N_p-1} B\Delta u(k) \\ &+ \ldots + A^{N_p-N_c} B\Delta u(k+N_c-1) \end{aligned} \tag{14}$$

where N_p and N_c are termed the prediction and control horizons, respectively.

The predicted output variables for the next *Np* samples can be expressed in the compact form as

$$Y = Fx(k) + \Phi\Delta U \tag{15}$$

$$Y = \begin{bmatrix} y(k+1) & y(k+2) & \cdots & y(k+N_p) \end{bmatrix}^T$$

$$\Delta U = \begin{bmatrix} \Delta u(k) & \Delta u(k+1) & \ldots & \Delta u(k+N_c-1) \end{bmatrix}^T$$

$$F = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix}; \Phi = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ CA^2 B & CAB & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & \ldots & CA^{N_p-N_c}B \end{bmatrix}$$

Then cost function used for the MPC is

$$J = (R_s - Y)^T \bar{Q}(R_s - Y) + \Delta U^T \bar{R} \Delta U \tag{16}$$

where $\bar{Q} = diag[Q, \ldots, Q]$ and $\bar{R} = diag[R, \ldots, R]$ are block diagonal matrices that have identical component matrices *Q* and *R* respectively. Here, *Q* is positive semi-definite matrix and R is a positive definite matrix. R_s is the reference signal vector.

Minimizing the cost function (16) yields the optimal control vector

$$\Delta U = \left(\Phi^T \bar{Q} \Phi + \bar{R}\right)^{-1} \Phi^T \bar{Q} \left(R_s - Fx(k)\right)$$

and applying receding horizon principle

$$\Delta u(k) = \begin{bmatrix} I & 0 & \cdots & 0 \end{bmatrix} \Delta U \tag{17}$$

3.2 Laguerre Based Model Predictive Control (LMPC)

MPC can be designed using orthonormal functions wherein the control signal ΔU is represented using Laguerre functions. Since the state and output signals can also be expressed in terms of ΔU, and then they too can be expressed using Laguerre functions.

In this paper, orthonormal basis Laguerre function [7] is used for modeling control signal of (17). The z-transform of jth Laguerre function is given by

$$\Gamma_j = \frac{\sqrt{1-a^2}}{z-a}\left[\frac{1-az}{z-a}\right]^{j-1} \quad (18)$$

where $0 \leq a \leq 1$ is the pole of Laguerre polynomial.

The control input variable can be expressed by the following Laguerre functions:

$$\Delta u(k_i+k) \approx \sum_{j=1}^{N} c_j(k_i) l_j(k_i) \quad (19)$$

where l_j is the inverse z-transform of Γ_j in the discrete domain. The coefficients c_j are unknowns and must be acquired from systems data. The parameters a and N are used for tuning and can be adjusted by accordingly. Generally, choosing larger value for N increases the accuracy of input sequence estimation [7] and the control horizon (N_c) is related to the parameters a and N [14] by

$$a \approx e^{-N/N_c} \quad (20)$$

Equation (19) can be rewritten as

$$\Delta u(k_i+k) = L(k)^T \eta \quad (21)$$

$$L(k) = [l_1(k) \quad l_2(k) \quad \ldots \quad l_N(k)]^{\mathrm{T}}, \eta = [c_1 \quad c_2 \quad \ldots \quad c_N]^{\mathrm{T}}$$

At an arbitrary future instant m, the state is described using Laguerre functions as

$$x(k_i+m \mid k_i) = A^m x(k_i) + \sum_{i=0}^{m-1} A^{m-i-1} B L(i)^T \eta \quad (22)$$

Similarly, the output is described as

$$y(k_i+m \mid k_i) = CA^m x(k_i) + \sum_{i=0}^{m-1} CA^{m-i-1} B L(i)^T \eta \quad (23)$$

The objective is to come up with the coefficient vector η that minimizes the cost function

$$J = \sum_{m=1}^{N_P} x(k_i+m \mid k_i)^{\mathrm{T}} Q x(k_i+m \mid k_i) + \eta^{\mathrm{T}} R_L \eta \quad (24)$$

where $\mathrm{Q} \geq 0$ and $\mathrm{R_L} \geq 0$ is the weighting matrices for tuning purpose.

Equation (24) can be rewritten as

$$x(k_i+m \mid k_i) = A^m x(k_i) + \Phi(m)^T \eta \quad (25)$$

where $\Phi(m)^T = \sum_{i=0}^{m-1} A^{m-i-1} B L(i)^T$

By substituting (25) in (24) and minimizing J ($\partial J/\partial \eta = 0$), the Laguerre coefficients vector

$$\eta = -\Omega^{-1} \Psi x(k_i) \quad (26)$$

with $\Omega = \sum_{m=1}^{Np} \Phi(m) Q \Phi(m)^T + R_L$, $\Psi = \Phi(m) Q A^m$

The objective function (24) is subjected to the constraints on input and output variable that can be defined in the following form

$$\begin{aligned} u_{\min} &\leq u(k_i+m) \leq u_{\max} \\ \Delta u_{\min} &\leq \Delta u(k_i+m) \leq \Delta u_{\max} \\ y_{\min} &\leq y(k_i+m) \leq y_{\max} \end{aligned} \quad (27)$$

where $\Delta u(k_i+m) = L(m)^T \eta$, $u_{\min}$ and $u_{\max}$ are lower and upper limits on control variable, $y_{\min}$ and $y_{\max}$ are lower and upper limits on the output variables and $\Delta u_{\min}$ and $\Delta u_{\max}$ lower and upper limits on incremental control variable respectively.

3.3 Stability

In practice, the choose of larger prediction horizon is limited by numerical problems, particularly in the process with high sampling rate. A well-established method to solve this problem is to use exponential data weighting in the objective function [12] an idea originally proposed by Anderson and Moore [13].

More specifically, we will focus on the discrete exponential factor $e^{\lambda \Delta t}$ for $t > 0$ and the discrete weights forming a sequence $\{\alpha_j, j = 0, 1, 2, \ldots\}$ in which we set $\alpha = e^{\lambda \Delta t}$ with Δt being the sampling interval.

The proposed objective function is equivalent the linear quadratic regulator (LQR) systems but with discrete weights included

$$\hat{J} = \sum_{j=1}^{N_P} \alpha^{-2j} x(k_i+j \mid k_i)^{\mathrm{T}} Q x(k_i+j \mid k_i) + \sum_{j=0}^{N_p} \alpha^{-2j} \Delta u(k_i+j)^T R \Delta u(k_i+j) \quad (28)$$

In compact form, the exponentially weighted cost function is

$$\hat{J} = \sum_{j=1}^{N_P} \hat{x}(k_i + j \mid k_i)^T Q x(k_i + j \mid k_i) + \sum_{k=0}^{N_p} \Delta\hat{u}(k_i + j)^T R \Delta\hat{u}(k_i + j) \quad (29)$$

with state equation

$$\hat{x}(k+1) = A_\alpha \hat{x}(k) + B_\alpha \Delta\hat{u}(k) \quad (30)$$

and $A_\alpha = A/\alpha$ and $B_\alpha = B/\alpha$

with $Q \geq 0$, $R > 0$, and $N_p \rightarrow \infty$, minimizing the cost function $\hat{J}$ is similar to the DLQR problem which is solved using algebraic Riccati equation (31). Then the state feedback control gain for the stabilization $\hat{K}$,

$$\hat{K} = (R + \alpha^{-2} B^T \hat{P} B)\alpha^{-2} B^T \hat{P} A \,,$$

$$\frac{A^T}{\alpha}\left[\hat{P} - \hat{P}\frac{B}{\alpha}\left(R + \frac{B^T}{\alpha}\hat{P}\frac{B}{\alpha}\right)^{-1}\frac{B^T}{\alpha}\hat{P}\right]\frac{A}{\alpha} + Q - \hat{P} = 0 \quad (31)$$

which makes the closed loop system stable with all its eigenvalues inside the unit circle and the closed loop system being described by

$$\hat{x}(k_i + j + 1 \mid k_i) = \alpha^{-1}(A - B\hat{K})\hat{x}(k_i + j \mid k_i) \quad (32)$$

From (32), the modified system has all its eigenvalues inside the unit circle by increasing $N_p \rightarrow \infty$. So

$$\alpha^{-1}\left|\lambda_{\max}(A - B\hat{K})\right| < 1 \quad (33)$$

Thus, by choosing $\alpha > 1$ it is possible to make stable. Several simulations on indicates the choice of α greater than unity stabilizes the system.

Table 1. Converter and controller parameters

Converter Parameters			
In S.I.		In p.u.	
L	27 µH	x_l	0.6786
C	4.7 µF	x_c	11.8124
R_c	0.025 Ω	r_c	0.0025
R_L	0.4 Ω	r_l	0.04
R_o	10 Ω	r_o	1
V_s	20 V	v_s	1
$V_{o,ref}$	10 V	$v_{o,ref}$	0.5
$I_{L,max}$	3 A	$i_{L,max}$	1.5
$V_{o,max}$	15 V	$v_{o,max}$	0.75

Controller Parameters	
N_p	200
T_s	25 µs

4. Simulation parameters and results

Simulations using MATLAB have been done in order to prove the effectiveness of proposed method. Table 1 shows the parameters of the DC-DC buck converter used in simulation. The constraints on the states input and incremental control inputs are chosen so as to assure the keep the signals at physically appropriate values as follows:

$$d_{\min} = 0 \leq d(t) \leq 1 = d_{\max}$$
$$\Delta d_{\min} = -0.5 \leq \Delta d(t) \leq 0.5 = \Delta d_{\max} \quad (34)$$

$$y_{\min} = \begin{bmatrix} i_{L\min} \\ v_{o\min} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \leq y(t) \leq \begin{bmatrix} i_{L\max} \\ v_{o\max} \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.75 \end{bmatrix} = y_{\max} \quad (35)$$

4.1 Use of Laguerre parameters

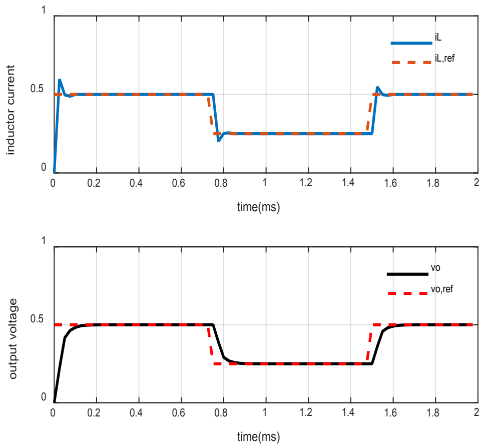
Assuming we want to attain a control horizon of 10. To grant stability for large prediction horizon (N_p=200), we used $\alpha = 1.6$ and $\lambda = 1$. The parameter N is an integer to specify the network order. We used (20) to find the corresponding scaling factor a as shown in Table 2.

Table 2. Scaling factor a for constant control horizon and varying N

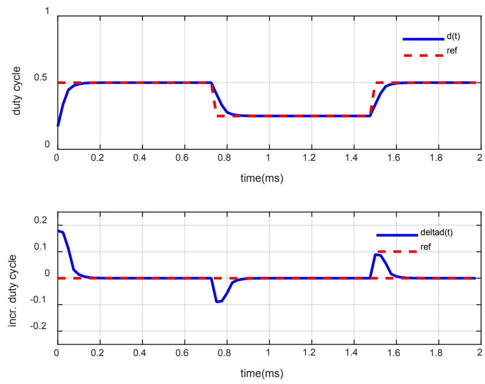
N_c	N	a
10	2	0.8187
10	4	0.6703
10	6	0.5488
10	8	0.4493
10	10	0.3679

Two cases are considered in order to analyze the system's response for square signal profile of the output voltage reference$V_{o,ref}$:10–5–10 V.

Case 1. Set the Laguerre Order N to be large value ($N = 8$). See Table 3 and Figure 2.



(a)



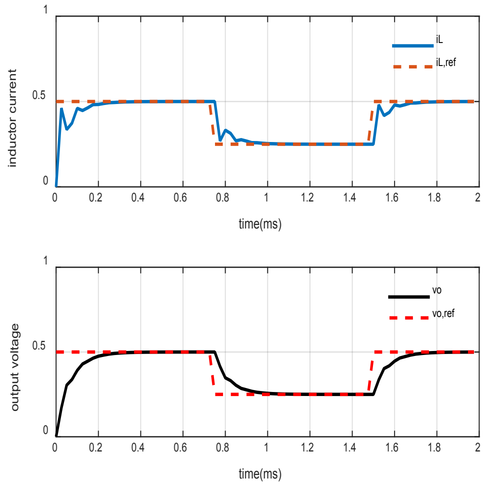
(b)

Figure 2. Simulation results for reference voltage variations $V_{o,ref}$: 10–5–10 V

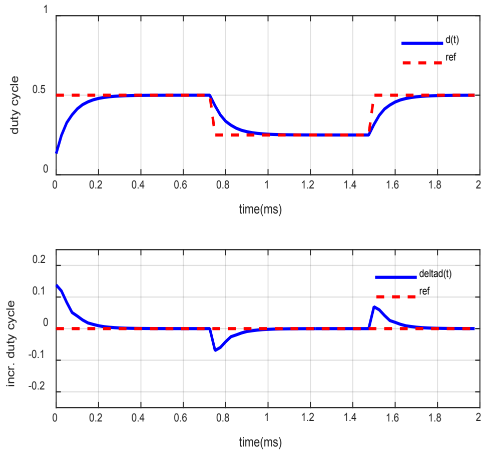
(a) Inductor current iL and Output voltage v_o obtained using LMPC for N=8

(b) Duty Cycle and incremental duty cycle obtained using LMPC for N=8

Case 2. Set the parameter smaller value ($N = 2$).



(a)



(b)

Figure 3. Simulation results for reference voltage variations $v_{o,ref}$: 10 –5–10 V

(a) Inductor current iL and Output voltage v_o obtained using LMPC for N=2

(b) Duty Cycle d(t) and incremental duty cycle Δ d(t) obtained using LMPC for N=2

It can be seen from Table 3 that the closed loop predictive control system is very similar to the DLQR system.

Table 3. Closed loop Eigen values and feedback gain vector when N=8

Performance Metrics	Control Strategy	
	LMPC	DLQR
Feedback gain	[-0.0464 -0.5118 0.1792 0.1792]	[-0.0463 -0.5118 0.1792 0.1792]
Eigen Values	-0.1011 ± j0.2268,1, 0.4135	-0.1012± j 0.2267,1, 0.2305

Table 4. Closed loop Eigen values and feedback gain vector when N=2

Performance Metrics	Control Strategy	
	LMPC	DLQR
Feedback gain	[-0.0424 -0.2860 0.1384 0.1384]	[-0.0463 -0.5118 0.1792 0.1792]
Eigen Values	-0. 2954 ± j 0.4628,1,0.6973	-0.1012 ± j0.2267,1, 0.4135

Table 5. Performances comparison of comparison LMPC and SS-MPC obtained in simulation for reference voltage $V_{o,ref}$: 10–5–10 V

Performance Metrics	Control Strategy	
	LMPC	SS-MPC
Feedback gain	[-0.1952 -0.0725 0.1534 0.3200]	[-0.4012 -0.1782 0.3774 0.5624]
Eigen Values	-0.3749 ± j0.4862, 0, 0.6888	-0.3815± 0.3798, 0, 0.4333
Overshoot	1.00 %	0.5008 %
Settling time ms	1.5919	1.7081

Table 6. Performances comparison of comparison LMPC and SS-MPC obtained in simulation for reference voltage $V_{o,ref}$: 10–5–10 V

Performance Metrics	Control Strategy	
	LMPC	SS-MPC
Feedback gain	[0.3966 -0.1715 0.3632 0.5584]	[-0.4012 -0.1782 0.3774 0.5624]
Eigen Values	-0.3754 ± j 0.3887, 0, 0.4346	-0.3815± 0.3798, 0, 0.4333
Overshoot	1.00 %	1.00 %
Settling time ms	1.5919	1.5919

For the same simulation, (different instance), one can observe in Figure 2(b) and (3b) the evolution of the duty cycle, paired with its incremental value to allow for a direct comparison. In addition, because of the presence of exponentially decaying factor in the Laguerre functions, the increment on duty cycle is granted to converge to zero after the transient time.

Comparing the results in table 3 and 4, it can be seen that the closed-loop control performance is more sensitive to the parameter *N* when is small (N=2). This is very helpful in the situations when the optimal DLQR system does not gives us with satisfactory performance.

4.2 Stability Analysis

Looking at the location the eigenvalue's of LMPC and checking whether they are within unit circle we can decide the stability of the system. We also make a comparison with those of the optimal Discrete Linear Quadratic Regulator (DLQR) system. To grant stability as explained previous section we use $\alpha = 1.6$, N_p=200 and N=8.

In Figure 4, we can see that all the eigenvalues appear inside the unit circle as required. In addition, it is noted that the eigenvalues of the LMPC coincides with that of the optimal DLQR. This not only shows the system is stable but also shows that the controller performs optimally. The exponentially weighted cost function removes the problem of stability (ill numerical condition) because the model used in the prediction is changed to be stable using the scaling factor α. As a result, the prediction horizon N_p can be selected to be sufficiently large without creating ill-conditioning (here N_p=200). Hence, closed-loop stability is guaranteed.

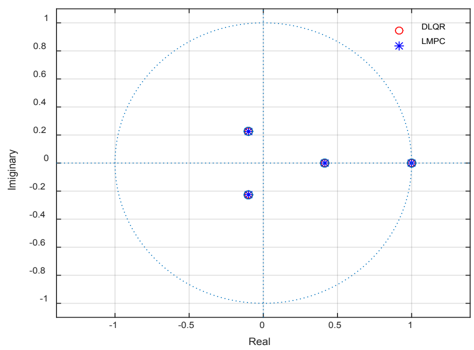


Figure 4. Eigenvalues for DLQR and LMPC

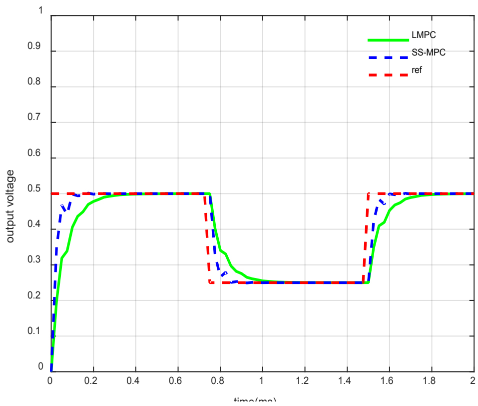
4.3 Performance of LMPC as compared to SS-MPC

In this section, we examine at the performance of LMPC when used to regulate the output voltage to reference signal. Also, LMPC is compared to SS-MPC.

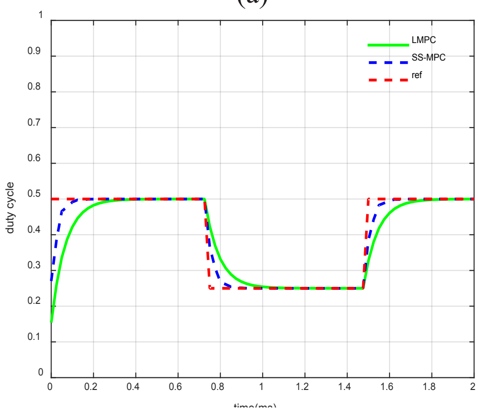
Here also we considered two cases:

(a) Performances comparison of Laguerre based Model predictive control(LMPC) and state space model predictive control(SS-MPC) simulated for N_c=10; N=2 , Q_y=1 and R=1. Using these parameters , the closed-loop poles of the DLQR system are -0.3815 ± j0.3798, 0.4333 and 0 with feedback gain:

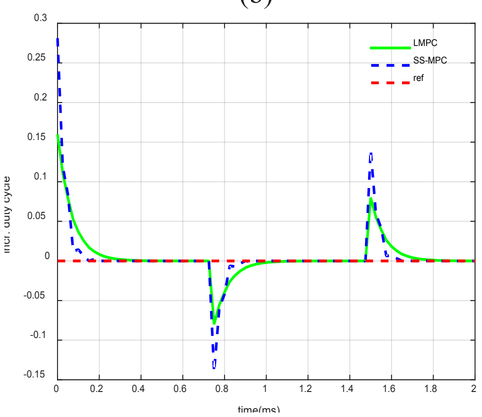
$$K_{lqr} = \begin{bmatrix} -0.4012 & -0.1782 & 0.3774 & 0.5624 \end{bmatrix}$$



(a)



(b)



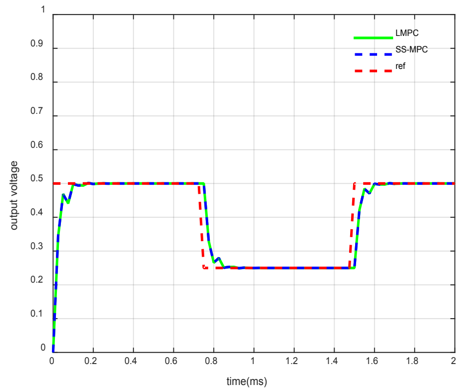
(c)

Figure 5. Simulation results for reference voltage variations $v_{o,ref}$: 10–5–10 V obtained using LMPC and SS-MPC for N=2

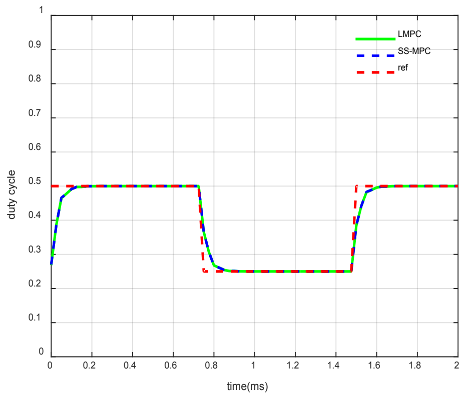
(a) Output voltage v_o; (b) Duty Cycle d(t); (c) Incremental duty cycle Δd(t).

(b) Performances comparison Laguerre based Model predictive control(LMPC) and state space model predictive control(SSMPC) simulated for N_c=10; N=5 , Qy=1 and R=1. Using these parameters , the closed loop poles of the DLQR system are -0.3815 ± j0.3798, 0.4333 and 0 with feedback gain:

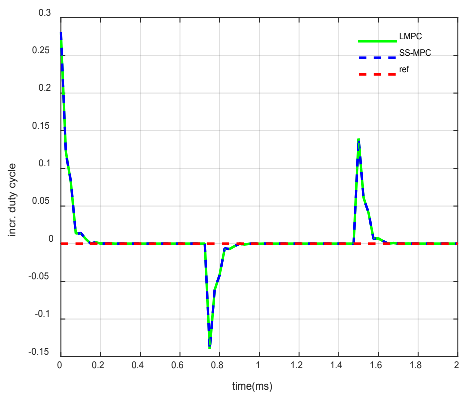
$$K_{lqr} = \begin{bmatrix} -0.4012 & -0.1782 & 0.3774 & 0.5624 \end{bmatrix}$$



(a)



(b)



(c)

Figure 6. Simulation results for reference voltage variations $v_{o,ref}$: 10–5–10 V obtained using LMPC and SS-MPC for N=5

(a) Output voltage v_o; (b) Duty Cycle d(t); (c) Incremental duty cycle Δd(t).

As it can be seen from figure 5, SS-MPC performs better than LMPC. This is because the LMPC controller used lower Laguerre order (N=2).

In Figure 6, both LMPC and SS-MPC regulate the output voltage to the reference very well. However, LMPC performs better than SS-MPC in that it requires only five parameters (N=5) to for the regulation compared to SS-MPC's minimum of 10(N_c = 10) parameters. Thus, we can choose a Laguerre network with lower number of terms N (that gives lower computation burden). The computational cost is lower if a smaller number of parameters are used.

5. Conclusion

Optimal control of the DC-DC buck converter was considered. We have presented state space averaging model and control approach for switch-mode buck DC–DC converters by formulating a constrained optimal predictive control problem. The proposed predictive controller uses orthonormal Laguerre functions to capture control signal which reduces computation largely in real time. Also, exponential data weighing is used to reduce numerical issue, particularly with large prediction horizon. A quadratic objective function equivalent the one used in discrete linear quadratic regulator (DLQR) has been used. In stability analysis LMPC has been compared to the optimal DLQR system. The LMPC gives extra advantage as compared to the more commonly used MPC(SS-MPC) that it can handle the buck DC-DC converter control where rapid sampling and more complicated process dynamics are required. Simulation results have been given to show that the proposed controller guided to a closed-loop system. Finally, it has been shown that the output voltage has been tracked to the reference.

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