A Modified Harmony Search Algorithm for the Economic Dispatch Problem

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Abstract: The paper presents a modified harmony search algorithm (MHS), useful for solving the economic dispatch (ED) problem assuming a non-linear cost function and various technical restrictions. The ED problem is a very important optimization problem for power systems, and technical restrictions must be considered: prohibited operating zones and ramp rate limits of power generating units, as well as transmission line losses. The MHS algorithm is based on harmony search (HS) algorithm, but a new harmony is obtained by inserting some features from artificial bee colony algorithm. The efficiency of MHS algorithm is tested against two systems consisting of 6 and 38 thermal power generating units. Results are compared with those obtained by applying other optimization techniques.

Keywords: economic dispatch; harmony search; transmission losses.

1. Introduction

Economic dispatch (ED) is an optimization problem of power systems that aims to determine the output power of thermal power generating units in order to have a minimal fuel cost for the entire system and, in the mean time, to satisfy some technical restrictions while operating the units.

The mathematical model of ED problem is non-linear, where both objective function and restrictions system may be non-linear. Classical methods were used for solving the ED problem: linear programming [1], non-linear programming [2], quadratic programming [3], Lagrangian relaxation algorithm [4] and dynamic programming [5]. Usually, these methods have got difficulties in finding a global optimum, they being able to offer only a local optimum point. Moreover, classical methods need a calculation of derivatives and some checking on continuity and derivability conditions of functions belonging to optimization model. To cover these drawbacks several artificial intelligence-based optimization techniques were applied. One of the most frequently used methods is based on the particle swarm optimization (PSO) applied in classical, enhanced or hybrid versions: PSO, PSO with time varying acceleration coefficients (PSO-TVAC) [6-8], new PSO (NPSO, NPSO-LSR) [9, 10], improved PSO [11], distributed Sobol PSO with tabu search algorithm (DSPSO-TSA) [12]. Other methods used for solving ED problems are: evolutionary programming (EPs) [13], biogeography-based optimization (BBO) [14], tabu search and multiple tabu search (TS, MTS) [15], differential evolution (DE) [16, 17], hybrid DE (DEPSO) [18], artificial bee colony algorithm (ABC) [19], incremental ABC with local search (IABC-LS) [20], harmony search (HS) [21], differential HS (DHS) [22].

Harmony search is a meta-heuristic algorithm inspired from a musical process of searching for a perfect state of harmony. The HS is an easy to implement algorithm, having good convergence characteristics and may be easily adapt to work with other algorithms [23, 24]. Thus, the HS algorithm or its versions were successfully used for solving mathematical [25-27] and engineering problems with continuous variables: reliability optimization [28], automatic parameter configuration [29], design of water distribution networks [30] etc.

In this paper, the HS classical algorithm is enhanced with some features specific to artificial bee colony algorithm in order to solve the economic dispatch problem. The new algorithm is called modified harmony search (MHS) algorithm. Its results are compared with others obtained by applying different optimization techniques.

2. ED Problem Formulation

Considering a power system where $\boldsymbol{n}$ thermal generating power units are operating, each unit having an output power P_i, i=1, 2,..., n. Output powers P_i define the solution vector $P=[P_1, P_2,\ldots, P_i,\ldots, P_n]^T$. The fuel cost $F_i(P_i)$, for each generator i, may be represented by a quadratic polynomial function such as:

$$F_i(P_i) = c_i P_i^2 + b_i P_i + a_i \qquad (1)$$

where a_i, b_i and c_i are fuel cost coefficients of generator i.

To solve an ED problem means determining the P_i output powers of the generating units, so that the total fuel cost (objective function) is minimal, considering a set of equality and inequality technical constraints. The objective function is:

$$\min F = \sum_{i=1}^{n} F_i(P_i) \qquad (2)$$

The equality and inequality constraints for the ED problem are given by (3)-(8) relations:

i) Minimum and maximum real power operating limits:

$$P_{i,min} \le P_i \le P_{i,max},\ i=1,2,\ldots,n \qquad (3)$$

where $P_{i,min}$ and $P_{i,max}$ represent the minimum and the maximum operating limits of a generator i.

ii) Generator ramp-rate limits:

$$P_i \le P_i^{Prev} + UR_i, \text{ if output power increases} \qquad (4)$$

$$P_i \ge P_i^{Prev} - DR_i, \text{ if output power decreases} \qquad (5)$$

where P_i^{Prev} is the previous hour output power of unit i. DR_i and UR_i are the down-ramp and up-ramp limits of the i unit.

Relations (3)-(5) can also be expressed by:

$$PO_{i,min} \le P_i \le PO_{i,max} \qquad (6)$$

where $PO_{i,min}$= max($P_{i,min}$, P_i^{Prev} -DR_i) and $PO_{i,max}$= min($P_{i,max}$, P_i^{Prev} +UR_i).

iii) Prohibited operating zones of the generator. Power generating units may have some prohibited operating zones, which, for a certain i generator, are given by:

$$P^L_{i,z} < P_i < P^U_{i,z},\quad z=1,2,\ldots, NZ_i \qquad (7)$$

where NZ_i is the number of prohibited zones of unit i. $P^L_{i,z}$ and $P^U_{i,z}$ are the lower and upper boundary of the z prohibited operating zone for the unit i.

iv) Real power balance constraint:

$$P_G - P_L - P_D = 0 \qquad (8)$$

where P_D is the load demand in the system, in MW. P_L represents the transmission loss, in MW.

The transmission losses P_L at the entire system level are calculated using constant B coefficient formula (Kron's relation):

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{0i} P_i + B_{00} \qquad (9)$$

where B_{ij} is an element of the loss coefficient matrix of size n x n, B_{0i} is i element of the loss coefficient vector of size n and B_{00} is the loss coefficient constant.

The total generated power (P_G) of the system by the n units is:

$$P_G = \sum_{i=1}^{n} P_i \qquad (10)$$

3. The Harmony Search Algorithm

Harmony search (HS) is a meta-heuristic population-based algorithm inspired from the musical process of searching for a perfect state of harmony. It was proposed by Geem et al. [23] and developed by Lee and Geem [24] for engineering optimization problems with continuous variables.

Generally, HS algorithm is used to solve optimization problems with continuous parameter. Thus, the optimization problem consists of searching the minimum of an $f(\boldsymbol{x})$ function considering the constraints $x_i^{min} \le x_i \le x_i^{max}$, i=1, 2, ..., n, where n is the number of decision variables, and x_i^{min} and x_i^{max} are lower and upper limits for the variable x_i. Function $f(\boldsymbol{x})$ represents the objective function, $\boldsymbol{x}=(x_1,x_2,..,x_n)$ is the n-dimensional solution vector, and x_i is the i^{th} component of $\boldsymbol{x}$ vector.

The HS algorithm is defined by the following parameters [24]: the harmony memory size (HMS), harmony memory considering rate (HMCR), pitch adjusting rate (PAR), distance bandwidth (bw) and maximum number of iterations (improvisations) (N_{max}). HMS is the number of vectors in harmony memory that satisfy the constraints of the problem, and HMCR and PAR are the parameters used for generating a new vector.

During the optimization process, solution vectors are being stored as a harmony memory HM=[$\boldsymbol{x}_1$, $\boldsymbol{x}_2$,..,$\boldsymbol{x}_{HMS}$]. Each vector $\boldsymbol{x}_j|_{j=1,2,..HMS}$, belonging to HM is a possible solution of the problem. The optimization procedure that uses

HS meta-heuristic algorithm consists of the following steps [23, 24]:

Step 1. *Initialize the optimization problem and algorithm parameters*. Here, the parameters of HS algorithm are initialized: HMS, HMCR, PAR, bw and N_{max}. Also, lower (x_i^{min}) and upper (x_i^{max}) limits for the variable x_i are set, as well as other specific parameters of the problem.

Step 2. *Initialize the harmony memory (HM).* Initially, HM matrix consists of HMS harmonies (column solution vectors). The elements of each vector $\boldsymbol{x}_j|_{j=1,2,..HMS}$, are randomly generated, inside $[x_i^{min}, x_i^{max}]_{i=1,2,..n}$, interval having a uniform distribution:

$$x_i=x_i^{min}+rand()\times(x_i^{max}-x_i^{min}),\ i=1,2,..,n \quad (11)$$

rand() is a uniformly distributed number inside [0, 1] interval.

Then, for each HM vector ($\boldsymbol{x}_j|_{j=1,2,..HMS}$) the value of the objective function $f(\boldsymbol{x}_j)$ is determined.

Step 3. *Improvise a new harmony*. The new harmony vector, $\boldsymbol{x}^{new}=(x_1^{new}, x_2^{new},.., x_n^{new})$ is generated from the HM by sequentially applying three rules: memory considerations, pitch adjustments, and random selection. Each element ($x_i^{new}|_{i=1,2,..n}$) belonging to vector $\boldsymbol{x}^{new}$ is generated using the following procedure [24, 27]:

If *rand()*≤HMCR **then begin** /*memory consideration*/

$$x_i^{new}=x_{ij},\ \text{unde}\ j\sim U[1, HMS]\ \text{and}\ i=1,2,..,n \quad (12)$$

If *rand()*≤ PAR **then** /*pitch adjustment*/

$$x_i^{new}= x_i^{new}\pm r\times bw,\ \text{unde}\ r\sim U[0,1]\ \textbf{end} \quad (13)$$

else /*random selection*/

$$x_i=x_i^{min}+rand()\times(x_i^{max}-x_i^{min}),\ i=1,2,..,n \quad (14)$$

r and *rand()* are uniformly distributed random numbers in the range [0,1].

Step 4. *Update the harmony memory*. If the new harmony vector $\boldsymbol{x}^{new}$ is better than the worst harmony vector ($\boldsymbol{x}^{worst}$) of HM, then $\boldsymbol{x}^{worst}=\boldsymbol{x}^{new}$. The objective function $f(\boldsymbol{x})$ is used for comparing $\boldsymbol{x}^{new}$ and $\boldsymbol{x}^{worst}$ vectors.

Step 5. *Check the stopping criterion*. The algorithm stops when then maximum number of iterations (N_{max}) is reached. If $k<N_{max}$ then go to Step 3 (*k* current number of the iteration), otherwise Stop and return the best harmony (solution) din HM.

4. The modified harmony search (MHS) algorithm

The MHS algorithm is built on the same steps like HS algorithm, its purpose being the enhancement of classical HS algorithm performances. The difference between MHS algorithm and HS algorithm is made by the generating way of the new harmony described at step 3. The following changes have been done to MHS algorithm:

– the rule regarding the "random selection" of a new harmony vector is eliminated;
– equation (12) regarding "memory consideration" is replaced with equation (15) that borrows from artificial bee colony algorithm some features for generating the solution [31, 32]:

$$x_i^{new} = x_{ij} + U(-1,1)\times(x_{ij} - x_{ik}),\quad j\sim U[1, HMS],\ k\sim U[1, HMS], j\neq k\ \text{and}\ i = 1,2,...,n \quad (15)$$

– equation (13) regarding "pitch adjustment" is replaced with equation (16). This searching strategy is based on the advantages offered by the best global solution and is similar with the one in ABC algorithm [20]:

$$x_i^{new} = x_i^{best} + U(-1,1)\times(x_{ij} - x_{ik}^{best}),\ i = 1,2,...,n\ \text{and}\ j\sim U[1, HMS], \quad (16)$$

x_i^{best} is the i^{th} component corresponding to the best solution vector resulted until the current iteration. U(-1,1) is a uniform random real number inside [-1, 1] interval. In must be pointed out that, according to the above-mentioned changes, MHS algorithm has got only three parameters (HMS, PAR and N_{max}), while (HMCR and bw) parameters are eliminated.

The MHS algorithm combines the HS algorithm with several features of ABC algorithm. Hybridizing HS algorithm with ABC algorithm has been done in other papers, too [26, 33], but this study has got some differences regarding the generating of a new harmony through memory consideration and pitch adjustment.

5. Simulation Results and Comparison

To test the efficiency of MHS algorithm two different test systems were studied: a 6-unit system, with power losses considered (test system 1), and a 38-unit system, without considering the power losses (test system 2). All case studies were implemented in MathCAD, on a personal computer having a 1.79 GHz processor and 896 MB of RAM. The solution's quality is evaluated through 100 or 200 trials.

For each trial the values of the following items are memorized: best total fuel cost F (B), average total fuel cost F (A), worst total fuel cost F (W) and standard deviation (SD). For each system, values of the parameters used for HS and MHS algorithm (HMS, HMCR, PAR, bw, N_{max}) were determined by performing experimental trials.

5.1 Test system 1: 6-unit with losses

This case study contains a system of 6 units with transmission losses, ramp rate limits and prohibited operating zones of the units taken into consideration. The tested system data related to the cost coefficients (a, b, c), power operating limits, ramp-rate limits, prohibited operating zones of the units, and also the loss coefficient B are taken from [6, 34].

The load demand is P_D=1263 MW. The characteristics of thermal power generating units and values of loss coefficient B are described in Table 1 and Table 2.

For the studied system, after a few experimental trials, the parameters were set to the following values: HMS=8, PAR=0.4 and N_{max}=1000 (for the MHS algorithm) and HMS=8, HMCR=0.9, PAR=0.3, bw=0.01 and N_{max}=1000 [27] (for the HS algorithm).

Table 3 presents the output powers of thermal power generating units (P_i, i=1, 2,..., 6) after running the MHS and HS algorithms. Values of B, A, W, SD items, generated power (P_G), power losses (P_L) and tolerance (TOL) are also shown. Both algorithms satisfy the equality constraint (8) with a very high precision ($TOL_{MHS}=-3.925\cdot10^{-13}$ MW and $TOL_{HS}=-9.315\cdot10^{-11}$ MW).

T-test is applied for a comparison between the results obtained by MHS and HS algorithms.

Considering the characteristics of the MHS and HS algorithms shown in Table 3 - average (A) and standard deviation (SD) - the value of $t_{calculated}$ is computed ($t_{calculated}$=3.113). Then, from the table with critical values "T-test", for a significance level of 1%, $t_{critical}$ is returned ($t_{critical}$=2.601).Since, $t_{calculated}=3.113>t_{critical}=2.601$, between the results obtained by MHS and HS algorithms there are significant statistical differences ($A_{MHS}<A_{HS}$).

Table 1. The characteristics of thermal power generating units and prohibited operating zones (6-units)

Unit	a ($/h)	b ($/MWh)	c ($/MW^{2}h)	UR_i (MW)	DR_i (MW)	P_i^{Prev} (MW)	$P_{i,min}$ (MW)	$P_{i,max}$ (MW)	Prohibited zone (MW)
1	240	7.0	0.0070	80	120	440	100	500	[210 240], [350 380]
2	200	10.0	0.0095	50	90	170	50	200	[90 110], [140 160]
3	220	8.5	0.0090	65	100	200	80	300	[150 170], [210 240]
4	200	11.0	0.0090	50	90	150	50	150	[80 90], [110 120]
5	220	10.5	0.0080	50	90	190	50	200	[90 110], [140 150]
6	190	12	0.0075	50	90	110	50	120	[75 85], [100 105]

Table 2. B-loss coefficient values (6-units)

B_{ij}=	17	12	7	-1	-5	-2	$\cdot10^{-4}$
	12	14	9	1	-6	-1	
	7	9	31	0	-10	-6	
	-1	1	0	24	-6	-8	
	-5	-6	-10	-6	129	-2	
	-2	-1	-6	-8	-2	150	
B_{0i}=	-0.3908	-0.1297	0.7047	0.0591	0.2161	-0.6635	$\cdot10^{-3}$

B_{00}=0.0056

Table 3. The best solution obtained using MHS and HS algorithms (6 units, P_D=1263MW, 200 trials)

Output	MHS	HS
P_1 (MW)	447.5038934324	447.5042986422
P_2 (MW)	173.3188266703	173.3204504696
P_3 (MW)	263.4628642464	263.462694264
P_4 (MW)	139.0649874081	139.0648289018
P_5 (MW)	165.4738752653	165.4731533959
P_6 (MW)	87.1338060426	87.1328232359
P_G (MW)	1275.9582530651	1275.9582489093
P_L (MW)	12.9582489093934	12.9582489093934
P_D (MW)	1263	1263
TOL_M (MW)	$-3.925 \cdot 10^{-13}$	$-9.315 \cdot 10^{-11}$
Best cost F (B) ($/h)	15449.8995248809	15449.8995249519
Average cost F (A) ($/h)	15449.8995250435	15449.8995486667
Worst cost F (W) ($/h)	15449.8995257499	15449.9007357696
SD ($/h)	$1.7628 \cdot 10^{-7}$	$1.0626 \cdot 10^{-4}$

The convergence characteristics for MHS and HS algorithms are shown in Figure 1. It can be seen that both algorithms are able to reach the optimal solution, but MHS algorithm is faster; it needs a smaller number of iterations (approximately 130, while HS algorithm needs approx. 380 iterations).

MHS and HS algorithms robustness was studied by performing 200 trials. For each trial the best cost F (item B) was stored, and it was represented in Figure 2. In Figure 2 and Table 3 we note that MHS algorithm is very stable ($SD_{MHS}=1.7628 \cdot 10^{-7} < SD_{HS}=1.0626 \cdot 10^{-4}$) and has the best items B_{MHS}=15449.8995248$/h < B_{HS}=15449.8995249$/h, $A_{MHS} < A_{HS}$ and $W_{MHS} < W_{HS}$).

Comparing MHS with other methods. When applying different optimization methods, results may be influenced by the level of satisfaction of the equality constraint (defined by (8) equation). Thus, all methods must be applied in similar conditions (tolerance value - equation (17) – must be approximately the same for all methods).

$$TOL_M=P_G-P_L-P_D \quad (17)$$

In order to verify if MHS algorithm works in similar conditions like any other method M, the following relation is used:

$$Dif\_TOL=|TOL_{MHS}-TOL_M|< \varepsilon \quad (18)$$

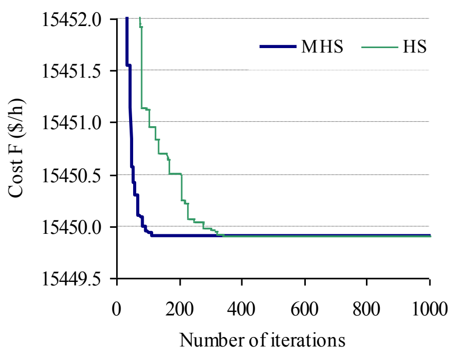

Figure 1. Convergence characteristics for MHS and HS algorithms, 6-units

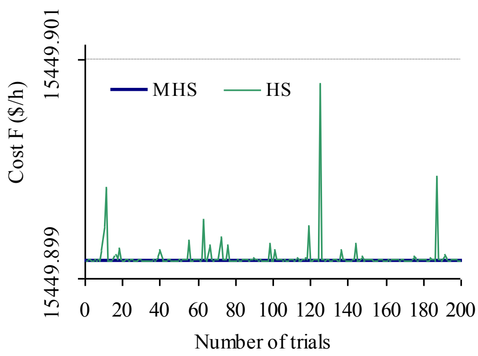

Figure 2. The best total cost *F* obtained with MHS and HS algorithms, for 200 trials, 6-units

where, TOL_M, TOL_{MHS} is the tolerance for a method M, respectively algorithm MHS.

For the test system 1, it is assumed that $\varepsilon=10^{-10}$ MW can be neglected, meaning that all methods are being applied in similar conditions with respect to satisfaction of equality constraint (8).

Table 4 shows a comparison between the results of MHS algorithm and three other optimization methods used for solving the same problem: particle swarm optimization (PSO) [6], multiple tabu search algorithm (MTS) [15] and differential evolution (DE) [16]. Thus, the set of methods M is: M={PSO, DE, MTS}. It must be said that MHS algorithm was applied for similar conditions as the indicated methods were (for all comparisons: MHS vs. PSO, MHS vs. DE and MHS vs. MTS, Table 4 indicates $Dif\_TOL< 10^{-10}$ MW).

Table 4. Comparative results after applying different optimization techniques (6 units, P_D=1263 MW).

Method	PSO [6]	MHS	MTS [15]	MHS	DE [16]	MHS
1. Result described according to the indicated references						
P_1 (MW)	447.4970	447.5034920879	448.1277	447.5054236915	447.744	447.5016941993
P_2 (MW)	173.3221	173.3168873323	172.8082	173.3179702901	173.407	173.3176508104
P_3 (MW)	263.4745	263.462198913	262.5932	263.4634652066	263.411	263.4600979625
P_4 (MW)	139.0594	139.065688249	136.9605	139.0657881467	139.076	139.0634606943
P_5 (MW)	165.4761	165.4732611341	168.2031	165.4726437196	165.364	165.471156291
P_6 (MW)	87.1280	87.1354007626	87.3304	87.1356138354	86.944	87.1327806056
Best cost F_M (B) ($/h)	**15450**	**15449.8822209778**	**15450.06**	**15449.9350750959**	**15449.766**	**15449.7480804051**
Average F_M (A)($/h)	15454	15449.8822211603	15451.17	15449.9350752945	15449.777	15449.7480806214
Worst cost F_M (W)($/h)	15492	15449.8822220211	15453.64	15449.9350759751	15449.874	15449.7480816701
SD_M ($/h)	-	$1.9053\cdot10^{-7}$	0.9287	$1.9534\cdot10^{-7}$	-	$2.3538\cdot10^{-7}$
Number of trials	50	100	100	100	100	100
2. The computation of the power loss (P_L), generated power(P_G) and TOL_M tolerance based on the best solution described in the indicated references						
P_G (MW)	1275.9571	1275.9569284789	1276.0231	1275.9609048899	1275.9460	1275.9468405631
P_L (MW)	12.9583778743	12.95820635316	13.0204746591	12.95827954902	12.9571840155	12.95802457851
TOL_M (MW)	-0.0012778743	-0.00127787426	0.0026253408	0.002625340879	-0.0111840155	-0.01118401541
Dif_TOL* (MW)	0	$3.999\cdot10^{-11}$	0	$7.899\cdot10^{-11}$	0	$9.000\cdot10^{-11}$

* $Dif\_TOL=|TOL_{MHS} - TOL_M|$; TOL_M, TOL_{MHS} - the tolerance for a method *M*, respectively algorithm MHS; "-"data not available.

Table 4 shows that MHS algorithm has got the best *Cost F* (item *B*) for all the three analyzed situations: B_{MHS}=15449.882220$/h<$B_{PSO}$=15450$/h, B_{MHS}=15449.7480804$/h<$B_{DE}$=15449.766$/h, B_{MHS}=15449.935075$/h<$B_{MTS}$=15450.06$/h).

Also, items A_{MHS}, W_{MHS} and SD_{MHS} obtained from MHS algorithm have got better values then similar items obtained from PSO, DE and MTS algorithms.

Moreover, the worst value obtained by MHS algorithm is lower than the best value obtained by the methods specified (in Table 4 $W_{MHS}<B_{PSO}$ (MHS vs. PSO), $W_{MHS}<B_{DE}$ (MHS vs. DE) and $W_{MHS}<B_{MTS}$ (MHS vs. MTS)).

Values of items *B*, *A*, *W* and *SD* were calculated using a higher or, at least the same, number of trials like for the other methods (PSO, DE and MTS). The average computation time is of 5.4sec.

From the items (B, A, W, SD) point of view, MHS algorithm proves to be superior comparing to HS, PSO, DE and MTS, as may be seen from the results presented in Table 3 and Table 4 and the characteristics from Figure 1 and Figure 2.

5.2 Test system 2: 38-unit without losses

A 38-unit test system is studied in solving the ED problem with the MHS algorithm. The tested system data related to the cost coefficients (*a*, *b*, *c*) and power operating limits are taken from [35]. The load demand is P_D=6000 MW.

For the system with 38-units, after a few experimental trials, the parameters used were set to the following values: *HMS*=15, *PAR*=0.4 and N_{max}=10000 (for the MHS algorithm) and HMS=15, HMCR=0.99, PAR=0.3, bw=10^{-6} and N_{max}=10000 (for the HS algorithm).

Table 5 shows the best solutions (P_i, i=1, 2, ..., 38) obtained from MHS and HS algorithms, as well as other solutions obtained from the following optimization technique PSO_TVAC [7],

Table 5. Best solution obtained using MHS and HS algorithms and comparison with other optimization techniques (38-units, P_D=6000 MW, 100 trials)

Output	MHS	HS	PSO_TVAC [7]	DE/BBO [36]	BBO [36]
P_1 (MW)	426.6055451709	426.600231683	443.659	426.606060	422.230586
P_2 (MW)	426.6047089764	426.5937015757	342.956	426.606054	422.117933
P_3 (MW)	429.6635302444	429.650913618	433.117	429.663164	435.779411
P_4 (MW)	429.6647862209	429.675753655	500.000	429.663181	445.481950
P_5 (MW)	429.6667821468	429.6545940263	410.539	429.663193	428.475752
P_6 (MW)	429.6628268036	429.6702425681	482.864	429.663164	428.649254
P_7 (MW)	429.6609836262	429.6691319879	409.483	429.663185	428.119288
P_8 (MW)	429.6631083527	429.6689470513	446.079	429.663168	429.900663
P_9 (MW)	114.0000000000	114.000001626	119.566	114.000000	115.904947
P_{10} (MW)	114.0000000000	114.0000000000	137.274	114.000000	114.115368
P_{11} (MW)	119.7673921028	119.7662422478	138.933	119.768032	115.418662
P_{12} (MW)	127.0724320711	127.076361834	155.401	127.072817	127.511404
P_{13} (MW)	110.0000000000	110.0000000000	121.719	110.000000	110.000948
P_{14} (MW)	90.0000000000	90.0000000000	90.924	90.0000000	90.0217671
P_{15} (MW)	82.0000000003	82.0000000000	97.941	82.0000000	82.0000000
P_{16} (MW)	120.0000000000	120.0000000000	128.106	120.000000	120.038496
P_{17} (MW)	159.5982110232	159.599299215	189.108	159.598036	160.303835
P_{18} (MW)	65.0000000000	65.0000000000	65.000	65.0000000	65.0001141
P_{19} (MW)	65.0000000000	65.0000000000	65.000	65.0000000	65.0001370
P_{20} (MW)	272.0000000000	272.0000000000	267.422	272.000000	271.999591
P_{21} (MW)	272.0000000000	272.0000000000	221.383	272.000000	271.872680
P_{22} (MW)	260.0000000000	259.9999997967	130.804	260.000000	259.732054
P_{23} (MW)	130.6483891957	130.6460481007	124.269	130.648618	125.993076
P_{24} (MW)	10.0000000000	10.0000006793	11.535	10.0000000	10.4134771
P_{25} (MW)	113.3051024999	113.3130828089	77.103	113.305034	109.417723
P_{26} (MW)	88.0671008506	88.0692392482	55.018	88.0669159	89.3772664
P_{27} (MW)	37.5056685997	37.5059577346	75.000	37.5051018	36.4110655
P_{28} (MW)	20.0000000000	20.0000000000	21.682	20.0000000	20.0098880
P_{29} (MW)	20.0000000000	20.0000000000	29.829	20.0000000	20.0089554
P_{30} (MW)	20.0000000000	20.0000000000	20.326	20.0000000	20.0000000
P_{31} (MW)	20.0000000000	20.0000000000	20.000	20.0000000	20.0000000
P_{32} (MW)	20.0000000000	20.0000000000	21.840	20.0000000	20.0033959
P_{33} (MW)	25.0000000000	25.0000000000	25.620	25.0000000	25.0066586
P_{34} (MW)	18.0000000000	18.0000000000	24.261	18.0000000	18.0222107
P_{35} (MW)	8.0000000000	8.0000000000	9.667	8.00000000	8.00004260
P_{36} (MW)	25.0000000000	25.0000000000	25.000	25.0000000	25.0060660
P_{37} (MW)	21.7817319076	21.7811436951	31.642	21.7820891	22.0005641
P_{38} (MW)	21.0616992228	21.0591058638	29.935	21.0621792	20.6076309
P_G (MW)	5999.9999990156	5999.9999990154	6000.005	5999.999992	5999.95286
P_D (MW)	6000	6000	6000	6000	6000
TOL_M(MW)	$-9.844\cdot10^{-7}$	$-9.846\cdot10^{-7}$	$5.000\cdot10^{-3}$	$-8.000\cdot10^{-6}$	$-4.713\cdot10^{-2}$
Best cost F_M ($/h)	9417235.78535618	9417235.78581085	9500448.307	9417235.786391673	9417633.63764437
Average F_M ($/h)	9417235.78606311	9417235.78707343	-	-	-
Worst cost F_M($/h)	9417235.78641317	9417235.79132093	-	-	-
SD_M ($/h)	$3.852\cdot10^{-4}$	$8.196\cdot10^{-4}$	-	-	-

"-" data not available

Biogeography-Based Optimization (BBO) [36], Hybrid Differential Evolution with Biogeography-Based Optimization (DE/BBO) [36]. Therefore, the set of methods M consists of: M={PSO_TVAC, BBO, DE/BBO}. At the end of Table 5 items *B*, *A*, *W* and *SD* are shown for all methods.

The values $t_{calculed}$ and $t_{critical}$, for a significance level of 1%, were determined by applying "T-test". Since, $t_{calculated}=11.44>t_{critical}=2.626$, between the results obtained by MHS and HS algorithms there are significant statistical differences ($A_{MHS}<A_{HS}$). Convergence characteristics for MHS and HS algorithms (for 38-units) are shown in Figure 3.

It is observed that MHS has a better convergence than HS algorithm. In Fig. 4 the best values of *Cost F* (B) function for MHS and HS algorithms are depicted (considering 100 trials). The data from Table 5 and Figure 4 show that MHS algorithm has a better stability than the HS algorithm ($SD_{MHS}=3.852\cdot10^{-4}$ $<SD_{HS}=8.196\cdot10^{-4}$). Also, the values of items B, A, W and SD are better in case of MHS algorithm, comparing to the values of similar items obtained by HS algorithm (Table 5). The average computation time is of 47.2 sec.

Comparing MHS with other methods. Analyzing Table 5 and focusing on item *B*, it can be seen that MHS algorithm is superior comparing to other optimization methods: PSO_TVAC, BBO. The worst value obtained by MHS algorithm is better than the best value obtained by PSO_TVAC ($W_{MHS}<B_{PSO\_TVAC}$) and BBO ($W_{MHS}<B_{BBO}$). Also, MHS algorithm has better results than HS algorithm ($B_{MHS}<B_{DE/BBO}$), when equality constraint (8) is satisfied with a higher precision: ($|TOL_{MHS}|=9.844\cdot10^{-7}<|TOL_{DE/BBO}|=8.000\cdot10^{-6}$).

6. CONCLUSION

In this paper, the MHS algorithm has been tested to solve the economic dispatch problem. The MHS algorithm is based on harmony search algorithm. Some features of HS algorithm were replaced with others belonging to artificial bee colony algorithm, in order to enhance its capacity to avoid premature convergence and to get high-quality solutions.

To solve a 6-units test system, several operational characteristics of thermal power generating units were considered (ramp-rate limits, prohibited operating zones, minimum and maximum power operating limits) that define a range of non-continuous values for the output powers of thermal power generating units. Transmission losses in electric line were also considered.

The MHS algorithm was successfully applied on two test systems consisting of 6 units and 38 units. Results show that MHS algorithm is better than HS algorithm for both case studied, if considering items B, A, W and SD. Also, MHS algorithm is better than other optimization techniques used for solving this problem (PSO, DE and MTS for 6-units), respective (PSO_TVAC, DE/BBO and BBO for 38-units). Considering these good results, it may be said that MHS algorithm has the ability to obtain high-quality solutions, guarantees stability and a good calculation time, both for the 6-units test system, and for the large-scale test system with 38-units.

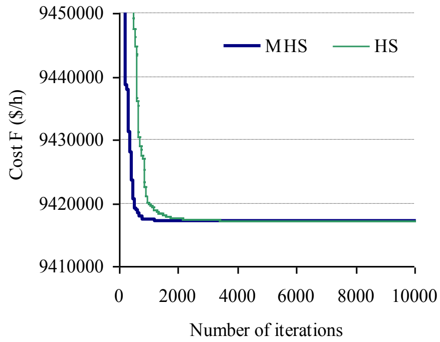

Figure 3. Convergence characteristics for MHS and HS algorithms, 38-units

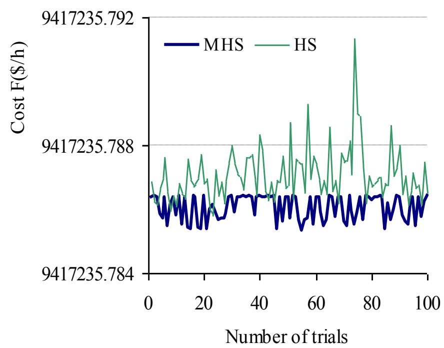

Figure 4. The best total cost *F* obtained with MHS and HS algorithms, for 100 trials, 38-units

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