Fault Diagnosis of Uncertain Systems Based on Interval Fuzzy PETRI Net

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Abstract: The purpose of the following article is a novel approach to modeling, diagnosis and control of discrete event systems. This approach uses the Petri net Fuzzy Interval (IFPN) to describe the diagnosis system. The Petri network is used to model the system which needs to be controlled. This work is a description of a case study applied to an ingredient mixing system where the concentration of ingredients has to be held within a valid range.

Keywords: Uncertain systems, Intervals Fuzzy Petri Nets, Modelling, Fault Diagnosis.

1. Introduction

Fault diagnosis in dynamic systems has been the subject of several research works. It is defined as a process of detecting any deviation or intended behaviour of a system and isolating the cause of this failure. There are many approach diagnosis and technical processes [1, 5, 10, 16, 17, 18, 23, 24, 29] that have been applied to fault diagnosis of dynamic systems. Their goals are focused on a timely diagnosis of defects in order to provide a precise judgment rule for distribution operators. Particularly when serious defects occur in a system, a lot of alarm information is transmitted to the control system. In such cases operators must quickly and accurately assess the causes, location and defect system components. Diagnosing faults with the right methods can provide accurate and effective diagnostic information for ship operators and help them take adequate action in certain faulty situations to ensure safety.

In the context of diagnosis more model-based systems have been used: finite state machine [17, 8], Petri nets [9, 31, 34, 35, 38]. Despite the various researches in recent years and the proposed modeling approaches for the fault diagnosis, several unresolved problems have remained.

The performances of the diagnostic approaches depend on the means of the model being used. Obtaining and using the model to construct a diagnosis system is a complex and difficult task more particularly for the uncertain systems

because of unforeseen and uncontrollable events that characterize them.

This paper focuses on fault diagnosis of uncertain discrete-event systems and tries to use Interval Fuzzy Constrained Petri Nets (IFCPN) as a tool of modelling, identification and isolation.

The proposed tool IFCPN model is introduced in order to extend some properties of the Interval Constraint Petri Nets [13, 14] which is considered as an extension the P-temporal Petri Net [26] and a sub-class of High Level PN [21, 25].

Our main contribution is to extend the functional range of ICPN applications to fault diagnosis of uncertain systems where the validity intervals of any parameter are fuzzy and characterized by the propagation of uncertain events [4, 15, 30].

This paper is organized as follows: In the first part of this work, we studied the modelling of a mixing system: we propose an approach using Statistical Process Control in order to build the validity ranges of fuzzy intervals and we compute robust control laws of this model.

In the second part we study the robustness of a system which is defined as its ability to maintain the concentration properties specified on the occurrence of disruptions. After having proved this robustness we use the tool built for the diagnosis of the defaults. The diagnosis method is based on fuzzy logic [37].

2. Interval Constrained Petri Nets (ICPN)

ICPN model allows modelling and guaranteeing a constraint on any parameter of dynamic systems.

Definition [14]:

An Interval Constraint PN is a t-uple:
$$< R, m, I_S, D, V, V_0, X, X_0 >$$

where:

- R is a marked Petri Net model

- m is the application: $m : P \to \mu\Theta$ that associates a token to places:

where Θ is a set of formulas (non-empty) and $\mu\Theta$ is a multiset defined on V.

- $I_S : P \to R \cup \{-\infty, +\infty\} \times R \cup \left(Q^+ \cup \{-\infty, +\infty\}\right)$ defines the static intervals associated to the places of PN.

- D defines an application:
$$D : m(p) \times P \to Va$$

that associates a rational $q \leq b_i$ to pair (place, token). It results from a variation of the value of a token which associated in a place.

- X defines an application: $X : Va \to Q$ that assigns to each variable a value: $va \to u \in Q$;

- X_0 defines the initial values.

- V associates a formula to each token (K) of rational values.

- V_0 is the initial formulas which is associated to the token.

A token in place p_i is taken into account in transition validations if and only if it has reached a value comprised between the limits of the static interval $[a_i, b_i]$.

After the firing of an upstream transition, tokens are generated in output places and their values will be equal to:

$$V(k) + q \tag{1}$$

We did not specify the meaning of parameter q and V(k) to generalize the model.

For more details see [14]

3. Interval Fuzzy Petri Nets

3.1 Definition:

The concept of an Interval Fuzzy Petri Nets (IFPN) [20, 36] is derived from the ICPN model where we define IS as a fuzzy interval associated with places [36]. This model is used to represent a fuzzy rule-based system that is capable of modeling the fuzzy production rules (type if p_j and p_k with certainty Factor μ_i) [3, 7, 27].

The theory of fuzzy interval has found applications for modeling and controlling uncertain presentation systems [2, 7, 19, 27] in the settings. Knowing that a conventional interval includes all possible values, in a fuzzy range we associate a degree of uncertainty for each of the possible values.

In this article, we introduce a method for calculating the limits of a given range. First we present in the following lines the basic concepts of conventional and fuzzy intervals.

3.2 Conventional interval

A number of the fuzzy interval **U** is defined as the set of real numbers **x** such that:

$$I_F = \left[i^-, i^+\right] = \left\{x \in R / i^- \leq x \leq i^+\right\}$$

Let's consider two closed intervals of reel numbers:

$$I_F = \left[i^-, i^+\right] \text{ and } L_F = \left[l^-, l^+\right]$$

The arithmetic operations in the interval are defined as follows [30, 32]:

- *Addition*: $I_F + L_F = \left[i^- + l^-, i^+ + l^+\right]$

- *Subtraction*:
$$I_F - L_F = \left[i^- - l^+, i^+ - l^-\right]$$

- *Multiplication*:
$$I_F \times L_F = \left[Min(I_F), Max(I_F)\right], \text{ where}$$
$$I = \left[i^- i^-, i^- i^+, i^+ i^-, i^+ i^+\right]$$

- *Division*: $\dfrac{1}{I_F} = \left[\dfrac{1}{i^+}, \dfrac{1}{i^-}\right]$ if $0 \notin I_F$

- $$\frac{I_F}{L_F} = \left[\begin{matrix} \min\left(\dfrac{i^-}{l^-}, \dfrac{i^-}{l^+}, \dfrac{i^+}{l^-}, \dfrac{i^+}{l^+}\right), \\ \max\left(\dfrac{i^-}{l^-}, \dfrac{i^-}{l^+}, \dfrac{i^+}{l^-}, \dfrac{i^+}{l^+}\right) \end{matrix} \right]$$

3.3 Fuzzy interval

A fuzzy interval I_F is defined as a conventional interval to which is associated a membership function denoted μ_I :

$$\mu_i = \begin{cases} R \rightarrow [0,1] \\ x \rightarrow \mu_i(x) \end{cases}$$

In a fuzzy interval each value has a degree of belonging. The higher it is the more plausible the value.

If the membership function is a trapezoidal or triangular function, this interval can be expressed by the following relations:

- Case 1: trapezoidal function:

$$I_F = \begin{bmatrix} i^-, i^+ \end{bmatrix} : Core(I_F) = \begin{bmatrix} (i^{\mathrm{mod}})^-, (i^{\mathrm{mod}})^+ \end{bmatrix}$$

- Case 2: triangular function:

$$I_F = \begin{bmatrix} i^-, i^+ \end{bmatrix} : Core(I_F) = \begin{bmatrix} I_F^{\mathrm{mod}} \end{bmatrix}$$

The Core represents the plausible values.

Fuzzy concepts [36] are based on more membership functions (trapezoidal, triangular, Gaussian …) and the rules if …then.

The trapezoidal and triangular membership functions are most frequently used. These two functions make it possible to transform the numerical values into fuzzy spaces.

In this work, we use the triangular function defined by l_m and I_h, respectively, the lower and the upper limit.

We also use the trapezoidal function with variable intervals depending on the slopes which vary between 0% and 100% (high function value).

In this analysis the probabilities are included in the representation of the numbers, so that the fuzzy intervals may be first described with a base and then with a larger interval. Figure 1 shows the three fuzzy intervals A, B and C of the level h with different bases and tops.
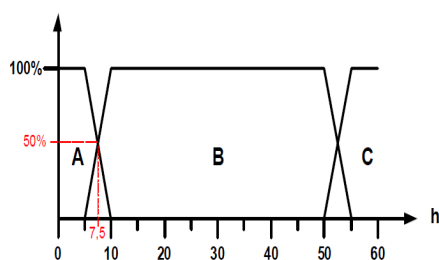

Figure 1: Example of Fuzzy intervals

h1 = [(0, 10): (0, 5)]
h2 = [(5, 55): (10, 50)]
h3= [(50, 60): (55, 60)]

Moreover, in the fuzzy theory, the rules must be expressed in terms of fuzzy variables and is to be placed in fuzzy space.

In this work we combine two methods: we use mathematical equations to construct the limits of intervals and secondly we use the rules to determine the faults. To do this, one must define the qualitative arithmetic which allows mathematical operations on qualitative numbers. The basic operations required are addition and subtraction.

Consider the operations of addition and subtraction of two fuzzy intervals h1 and h2 of figure. 2. They are performed as the operations on the conventional intervals.
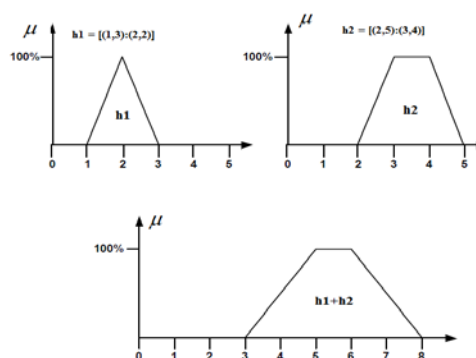

Figure 2: Fuzzy interval operations addition

Fuzzy IF - THEN Rules

The concept of fuzzy IF-THEN [15, 36] is frequently used for describing logical dependence between variables of the following form:

IF V_1 is K_1 AND V_n is K_n THEN U is M.

We define with K_1, ..., K_n and M as the predicates which characterizes the variables V_1, ..., V_n and U. These variables have often specified as typically linguistically. Our work is based on these specific linguistic expressions. The variables V_1, ..., V_n are often called independent inputs or variables. M is the output variable (dependent variable).

Example of the linguistic description:

 Rule1 = IF V₁ is X₁₁ AND ...AND Vₙ is X₁ₙ
THEN U is M₁...
Rule m= IF V₁ is Xₘ₁ AND ...AND Vₙ is Xₘₙ
THEN U is Mₘ

The relation between fuzzy PN and the linguistic description "IF THEN rules" is obvious. Figure 3 shows this correspondence between fuzzy PN and "IF THEN" rule of the above previous form:
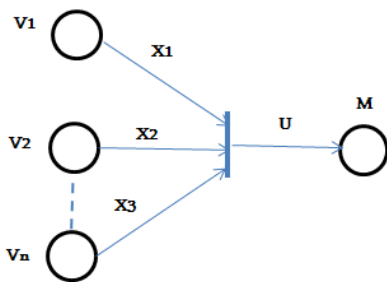

Figure 3. Correspondence between linguistic description and one rule "IF-THEN".

4. Workshop modeling

Modelling with IFPN will be illustrated on example of mixing system (Figure 4). We simplify it as much as possible to show described modeling technique. Two ingredients Q1 and Q2 are filled in the mixer tank and then mixed with water. The quantity of each ingredient as well as that of the water is adjusted by a modulating valve. The total quantity must not exceed 95% of the total capacity of the tank. After the mixing operation the product will be filled in bottles to be marketed. The quality of the product depends on the concentration limits of mixture's ingredients that we want to keep constant.

There are 5 steps in the mixing process:

step1: Add Ingredient Q1

step2: Add Ingredient Q2

step3: Add Water
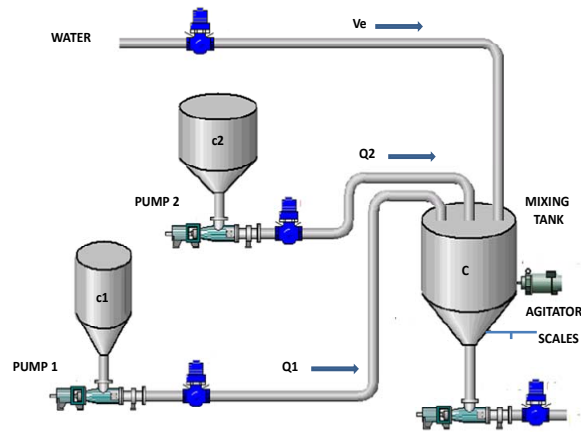
step4: Mix the products

step5: Product filling

Figure 4. Mixing system

Considering the following relationship:

$$C = \frac{m_1 + m_2}{Ve} \qquad (2)$$

Where:

m_1 is the masse of product 1 in kg

m_2 is the masse of product 2 in kg

Ve: volume of water in mm³

Such that:

$m_1 = C_1 V_1$ and $m_2 = C_2 V_2$ where Ci an V_i are respectively the concentration (in Kg/mm³) and the volume (in mm³) of the product Qi (i= 1, 2).

The relation (2) can be written as following:

$$C = \frac{C_1 h_1 S + C_2 h_2 S}{Ve} \qquad (3)$$

C is the concentration of the final product in Kg/mm³. There is a validity range of this parameter:
C $\in$ [**C_min, C_max**].

An out of range value of the concentration **C,** the product will be rejected.

The variation of C depends on those of C1 and C2. Our aim is to control the specifications required for concentration C and to maintain these values within a validity interval:

C₁ is the concentration of the final product in Kg/mm³. And there is a validity range of this parameter:
C₁ $\in$ [**C_{1min}, C_{1max}**].

C₂ is the concentration of final product in K/mm³. There is a validity range of this parameter:
C₂ $\in$ [**C_{2min}, C_{2max}**].

Ve is the volume of water in mm^3:
Ve $\in$ [**Ve$_{min}$, Ve$_{max}$**]

S: is a section of tank in mm^2

$$C_{min} \leq \frac{C_1 h_1 S + C_2 h_2 S}{Ve} \leq C_{max} \qquad (4)$$

4.1 Possibility of simulation:

The construction of the model requires a prior calculation of the tolerances on each parameter. For this purpose it is possible to use the simulation on real values from statistical data on production [11, 22, 28]. The following paragraph describes the steps of this method of calculation and the assumptions taken into consideration

4.2 Linearization approximation

We suppose that the variation of the parameters around the mean is very small. Then we can approximate the relationship (eq. 3) by the first order linearization in the neighborhoods of the reference setting:

$\Delta C \in [\Delta C_{min}, \Delta C_{max}]$

Either C_0, C_{10}, h_{10}, C_{20}, V_{20} and V_{e0}, are respectively the values targets of parameters C, C_1, h_1, C_2, h_2 and V_e. After linearization of equation (3) around the operating point we can write:

$$C \approx C_0 + \frac{\delta C}{\delta c_1}(c_1 - c_{10}) + \frac{\delta C}{\delta h_1}(h_1 - h_{10}) + \frac{\delta C}{\delta c_2}(c_2 - c_{20}) +$$

$$\frac{\delta C}{\delta h_2}(h_2 - h_{20}) + \frac{\delta C}{\delta Ve}(Ve - Ve_0)$$

$$\frac{1}{2}\frac{\delta^2 C}{\delta c^2}(c_1 - c_0)^2 + \frac{1}{2}\frac{\delta^2 C}{\delta h_1^2}(h_1 - h_0)^2 +$$

$$\frac{1}{2}\frac{\delta^2 C}{\delta c_2^2}(c_2 - c_{20})^2 + \frac{1}{2}\frac{\delta^2 C}{\delta h_2^2}(h_2 - h_{20})^2 +$$

$$\frac{1}{2}\frac{\delta^2 C}{V_e^2}(V_e - V_{e0})^2 \qquad (5)$$

This relation can be written down as:

$$C \approx C_0 + \sum_{i=1}^{5}[b_i X_i + b_{ii} X_i^2] \qquad (6)$$

with:

$$X_1 = \frac{(c_1 - c_0)}{\sigma_1} \; ; \; X_2 = \frac{(h_1 - h_0)}{\sigma_2} ; \; X_3 = \frac{(c_2 - c_{20})}{\sigma_3}$$

$$X_4 = \frac{(h_2 - h_{20})}{\sigma_4} ; \; X_5 = \frac{(V_e - V_{e0})}{\sigma_5} ;$$

and $b_i = c_i \sigma_i$; $b_{ii} = c_{ii} \sigma_i^2$

where:
$$b_1 = \frac{\delta C}{\delta c_1} ; \quad b_2 = \frac{\delta C}{\delta h_1} ; \qquad b_3 = \frac{\delta C}{\delta c_2} ;$$

$$b_4 = \frac{\delta C}{\delta h_2} ; \quad b_5 = \frac{\delta V_e}{\delta V_e} ; \quad b_{11} = \frac{\delta^2 C}{\delta c_1^2} ;$$

$$b_{22} = \frac{\delta^2 C}{\delta h_1^2} ; \quad b_{33} = \frac{\delta^2 C}{\delta c_2^2} \quad b_{44} = \frac{\delta^2 C}{\delta h_2^2} \text{ and}$$

$$b_{55} = \frac{\delta^2 C}{\delta V_e^2}$$

where σ_i is the standard deviation of the considered parameter.

If we assume that the variations of the parameters follow the normal distribution the first approximation can be writing [11, 30]:

$$\sigma_C^2 = \sum_{i=1}^{5} b_i^2 = \sum_{i=1}^{5}\left(\frac{\delta C}{\delta x_i}\right)\sigma_i^2 \qquad (7)$$

The application of the relation (7) gives us:

$$\sigma_C^2 = \frac{S^2 h_1^2}{Ve^2}\sigma_{c1}^2 + \frac{S^2 C_1^2}{Ve^2}\sigma_{h1}^2$$
$$+ \frac{S^2 h_2^2}{Ve^2}\sigma_{c2}^2 + \frac{S^2 C_2^2}{Ve^2}\sigma_{h2}^2 + \frac{S^2}{Ve^4}(C_1 h_1 + C_2 h_2)\sigma_{Ve}^2 \qquad (8)$$

And after simplification of this relation (8) we obtain:

$$\sigma_{Ve} = \sqrt{\frac{Ve^4}{S^2(C_1 h_1 + C_2 h_2)}\sigma_C^2 - \frac{h_1^2 Ve^2}{(C_1 h_1 + C_2 h_2)}\sigma_{c1}^2 - \frac{C_1^2 Ve^2}{(C_1 h_1 + C_2 h_2)}\sigma_{h1}^2 - \frac{h_2^2 Ve^2}{(C_1 h_1 + C_2 h_2)}\sigma_{c2}^2 - \frac{C_2^2 Ve^2}{(C_1 h_1 + C_2 h_2)}\sigma_{h2}^2} \qquad (9)$$

4.3 Application: design of experiments

In the previous chapters we defined reasons for a concentration values:

- Factor C_1: the concentration of ingredient Q1

- Factor C_2: the concentration of ingredient Q2

- Factor V_e: the quantity of water

For a traditional experimental design, we research the response system facing varying input values. In this case we adjust these factors to give the corresponding response of system.

In our case we consider uncertain parameters values:

- the concentration values C1 and C2 depend on several parameters as certain treatments and there are uncertain and uncontrollable.

- Levels h1 and h2 are adjustable while acting on the quantity or weights of products Q1 and Q2.

- the volume of water Ve depends on the values of C1 and C2.

To resolve this problem we will use in this paper the statistical real data of the production. Afterwards we are going to calculate tolerances for these parameters. For a more details see the results of this study.

Statistical data of the process gives us the standard deviation of concentrations C, C1, C2: and quantity of water Ve. These are respectively:

$$\sigma_c = 0.2, \ \sigma_{c1} = 0.6, \ \sigma_{c2} = 0.6; \ \sigma_{Ve} = 0.5$$

Let consider that the value targets for a good concentration are following:

$m_1 = 200$ kg with a concentration $C_{10} = 0.25$

$m_2 = 80$ kg with a concentration $C_{20} = 0.07$

Target volume of water: $Ve_0 = 650$ L

Capacity of a mixing tank = 1 m^3

Section of mixing tank S = 1m^2

Under these conditions the standard deviation of the concentration can be determined while applying the following equation: $\sigma_C = 0.427$

The tolerance in the volume of water is:

$IT_{Ve} = 0.26$

The tolerances of the parameters are given by the following table.

Table I: Limit values of parameters

Parameter	Unit	Value
c_{1min}	g/mm^3	0.22
c_{1max}	g/mm^3	2.27
c_{2min}	g/mm^3	0.06
c_{2max}	g/mm^3	0.08
h_{min}	M	0.2
h_{max}	m	0.2
V_{emin}	l	800
V_{emax}	l	950
C_{min}	g/mm^3	0.5
C_{max}	g/mm^3	0.7
Target value of C1	g/mm^3	0.25
Target value of C2	g/mm^3	0.07
Target value of C	g/mm^3	0.6

The figure 5 represents the network ICPN model of mixing system (figure 4). In this model place P_{01} and P_{02} represent stocks to the entries of system. Places P_{c1}, P_{h1}, P_{c2} and P_{h2} are respectively the quantity of product C_1 in tank 1, the level in tank 1, the quantity of product C_2 in tank 2, the level in tank 2. The places P_e and P_c represent respectively the quantity of water and the concentration of finished mixing operation. Place Pc is a place of control which prevents the simultaneous crossing of the transitions T_1 and T_4.

Finally, after the ICPN process model is building, we can analyze its structural properties and we prove that most P-time PN structural properties can be extended to IFCPN [15].

When using information on production data, the method was applied in order to calculate the limits of the intervals (Figure 5).
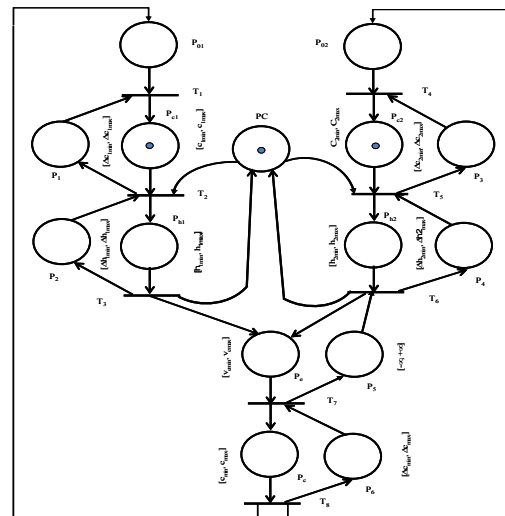

Figure 5. Validity intervals of the ICPN model

5.   Computing a robust command

The process model is defined by valid fuzzy intervals. Under some particular assumptions, we can usually extend the structural properties of ICPN to analyse the structural properties Interval Fuzzy PN model. Our goal is to provide the internal and external robustness system [6].

The robustness of a dynamic system is defined as its capacity to maintain the properties specified in the face any deviations. The robustness characterizes the ability of system with input fluctuations. For each system two types of disturbances are defined [11, 12, 33] external and internal: the internal disturbance is due to the change of the states of the organs of the machine. The external derivate is due to input fluctuation.

So we define two types of robustness [11, 26]:

A passive and an active robustness. The first one is insured when reaching the objectives without adjustment. On the other hand the second type is insured when it can not reach objectives without adjustment and we need to calculate a new command.

The figure 6 illustrates the notion of robustness and their margins. The determination of which is crucial for the calculation of a new command in the event of violation of the constraints.

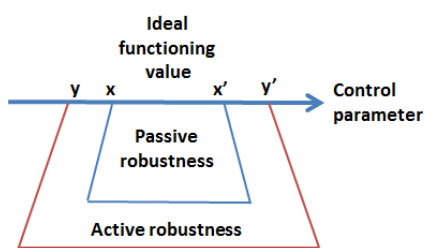Exceeding these values involves a violation of the constraints.

Figure 6. Margins of robustness

This study concerns the problem of the passive robustness of the mixing system in the case of derivatives of the values of the input products C1 and C2. In this case the calculated control system must be able to compensate for these deviations and specifications on the concentration C of the final product must not be violated.

Indeed, the mixing systems are subject to disruptions. Their causes vary: changes in

operating times, variations in the quality of raw materials, state of the process...

In this work we propose to build a robust control synthesis tool. The tool uses static intervals constraint on the components. Thus, this control is ensured through a few real-time adjustment operations. The calculation of these settings is made as a function of the static intervals assigned to the parameters and using the developed graph (Figure 7) relating to the Fuzzy Petri Net model.

In this developed graph the nodes correspond to the transitions of the Petri net of the figure 5.

For each place p of PN model we associate two arcs. The value of first arc from place to transition is obtained by:

$$X_p = a - C.m$$

The value of second one from transition to place is calculated by:

$$Y_p = -b + C.m$$

Where C represents a periodic control of the parameters that is obtained with implementation of an algorithm [26].

In our case we for the IFPN of the Figure 5 we suppose that C = 1 then we obtain the developed graph (figure 7).
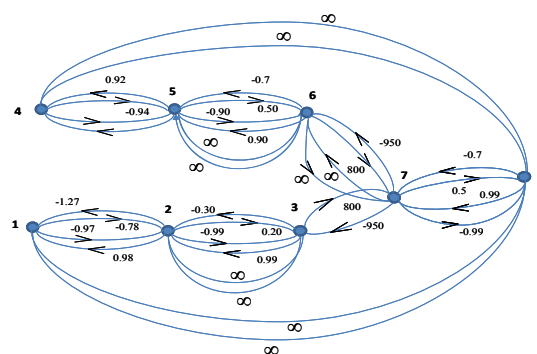

Figure 7. Developed Graph

6. Simulation and results

The monitoring of the system during a period of one month allowed us to operate the variations of the output measures: concentration C1, concentration C2 and the final product C.

The Fuzzy PN model has been simulated with the values of Table 1. Our aim was to preserve the specifications on these parameters. The results obtained (figure 8) show the robustness

of our model. Indeed the variation of the measured concentration values do not exceed must respect the constraint specification. In these conditions the process is robust.

The following production sequence takes place: first phase is preparation of the mixing product.

Firstly, the valve V1 is open, an ingredient Q1 flows into tank. When the level h1 in the tank is
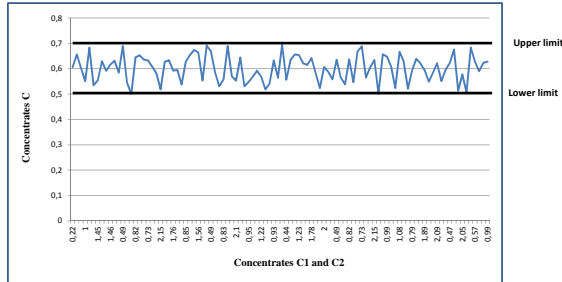

Figure 8. Variation of concentration value

reached ($h_{1\min} \leq h_{10} \leq h_{1\max}$), the valve V1 is closed and the valve V2 is opens, ingredient Q2 flows into the tank. When the h2 in the tank is reached ($h_{2\min} \leq h_{20} \leq h_{2\max}$) and valve V2 closes.

After the activation of sensors h1 and h2, valve V3 is open, water Qe flows into the mixing tank. When the level h3 in the tank is reached ($h_{3\min} \leq h_{30} \leq h_{3\max}$), valve V3 is closed and the agitator will run for 3 mn until the batching cycle is complete.

Control sequence:

(1) S0: initialization phase (the mixing tank is empty).
(2) S1: V1 is open, the Fill with Q1.
(3) S2: When the level h1 is reached then V1 is closed and V2 is opened.
(4) S3: If Level h2 is reached then V2 is closed.

During every phase of the preparation of the product, the ingredient level of must be controlled. If the total level rises to greater than **95%** of tank's mixing capacity, the command will generate an alarm and the process must be stopped.

Considered faults:

We considered three faults _f = {f1, f2, f3}, where:

- f1: valve V1 stuck close. This means that the tank is empty. Controller waits for event h1 which cannot occur because of the fault.

- f2: valve V2 stuck close. Controller waits for event h2 which cannot occur because of the fault.

- f3: valve V3 being stuck open. This fault causes an overflow.

- f3: The third considered fault is sensor h3 which stays in close position. It means there is an overrun of 95% of the tank capacity but the sensor is faulty.

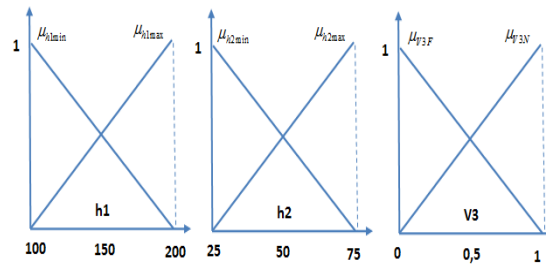Figure 9 shows the three membership functions associated to h1, h2 and V3.

Figure 9. Membership functions

$$\begin{cases} \mu_{h1\min} = \dfrac{-1}{100} h1 + 2 \\ \mu_{h1\max} = \dfrac{1}{100} h1 - 1 \end{cases},$$

$$\begin{cases} \mu_{h2\min} = \dfrac{-1}{50} h2 + \dfrac{3}{2} \\ \mu_{h2\max} = \dfrac{1}{50} h2 - \dfrac{1}{2} \end{cases}; \begin{cases} \mu_{V3F} = -V3 + 1 \\ \mu_{V3N} = V3 \end{cases}$$

Fuzzy Rules:

R1: if V1 is max and V2 is min then V3 is ok

R2: if V1 is max and V2 is maw then V3 is fault.

In these conditions we can write:

$$\mu(V3) = \max \begin{cases} \min\left(\mu_{\max V1}, \mu_{\min V2}, \mu_{V3F}\right), \\ \min\left(\mu_{\max V1}, \mu_{\max V2}, \mu_{V3N}\right) \end{cases}$$

Suppose that at a given instant the quantities measured are the following V1 = 170 and V2 = 40. Under these conditions we shall have (Figure 10) $\mu_{V1\max} = 0.8, \mu_{V2\min} = 0.4, \mu_{V2\max} = 0.8$, from where:

$$\mu(V3) = \max\left\{\min\left(0.8, \mu_{V3F}\right), \min\left(0.4, \mu_{V3N}\right)\right\}$$

The defuzzification by the method of the first maximum gives V3 = 0, we conclude that the valve V2 is in default.
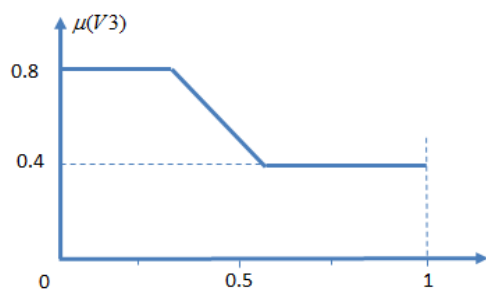

Figure 10. Membership function of the fuzzy sub-set

7. Conclusion

In this paper we present an approach dealing with automatic control specification of an industrial process. We propose a control loop which is able to maintain a constant concentration value of product despite the fact that the concentration values of C1 and C2 are not deterministic.

The applied methodology is based upon the use of an IFCPN model. The design experiments were used in order to compute the validity intervals of the critical parameters. The manual control generated a great variety of different settings but they were not all taken into account.

The IFCPN model with its completely defined intervals has been built. Our approach is validated through an industrial application and for this validation we have chosen the simplest case. However, the proposed approach can be extended to the diagnosis analysis of the multiple causes and common cause failures.

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