Solving a Distribution Network Design Problem by Combining Ant Colony Systems and Lagrangian Relaxation

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Abstract: Distribution network design (DND) attempts to integrate tactical issues such as inventory policies and/or vehicle routing decisions with strategic ones such as the problem of locating facilities and allocate customers to such facilities. When inventory policy decision making is considered the problem is also known as inventory location modelling (ILM) problem. During the last two decades, mathematical programming as well as (meta-)heuristic approaches have been considered to address different DND problem. In this article we consider a hybrid algorithm of Lagrangian Relaxation and artificial ants to solve an ILM problem previously proposed in the literature. We use ACS to allocate customers to a subset of warehouses that is previously generated by the Lagrangian relaxation. Results show that the hybrid approach is quite competitive, obtaining near-optimal solutions within an acceptable time.

Keywords: Distribution Network Design, Matheuristics, Ant Colony Optimization, Lagrangian Relaxation.

1. Introduction

The location-allocation problem is an important problem in logistics. The main goal is to find the best possible combination of facility locations (among a set of possible locations) to serve a set of customers, which must be allocated so that their demand may be fully satisfied by the installed facilities. This problem is, clearly, at strategic level as once the decision on what locations should be installed is made, it is very difficult to reverse such a decision because of the time and cost that such changes would imply. These class of problems have been widely studied in the literature ([1, 3, 8, 9, 10, 14, 34), and a variety of techniques have been proposed so far to solve it (for a comprehensive survey please see [21]). During the last two decades, several authors have pointed out the importance of including tactical aspects such as inventory and transportation decisions within the same model that is used to solve the strategic problems [24, 30, 31, 32, 15, 35, 36, 39]. This is because sequential approaches, i.e. those that solve sequentially the strategic, tactical and operational problems, have been proved to lead to sub-optimal solutions, as the quality of the solution at each level will always depends on the quality of the solutions obtained in previous levels.

When inventory policies are included within the location-allocation model, the resulting model is known as Inventory Location Model (ILM).

Several ILM have been proposed in the literature, being the main difference between them the inventory policy that is considered (see Section 2 for a short survey on ILM). In this article, we have considered the ILM proposed by [32]. In their article, authors propose a cross-level model that incorporates a continuous review inventory policy. In their proposed model storage space at each facility is limited and a capacity constraint over the distribution vehicles is applied. These conditions make the problem more realistic as they model common situations in logistics. It also makes this problem harder to solve though.

The resulting model is nonlinear and includes both integer and continuous decision variables. The model considers inventory control as well as facility location decision making. Two capacity constraints are considered: a limit over the size of the lot for the incoming orders to each facility and; a stochastic bound to the inventory capacity. In their article, authors solution approach consists of a Lagrangian relaxation and the well-known sub-gradient method, combined with a simple *2k*-opt heuristic. Miranda et. al. [32] highlights the fact that reducing capacity of inventory does not necessarily means that the number of facilities to be installed must increase. In fact, order size reduction leads to an optimal customer allocation, reducing total system cost.

The mathematical model presented in [32] we consider in this study is as follows. Let W be the set of possible warehouse locations and $i \in W$ be the i-th available warehouse in W. Let C be the set of customers to be allocated to selected warehouses $i \in W^+ \subset W$. As we mentioned before, a (Q_i, RP_i) inventory policy is considered. When the inventory level falls below RP_i level in warehouse i, Q_i items are ordered. LT_i time units after the order Q_i is triggered the items are received. This is known as *lead time*. A probability of $1-\alpha$ is considered to ensure a *level of service* for the system. This means that, with probability of $(1-\alpha)$, units below RP_i are enough to satisfy customers demand during the lead time. Stock-out is produced otherwise. This leads to the following constraint:

$$Prob(SD(LT_i) \leq RP_i) = 1-\alpha \tag{1}$$

where stochastic demand during lead time at warehouse i is denoted by $(SD(LT_i))$. Miranda et. al. [32] assumes a normally distributed demand which allows them to determine RP_i as

$$RP_i = D_i LT_i + Z_{(1-\alpha)} \sqrt{LT_i} \sqrt{V_i} \tag{2}$$

$Z_{1-\alpha}$ corresponds to standard normal distribution, D_i and V_i are the total demand and the total variance at warehouse i, respectively. Let HC_i be the holding cost and OC_i be the order cost associated to warehouse i. Thus the inventory cost is

$$HC_i Z_{1-\alpha} \sqrt{LT_i} \sqrt{V_i} + HC_i (\frac{Q_i}{2}) + OC_i (\frac{D_i}{Q_i}) \tag{3}$$

F_i denotes the fixed cost of opening warehouse i while RC_{ij} and TC_{ij} are the transportation unit cost and the fixed transportation cost between warehouse i and customer j, respectively. Thus, the goal is to minimise the total system cost that can be expressed as follows

$$\begin{aligned} \sum_{i=1}^{N} F_i X_i + \sum_{i=1}^{N} \sum_{j=1}^{M} (RC_i \mu_j + TC_{ij}) Y_{ij} \\ + HC_i Z_{1-\alpha} \sqrt{LT_i} \sqrt{V_i} \\ + HC_i \left(\frac{Q_i}{2}\right) + OC_i \left(\frac{D_i}{Q_i}\right) \end{aligned} \tag{4}$$

where μ_f is the customer j mean demand, N and M are the number of available warehouses and the number of customers, respectively.

Miranda et. al. [32] proposes two constraints associated to order quantity Q_i. The first one states the Q_i must be less or equal that an arbitrary value Q_{ub}. The second one fixes the maximum inventory level for each warehouse as follows

$$Prob\left(RP_i - SD_i(LT_i) + Q_i \leq I^{cap}\right) = 1-\beta \tag{5}$$

which can be restated as

$$Q_i + \left(Z_{(1-\alpha)} + Z_{1-\beta}\right) \sqrt{LT_i} \sqrt{V_i} \leq I^{cap} X_i \tag{6}$$

Additional constraints are:

$$\sum_{i=1}^{N} Y_{ij} = 1, \qquad \forall\ j=1,\dots,M \tag{7}$$

$$Y_{ij} \leq X_i, \quad \forall\ i=1,\dots,N; \forall\ j=1,\dots,M \tag{8}$$

$$0 \leq Q_i \leq Q_{ub}, \quad \forall\ i=1,\dots,N; \forall\ j=1,\dots,M \tag{9}$$

$$\sum_{j=1}^{M} \mu_j \cdot Y_{ij} = D_i, \qquad \forall\ i=1,\dots,N \tag{10}$$

$$\sum_{j=1}^{M} \sigma_j^2 Y_{ij} = V_i, \qquad \forall\ i=1,\dots,N \tag{11}$$

$$X_i, Y_{\{ij\}} \in \{0,1\}, \quad \forall\ i=1,\dots,N, \forall\ j=1,\dots,M \tag{12}$$

Equation (7) states that demand of customer i is fully satisfied by the system. Equation (8) ensures that customers are allocated only to installed warehouses. Equation (9) provides a valid range for Q_i. Equations (10) and (11) set the total demand D_i and total variance V_i for warehouse i based on the allocation variable Y_{ij}. Finally Equation (12) states that decision variables X and Y are binary variables, which make this problem much harder to solve.

2. Inventory Location Model: Literature Review

Many authors have been focused on the DND problem and on ILM in particular during the last two decades. For instance [37, 33, 12, 29, 30] all analyse decision making levels in both distribution network design and supply chain management (SCM). A detailed FLP reviews and analysis is presented in [16, 38 and 21]. However, commonly used FLP models neither consider interactions between facility location and inventory policy nor the impact of the latter on distribution network design. An example of such interaction is the effect on the final distribution network design produced by the so called *risk pooling effect*. It states that the safety stock required by the whole distribution system will decrease as fewer warehouses are installed. Daskin et. al. [17] and Shen et. al. [36] consider a continuous inventory policy, namely (Q, RP), and the well known uncapacitated facility location problem (UFLP). A safety stock at each site is considered. While authors in [17] employ Lagrangian Relaxation to solve the model, Shen et. al. [36] reformulates the model as a Set-Covering problem and solves it using a column generation method.

Continuous inventory policies have been widely used in DND models. Authors in [30] consider order quantity Q as a decision variable and, additionally, they add a capacity constraint to their model. A similar capacity constraint is considered in [31] and [35].

More recently, in [27] authors have presented an ILM that incorporates the risk pooling effect for both safety stock and running inventory. Additionally, in their model, the authors consider the effect produced when warehouses and end customers work jointly.

Tancrez et. al. [39] has presented a three-level supply chain non-linear ILM that integrates location, allocation, and shipment sizes. Firoozi et. al. [22] develops a model for the DND problem that considers the short lifetime of perishable products. To solve their model, the authors implemented a Lagrangian relaxation. One paper that has addressed the DND problem using periodic inventory review is presented in [11]. In their paper, the authors consider a *(R, S)* inventory policy. A similar approach is presented in [15], where authors considered a *(R, s, S)* inventory policy.

Different techniques to solve these models have been considered so far in the literature. For instance, Bard et. al. [7] proposed a branch and price algorithm for an integrated production and inventory routing problem. In Badri et. al. [5], Lagrangian relaxation was used to solve a mixed integer linear programming model for multiple echelon and multiple commodity supply chain network design.

Heuristics have also been used to solve DND problems. For instance, Aremtano et. al. [2] solves a model that integrates production and distribution decisions by considering the capacity constraints of the plant. They solve such a model by means of the well known TS algorithm. Askin et. al. [4] implements an evolutionary algorithm to solve an ILM considering multi-commodity and distribution planning decisions. In their paper, the authors present a very comprehensive description of their genetic algorithm. In [15] authors solve the resulting non-linear non-convex problem by mean of the well known Tabu Search heuristic. To the best of our knowledge no hybrid algorithm such as the one presented here has been applied to the ILM that is tackled in this article.

3. Solution Approaches

In this section both ACO and Lagrangian relaxation techniques are studied.

3.1 Ant Colony Optimization.

Ant colony optimization (ACO) algorithms were firstly proposed by [18]. They are natural inspired approaches that try to mimic the behaviour of natural ants. In general we can classify ACO algorithms into two different groups:

Ant Systems (AS). This metaheuristic is inspired in the behaviour of real ants which use pheromones as a communication medium. It has been shown to be very effective in solving complex combinatorial optimisation problems. Indirect communication between simple agents (ants) is the cornerstone of AS. This communication is implemented by means of pheromone trails. In AS, these pheromone trails serve as distributed, numerical information that is used by the (artificial) ants to probabilistically construct solutions to the problem that is being solved [13]. AS has been shown to be very effective in solving many combinatorial optimization problems. It performs particularly well in routing problems such as travelling salesman problem and VRP ([13] and [20] respectively). AS extracts three main ideas from natural ant behaviour [19]:

1. Ants prefer paths with higher pheromone levels,
2. Higher rate of growth of the amount of pheromone deposited on shorter paths, and

3. Trail mediates communication among ants.

The functioning of an AS algorithm can be described as follows [28]. A colony of artificial agents called *ants* moves from one state of the problem to the next one building a solution to the corresponding problem. A particular ant moves independently from the others. Such a movement is done by applying a stochastic rule based on the so called trails and attractiveness. A state transition rule guides ants movements. Ants prefer to move to states which are cheaper in terms of their associated cost. Such states have a high amount of pheromone trials. By moving, each ant incrementally constructs a solution to the problem. After an ant built a solution up (and also during the construction phase), it evaluates its solution and modifies trail value on components that are used in its solution.

Trails load will guide future artificial ants. AS also considers two additional tools: trail evaporation, which diminishes trail loads for each iteration avoiding large trail loads in some specific state; and daemon actions, that can be used to implement centralized actions which cannot be performed by single ants, such as the invocation of a local optimization procedure, or the update of global information to be used to decide whether to bias the search process from a non-local perspective.

The probability $p_k(r,s)$ of ant *k* moving from state *r* to *s* will be set to 0 if such a movement is infeasible. A movement is said to be infeasible if either it leads to an infeasible solution of the problem or it is in the *tabu list* of ant *k*, $tabu_k$. Tabu list is a list containing all moves which are infeasible or banned according to some criteria. If movement is not infeasible, probability $p_k(r,s)$ is computed as follows

$$p_k(r,s)=\begin{cases}\dfrac{[\tau(r,s)][\eta(r,s)]^{\beta}}{\sum_{u\notin tabu_k}[\tau(r,u)][\eta(r,u)]^{\beta}}, & if\ s\notin tabu_k\\ 0 & Otherwise\end{cases} \quad (13)$$

In Equation (13), the relative "weight" of pheromone trails w.r.t. the cost associated to a particular movement is denoted by β . Once all artificial ants have built a solution pheromone loads are updated according to equations (14) and (15)

$$\tau(r,s)=(1-\alpha)\tau(r,s)+\sum_{k=1}^{m}\Delta\tau_k(r,s) \quad (14)$$

where,

$$\Delta\tau_k(r,s)=\begin{cases}\left(\dfrac{1}{L_k}\right), & if\ (r,s)\in current\ tour\ of\ ant\ k\\ 0 & Otherwise\end{cases} \quad (15)$$

is the cumulative contributions of all artificial ants that include within their final solution the movement (r,s). *Evaporation coefficient*, a user defined parameter, is denoted by α , with $0\leq\alpha\leq 1$, and L_k is the length of the tour performed by ant k , that is its associated cost. The number of ants is denoted by m .

How much pheromone is deposited by an ant in a particular state depends on the quality of the solutions achieved. If the solution reached by an ant is a high quality one, then the contribution will be also high. This means that the movement the ant is selecting will be more attractive for the remaining ants in next iterations of the algorithm.

Ant Colony Systems (ACS). ACS has three variants w.r.t. the AS algorithm [18]. Firstly, ACS incorporates a state transition rule that allows the algorithm to explore new solutions with certain probability while more promising solutions are preferred. In this way, knowledge accumulated during previous iterations can be used to guide other ants decision making. As a second variant, a global updating rule is applied to the best tour. Finally, ACS implements a local updating rule at each step of the solution generation.

$$s=\begin{cases}argmax_{u\notin tabu_k}\{[\tau(r,u)][\eta(r,u)]^{\beta}\}, & if\ q\leq q_0\\ S, & otherwise\end{cases} \quad (16)$$

where q is a random number uniformly distributed [0,1]. A user defined parameter q_0 with $0\leq q_0\leq 1$ is set. If q_0 approximates to 0, then algorithm will prefer exploration over exploitation. Otherwise (i.e. q_0 approximating 1), algorithm will prefer exploitation over exploitation. A random value *S* is also defined.

$$\tau(r,s)=(1-\alpha)\tau(r,s)+\alpha\Delta\tau_k(r,s) \quad (17)$$

where

$$\Delta\tau(r,s)=\begin{cases}\dfrac{1}{L_{gb}}, & if\ (r,s)\in global\ best\ tour\\ p_k(r,s), & Otherwise\end{cases} \quad (18)$$

L_{gb} is the associated cost of the best solution found by the algorithm so far.

As mentioned before, ACS implements a local updating rule that is as follow.

$$\tau(r,s)=(1-\rho)\tau(r,s)+\rho\Delta\tau(r,s) \quad (19)$$

with ρ a user defined parameter ($0 \le \rho \le 1$). The general ACS structure as in [18, 40, 41] is presented in Algorithm 1.

As we mentioned before, we use ACS in our hybrid algorithm in order to allocate the set of customers to a subset $W^{+} \subset W$ of open warehouses. In our algorithm, each ant starts positioned in a random customer. Then the ant must to allocate the current customer *j* to a warehouse *i* among all possible warehouses in W^{+}. Once the current customer has been allocated, the ant 'jump' to the next random customer and repeat the same procedure until all customers have been visited or no feasible solution can be reached from the current position. In our representation, a 'path' corresponds to the link between a customer and a warehouse. Thus, when an artificial ant allocates customer *j* to warehouse *i*, pheromone trails are deposited in such a link. If that allocation results in a good solution, ants from next iterations will be more influenced to use that link, i.e. to allocate customer *j* to warehouse *i*. The opposite happens if the quality of the resulting solution is not as high as the ones obtained by other ants.

3.2 Lagrangian relaxation

Lagrangian relaxation, or Lagrangian heuristic (LH), has been widely used to solve location-allocation problems in the literature [6, 10, 23, 25, 26}. One key element in the LH is the chosen relaxation. Several different relaxations have been proposed so far (for a survey see [38]). In this article, we use the same relaxation as proposed by [32]. Applying a relaxation over the primal problem leads to a sub-problem known as the dual problem which, when solved, usually provides lower bounds for the primal problem. LH relies on the fact that obtained sub-problem must be easier to solve than the primal one. If not the case, other relaxations should be considered. Depending on the mathematical properties of both, the primal and dual problems, solving the dual problem could lead to feasible and (in some cases) optimal solutions of the primal problem. In most of the cases, primal heuristics still needed, and the quality of the obtained solutions is measured as the gap between dual solution value and primal solution value. This gap is known as *duality gap*. A duality gap equal to zero means the optimal solution of the primal problem has been found. To solve the dual problem, different techniques have been proposed. In this article a sub-gradient procedure is used to solve the dual problem, yielding improvements of the obtained lower bounds.

Algorithm 1:
General algorithmic frame for ant colony system

Begin
 Do{
  For Each (ant $k \in K$)**{**
   Initialise solution;
  }
  Do{
   For Each (ant $k \in K$)**{**
    Apply Rule $\left(\eta(r,s), \tau(r,s)\right)$;
    LocalUpdate();
   }
   GlobalUpdate $\left(\tau(r,s)\right)$;
  }while(all ants completed their solutions**)**
 }while(stop criterion is reached**)**
End

As we mentioned above, the relaxation used in this article is the one used in [31]. In their work, authors relaxed constraints (7), (10) and (11). This means that solutions of the dual problem are not necessarily feasible as they do not ensure that total demand of every customer is fully served. Then the objective function of the dual problem considered in this study is as follows [32], subject to constraints (6), (8), (9) and (12).

$$
\begin{aligned}
& Min \sum_{i=1}^{N} \left(F_i X_i\right) \\
& + \sum_{i=1}^{N} \sum_{j=1}^{M} \left(RC_i \mu_j + TC_{ij} + \lambda_i \mu_j + \omega_i \sigma_j - \psi_j\right) Y_{ij} \\
& + \sum_{i=1}^{N} \left(HC_i Z_{1-\alpha} \sqrt{LT_i} \sqrt{V_i}\right) \\
& + \sum_{i=1}^{N} HC_i \left(\frac{Q_i}{2}\right) + OC_i \left(\frac{D_i}{Q_i}\right) \\
& - \sum_{i=1}^{N} \left(\lambda_i D_i + \omega_i V_i\right) + \sum_{j=1}^{M} \psi_j
\end{aligned} \tag{20}
$$

Once this problem has been solved by mean of the well-known sub-gradient method, the set of open facilities W^{+} is stored in memory so the ACS algorithm can solve the associated allocation sub-problem. As we mentioned before, we need to be sure that the installed capacity, i.e. the total capacity of the system considering only W^{+} facilities, is enough to satisfy the entire customer demand. If not, we need to include additional facilities into W^{+}, so the ACS method can find feasible solutions for the primal problem.

3.3 Proposed hybrid approach

In the proposed method, LH is used to find a promising subset of possible locations $W^{+} \subset W$. This leads to an associated allocation problem that is easier to solve. We solve such a sub-problem by mean of ACS. This means that artificial ants can only allocate customers to those locations $i \in W^{+}$. Algorithm 2 shows the general framework of our hybrid approach.

Algorithm 2:
Algorithmic Frame for the Hybrid ACS-LH method

Begin
Do{
(W^{+}, Z_{lb}) = solveDualProblem();
If (W^{+} is not feasible**){**
W^{+} = addFacilities($W \backslash W^{+}$);
}
(X_i, Y_{ij}, Z_{ub}) = ACS(W^{+});
(λ, ω, ψ) = update Lagrange Multipliers (Xi, Y_{ij}, Z_{ub});
}until(stop criterion is reached**)**
End

As we can see in Algorithm 2, at each iteration Lagrange multipliers are updated based on the solution obtained by the ACO algorithm, helping to speed up the convergence of our hybrid approach.

4. Computational Results

In order to test our hybrid ACO-LH algorithm, two instance classes are considered. The first class of instances corresponds to a set of 8 small instances with 15 customers and 4 possible facilities. Optimal solutions for these instances are known, so we can check if our hybrid algorithm is able to find the optimal solution for our small instances. The second class of instances consists of 98 instances generated following the same procedure as in [32]. All these instances have 40 customer and 20 possible facilities.

We first make some experiments in a subset of instances in order to establish the best values for our algorithm parameters. In particular, we try different values for *h* (number of ants in the ACO method) and q_0 (exploration / exploitation index).

We perform 10 runs for each studied value. Reported values correspond to the average values obtained in those runs. Figures 1 and 2 show the obtained results of the corresponding sensitivity analysis.

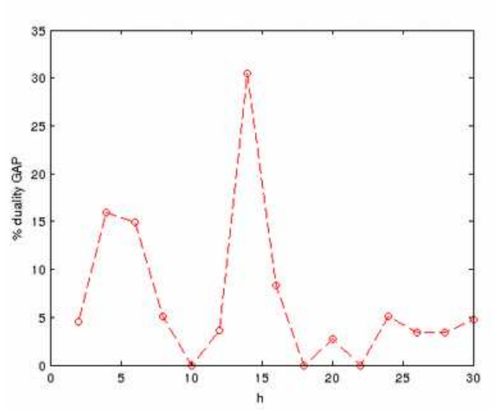

Figure 1. Sensitivity analysis for ACS parameter *h*, number of ants.

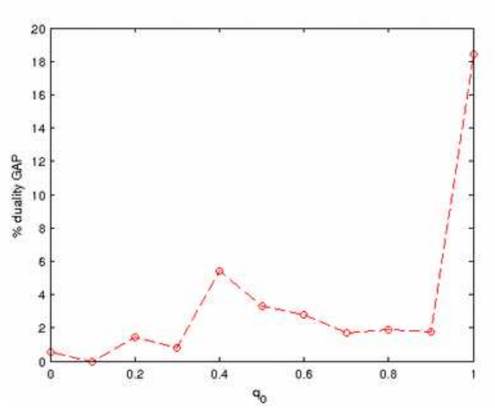

Figure 2. Sensitivity analysis for ACS parameter q_0, exploration-exploitation index.

As we can see in Figures 1 and 2 above, the value used for these two parameters can have a great impact on the performance of our algorithm. It is also interesting to note that best obtained values ($h=10$ and $q_0=0.1$) corresponds to the same values proposed for a completely different problem by [18].

After set parameter values, we test our algorithm over the set of small instances. For all instances the obtained *dualityGap* value is equal to zero, i.e. the optimal solution was found. We need to point out at this point that ACS and LH separately are also able to find the optimal solutions for all the small instances. Our hybrid algorithm needs less than 1s to find the optimal solution though.

We then apply our algorithm to the medium size instances. We compare the results with the ones reported in [32] where the same LH is used although a simple 2 k-opt heuristic is implemented instead our ACS algorithm. Table 1 shows the obtained results for the all 98 medium size instances considered in this study.

As we can see in Table 1, our hybrid ACS-LH algorithm is quite competitive in terms of the primal solution values. In almost half of the instances, our algorithm was able to find the same or even better results than its counterpart. Moreover, our algorithm exhibits a very good exploration level, which can be seen in the variety of solutions visited during the algorithm execution.

Table 1. Results for all the 98 instances. Solution values for both primal and dual problems are reported as well as its duality Gap.

Zlb LH	Zub LH	gap (%)	Zlb ACS-LH	Zub ACS-LH	gap (%)
171,174	180,040	4.92	171,201	180,135	4.96
171,122	178,604	4.19	171,146	178,208	3.96
175,092	184,029	4.86	175,026	183,938	4.85
174,941	182,278	4.03	175,064	182,449	4.05
179,103	187,882	4.67	179,193	188,135	4.75
178,948	186,131	3.86	179,118	186,603	4.01
182,929	191,735	4.59	183,197	192,134	4.65
183,026	189,984	3.66	183,131	190,603	3.92
186,485	195,587	4.65	187,075	196,134	4.62
186,414	193,836	3.83	187,018	194,602	3.9
190,229	199,440	4.62	191,108	199,931	4.41
191,050	197,689	3.36	191,070	198,547	3.77
194,494	202,942	4.16	194,961	202,942	3.93
194,459	201,542	3.51	194,919	201,558	3.29
172,679	180,692	4.43	172,737	180,692	4.4
172,534	179,359	3.8	172,664	179,000	3.54
176,696	184,616	4.29	176,625	184,692	4.37
176,528	183,070	3.57	176,682	183,172	3.54
180,513	188,469	4.22	180,294	188,692	4.45
180,658	186,923	3.35	180,636	187,358	3.59
184,511	192,321	4.06	184,640	192,691	4.18
184,402	190,775	3.34	184,645	191,358	3.51
188,391	196,174	3.97	188,482	196,174	3.92
188,194	194,628	3.31	188,334	195,357	3.6
192,524	200,027	3.75	192,581	200,413	3.91
192,533	198,481	3	192,490	199,518	3.52
196,399	203,424	3.45	196,621	203,424	3.34
196,355	202,333	2.95	196,614	202,552	2.93
174,216	181,216	3.86	174,216	181,216	3.86
174,212	180,018	3.22	174,212	180,114	3.28
178,215	185,167	3.75	178,215	185,215	3.78
178,212	183,870	3.08	178,212	184,114	3.21
182,184	189,020	3.62	182,215	189,215	3.7
182,181	187,723	2.95	182,211	188,113	3.14
186,213	192,872	3.45	186,206	193,215	3.63
186,180	191,575	2.82	186,211	192,113	3.07
190,161	196,725	3.34	190,105	197,214	3.6
189,911	195,428	2.82	190,207	196,113	3.01
194,035	200,578	3.26	194,158	200,881	3.35
194,210	199,281	2.54	194,066	200,122	3.03
198,106	203,892	2.84	198,112	203,892	2.83
197,825	203,133	2.61	198,167	203,534	2.64
175,641	181,710	3,34	175,641	181,710	3.34
175,641	180,619	2,76	175,641	180,851	2.88
179,641	185,689	3,26	179,641	185,589	3.21
179,641	184,684	2,73	179,641	184,765	2.77
183,547	189,541	3,16	183,640	189,710	3.2
183,606	188,537	2,62	183,640	188,851	2.76
187,640	193,394	2,98	187,640	193,709	3.13
187,538	192,390	2,52	187,640	192,850	2.7
191,640	197,246	2,84	191,640	197,709	3.07
191,554	196,242	2,39	191,640	196,850	2.65
195,597	201,099	2,74	195,639	201,709	3.01
195,595	200,095	2,25	195,639	200,850	2.59
199,570	204,856	2,58	199,639	206,712	3.42
199,236	203,948	2,31	199,639	204,532	2.39
177,016	182,212	2,85	177,017	182,212	2.85
177,016	181,652	2,55	177,017	181,591	2.52
181,016	186,211	2,79	181,016	186,211	2.79
181,014	185,590	2,47	181,016	185,590	2.46
185,016	190,211	2,73	185,016	190,211	2.73
185,016	189,371	2,3	185,016	189,590	2.41
189,016	193,890	2,51	189,016	194,211	2.68
189,015	193,224	2,18	189,016	193,590	2.36
193,015	197,742	2,39	193,015	198,210	2.62
192,396	197,077	2,38	193,015	197,589	2.32
197,015	201,595	2,27	197,015	202,210	2.57
197,014	200,929	1,95	197,015	201,589	2.27
201,014	205,447	2,16	201,014	205,958	2.4
200,807	204,782	1,94	201,015	205,575	2.22
178,346	182,860	2,47	178,346	182,860	2.47
178,346	182,448	2,25	178,344	182,225	2.13
182,346	186,859	2,42	182,346	186,859	2.42
182,344	186,448	2,2	182,346	186,353	2.15
186,344	190,716	2,29	186,345	190,859	2.36
186,344	190,447	2,15	186,345	190,352	2.11
187,302	194,569	3,73	190,345	194,859	2.32
189,386	194,087	2,42	190,322	194,352	2.07
194,344	198,421	2,05	194,345	198,858	2.27
194,168	197,939	1,91	194,345	198,352	2.02
198,335	202,274	1,95	198,344	202,858	2.23
198,344	201,792	1,71	198,344	202,351	1.98
202,344	206,126	1,83	202,342	206,886	2.2
202,256	205,645	1,65	202,344	206,351	1.94

179,635	183,672	2,2	179,635	183,654	2.19
179,633	183,142	1,92	179,601	183,085	1.9
183,543	187,654	2,19	183,630	187,654	2.14
183,576	187,270	1.97	183,633	187,142	1.87
186,818	191,433	2.41	187,634	191,654	2.1
187,595	191,141	1.86	187,635	191,141	1.83
191,613	195,286	1.88	191,632	195,653	2.06
190,882	194,982	2.1	191,632	195,141	1.8
195,597	199,139	1.78	195,623	199,653	2.02
195,569	198,835	1.64	195,634	199,141	1.76
199,615	202,991	1.66	199,633	203,653	1.97
199,629	202,688	1.51	199,632	203,140	1.73
203,619	206,844	1.56	203,616	207,652	1.94
203,627	206,540	1.41	203,632	207,140	1.69

Figures 3 and 4 show the networks obtained by ACSLH and LH respectively. In this particular instance, ACSLH obtained a solution value better than the one obtained by LH algorithm.

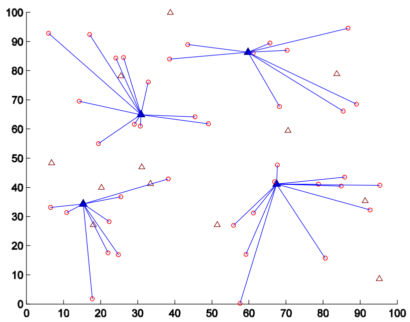

Figure 3. Network obtained by ACSLH algorithm for a particular instance.

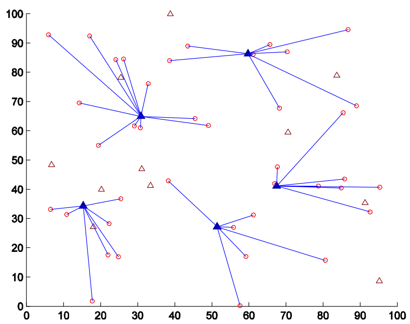

Figure 4. Network obtained by LH algorithm for a particular instance.

Beside the fact that our hybrid algorithm obtained a better solution for the case showed in Figures 3 and 4, we can see that the final obtained networks are quite similar one to each other in terms of the warehouses that are open as well as in terms of the customer allocation. However, in this particular case, the exploitation ability of our approach allows us to better inspect the solution space around the current solution and consequently to find better solution within the 'neighbourhood'.

4. Conclusions and Future Work

In this paper a hybrid approach combining ACS and Lagrangian method is presented and tested on an Inventory Location Model. Obtained results show that the hybrid approach is quite competitive w.r.t. other techniques proposed previously in the literature. While LH provides a promising subset of warehouses by means of the solution of a relaxed sub-problem, the implemented ACS allows us to allocate the set of customers to those warehouses provided by the LH. Furthermore, solutions obtained by ACS help back LH to fine tune values of its multipliers.

As a future work, our approach could be applied to other combinatorial problems such as Inventory Routing problems, Vehicle Routing problems, among others. Moreover, larger instance could be tested as size of the problem should not be a major issue for the hybrid algorithm performance.

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