

# Algorithms for the Computation of Passive Robustness Margins in Railway Transport Systems

Anis MHALLA<sup>1\*</sup>, Simon Collart DUTILLEUL<sup>2</sup>

<sup>1</sup> Laboratory of Automation, Electrical Systems Environment (LAESE), National Engineering School of Monastir (ENIM), Monastir, Tunisia  
anis.mhalla@enim.rnu.tn (\*Corresponding author)

<sup>2</sup> COSYS Department, ESTAS Laboratory, University of Gustave Eiffel, France  
simon.collart-dutilleul@univ-eiffel.fr

**Abstract:** This paper contributes to the analysis of the robustness of railway transport networks. With a focus on the maximum time constraints related to the interval when a metro is stopped at a station, robustness is analysed without control modifications against temporal disturbances. A rigorous axiomatics was elaborated for the analytical characterization of this kind of robustness in the context of manufacturing sites. This paper introduces a new semantic interpretation of the classical model of a manufacturing site with maximum time constraints in order to compute a passive robustness lower bound for a railway network. The aim is to avoid time constraint violations. Finally, in order to illustrate the effectiveness and accuracy of the proposed computational algorithms, their implementation in the context of the Sahel Railway Network is presented.

**Keywords:** Passive robustness, Temporal disturbances, Railway network, Computation algorithm.

## 1. Introduction

Important scientific and technological efforts have been made in order to build new services of smart manufacturing following the concepts of industry 4.0: the norm IEC 62264 (ISO, 2013) structures and defines main functions of the production processes and controls. This document claims that “IEC 62264 is a multi-part standard that defines the interfaces between enterprise activities and control activities”.

Similar formalization efforts were not made in the context of railway systems, but railway systems can be seen as a production system providing some services: transport of goods and persons. This paper aims to present railway systems as particular production systems, which would allow one to benefit from the huge existing conceptual framework.

Actually, railway systems are really special kinds of production systems involving specific constraints and goals. For this reason, a specific pragmatic interpretation must be carried out in order to obtain useful results.

The present paper aims to reuse the state of the art in time-critical manufacturing sites in order to prove robustness properties of metro line controls.

The remainder of this paper is structured as follows. A description of Tunisian railway transport system is detailed in Section 2. In Section 3, an algorithm for computing a lower bound of the passive robustness margin is presented. In

Section 4, the passive robustness margin for a real railway example is obtained, by applying the previous algorithm. Section 5 presents the conclusion based on the obtained results and outlines the prospects concerning diagnosis and future maintenance management for this kind of metro lines.

## 2. Literature Analysis

Many papers were devoted to the robustness of transport systems in an attempt to save time and to improve the quality of traffic (Mhalla & Gaied, 2018; Restel & Haladyn, 2022). (Müller-Hannemann et al., 2022) presents an approach to network modelling and robustness evaluation of multimodal freight transport networks, where nodes represent connections, terminals and crossings, and links are tracks. The network model reflects the characteristics of interconnection and interdependence. The freight can move from one modality to another in interconnected terminals, while the disruption of a single interdependent node (e.g. bridge, tunnel, crossing) affects several modalities. Considering infrastructure element disruptions and track capacity degradation as disruptions, the robustness of the network is assessed as the increase in total travel time caused by these disruptions. The robustness assessment model is applied to Dutch freight transport, considering three modalities: waterway, road and railway.

The work of Cats & Krishnakumari (2020) investigates the issue of railway networks robustness with very different structures and development patterns in the event of random and targeted attacks. An approach based on the theory of complicated networks is applied, and network performance is analysed under alternative sequential disruption scenarios corresponding to the successive closures of railway stations and track segments. An aggregate robustness indicator based on the integral deterioration of the network performance is adopted. Three exemplary networks are considered, the urban railway networks of London, Shanghai and Randstad. These three networks provide showcases of short- and long-term development models, mono- and polycentric agglomeration structures, including largest and oldest metropolitan heavy rail networks.

Another work dealing with the robustness of railway transport networks is that of Zhang & Ng. (2022). In this paper an Urban Railway Network (URN) is constructed in order to study the network dynamic robustness against the cascading failure fluctuation induced by the passengers' flow under various failure modes. The propagation of cascading failures is imitated by a Linear Threshold (LT) model, where the edge influence parameter is specified. Considering the network topology and functionality, two robustness indices, namely the edge size change in the most connected component ( $R_{GCS}$ ) and the operational efficiency (ROE) are employed. By fusing these two indicators, a synthetic operator is provided to comprehensively quantify the dynamic robustness of URN. The simulation results show that the robustness of URN changes over time. Additionally, an increasing passenger flow volume can exacerbate the cascading failure sizes and the network robustness impacts in several scenarios.

Other robustness studies applied to railway networks are those of Wang et al. (2019) and Cats et al. (2019).

The contributions above did not consider the time as a critical safety parameter. Nevertheless, on a urban metro line, a delay can lead to a nonacceptable solution. As an example, it may happen that many children take the train to go to school. If the delay does not allow them to take the

dedicated bus that carries them to school, about fifty children may be lost 10 kilometers far from their school.

The fact that a given value of a time disturbance can be an definitive violation of a constraint is rarely taken into account in works concerning wide transportations networks. This kind of requirement is rather analysed in the time- constrained workshop of the manufacturing sites (Long & Descotes-Genon, 1993). The following section extends the semantic interpretation of the analysis devoted to the classical model of a manufacturing site with maximum time constraints in order to introduce this new kind of robustness analysis for railway networks.

### 3. Industrial Context

Manufacturing Entreprise Solutions Association (MESA) initially identified eleven main functions:

1. Resource management;
2. Scheduling;
3. Flow of products and batches;
4. Document management;
5. Data collection and acquisition;
6. Staff management;
7. Quality management;
8. Process management;
9. Maintenance management;
10. Product traceability and genealogy;
11. Performance analysis.

This paper rather focuses on the first three items, assuming that the metro line case study on a particular kind of available resources is the result of the association of a driver with a given rolling stock. This paper assumes that drivers are not limiting resources, but anybody holding a little experience in the railway domain knows that this is not really true.

## Sahel Railway Network

Tunisian National Railways Company (TNRC) is a state-owned company, which operates the Sahel railway line, linking the main cities of the Tunisian Sahel region. It starts from Monastir station, goes on to the airport and up to the last station Sousse Bab Jdid, as it can be seen in Figure 1. The Sahel subway system has an overall average service frequency of 40 minutes and a 16-hour daily traffic. The analysed railway transport networks are assumed to be:

- scheduled,
- with single-period cyclic functioning,
- with shared equipment among the various trips,
- with a limited number of transportation resources.

## Modelling of the Railway Transport System

In the analysed transportation system, travel times must be met. P-time Petri nets (P-TPN) are handy for modelling the reviewed railway system with imprecise travel times.

**Definition 1** (Khansa et al., 1996): A P-TPN system is a triplet of the form  $\langle R, IS \rangle$ , where:

- $R$  is a Petri net system,
- $IS : P \rightarrow Q^+ \times (Q^+ \cup \{+\infty\})$  such that  $IS_i = [a_i, b_i]$  with  $0 \leq a_i \leq b_i$  is the static interval associated to the place  $p_i$ .

The main aim is to elaborate a new model in order to reproduce the railway traffic behavior. Following the measures retrieved from TNRC's SCADA (Supervision Control And Data Acquisition) system, a P-TPN model ( $M$ ) was developed, as it can be seen in Figure 2.

In this model, the token denotes the railway traffic on the considered network and for each place;  $[L_{ij}, q_{ij}^e, H_{ij}]$  designate the lower time window boundary, the expected token residence time and the higher time window boundary, respectively. As the sojourn times in different places do not have the same functional meanings, based on Long & Descotes-Genon (1993) a split of the P-TPN model into four sets was done, as in Figure 2:

- $R_U$ : set of places representing used / occupied transport resources (track section),
- $R_N$ : set of places modelling unused / free transport resources (track section),
- $Trans_C$ : ensemble of places indicating the use/occupation of a route/path by a transport resource for passenger transfer,
- $Trans_{NC}$ : set of places containing unexploited transport resources (parking metro at main station).

This functional decomposition was performed in order to analyse the robustness of the railway transportation network. It is not intended to be a valid proof for all transportation systems. It can only be asserted that it corresponds to a suitable decomposition for numerous railway systems.

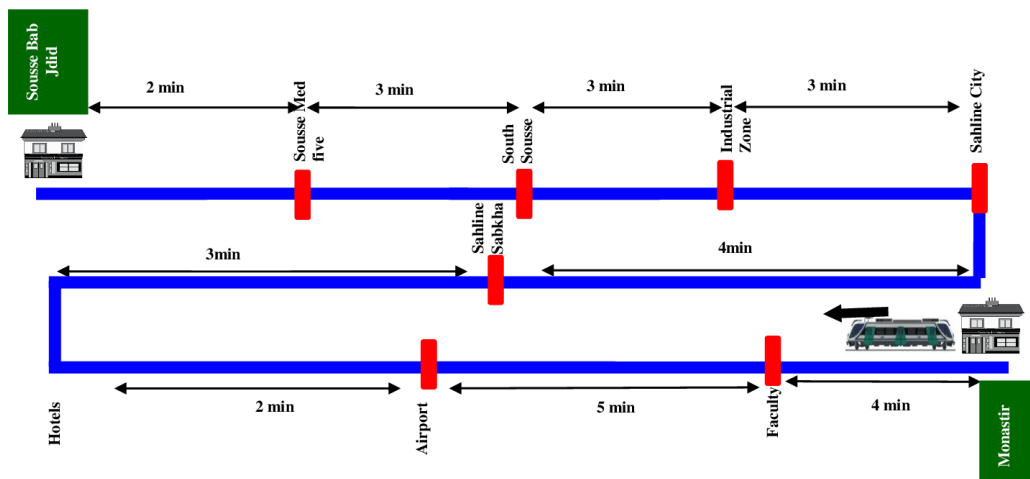


Figure 1. The Sahel Railway Network

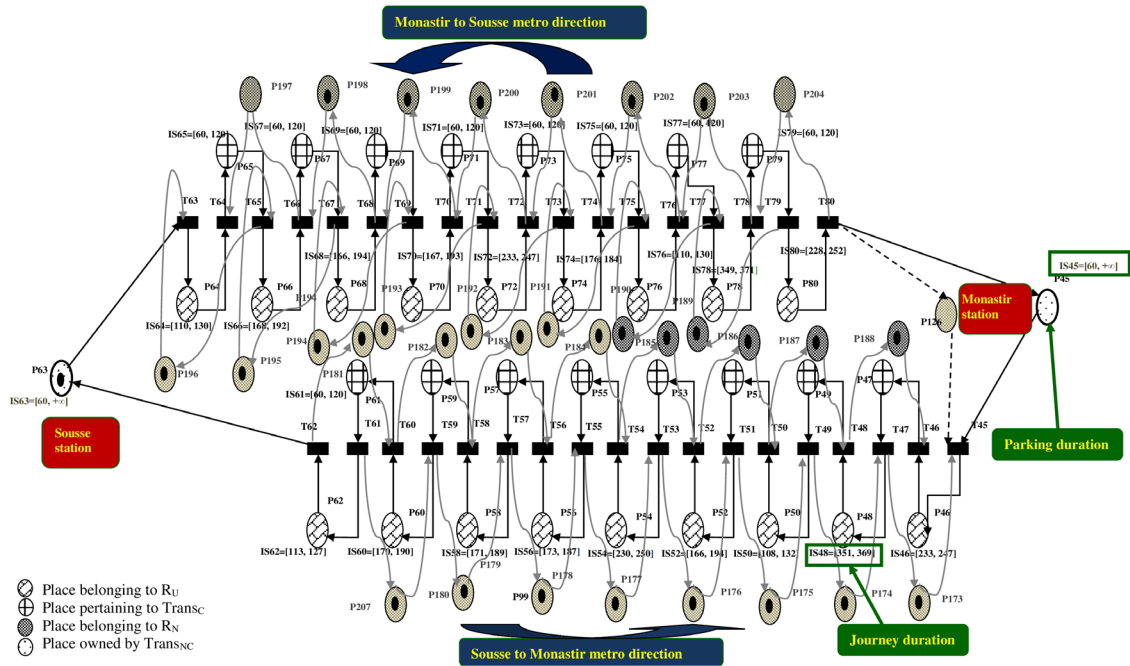


Figure 2. P-time Petri Net modeling of the Sahel railway network

## 4. Robustness Analysis for Transport Systems

### Notations

- $t_i^\circ$  (respectively, “ $\circ t_i$ ”) the output (respectively, the input) places of the transition  $t_i$ ;
- $p_i^\circ$  (respectively, “ $\circ p_i$ ”): the output transitions of the place  $p_i$  (respectively, the input transitions of the place  $p_i$ );
- $q_{ic}$ : the expected sojourn time of the token in the place  $p_i$  (computed by scheduling layer);
- $q_i$ : the effective sojourn time of the token in the place  $p_i$ ;
- $St_c(n)$ : the  $n^{\text{th}}$  expected firing instant of the transition  $t$ ;
- $IN(Sp_{th})$ : the first node of the path  $Sp_{th}$ ;  
 $OUT(Sp_{th})$ : the last node of the path  $Sp_{th}$ ;
- $T_c$ : the set of controllable transitions
- $T_o$ : the set of observable transitions
- $T_s$ : the set of synchronization transitions
- $St(n)$ : the  $n^{\text{th}}$  expected firing instant of the transition  $t$  (computed by scheduling layer);
- $C_{ms}$ : set of mono-synchronized subpaths

- $C_{se}$ : Set of elementary mono-synchronized subpaths.

### Basic definitions

**Definition 2:** A path ( $p_{th}$ ) in a P-time PN is defined as an orderly and oriented succession of places ( $p \in P$ ) and transitions ( $t \in T$ ), such as:  $Succ(t) = p$  and  $Succ(p) = t$  ( $Succ(k)$ : denotes the successor of  $k$  in the path  $p_{th}$ ).

**Definition 3:** (Mhalla et al., 2016): A mono-synchronized subpath  $Sp_{th}$  is a path that contains one and only one synchronizing transition as its last node. A disturbance  $\Omega$  is locally dismissed by a “ $p_{th}$ ” path if its final transition is fired as predicted.

**Definition 4:** Robustness is defined as a system’s ability to sustain its specified properties against anticipated or unexpected variations. Robustness refers to the system’s overall capability to withstand disturbances. A disturbance is defined as any unanticipated operational event that affects the performance of a generated schedule.

Based on the reaction type facing a disturbance, robustness can be of two types: Passive and Active robustness.

**Definition 5:** Passive robustness, refers to the case where no changes in the control system are required to preserve the properties specified by the technical requirements in presence of variations. By contrast,

active robustness is the robustness ensured by the temporal control of the process transitions. The Passive robustness is based upon variations included in static time intervals.

Using passive robustness, there is no control modification to preserve the required specifications. In this case, two concepts are introduced: compensable and transmissible margins. The compensable (respectively, the transmissible) margins denoted by  $\Psi_c$  (respectively  $\Psi_t$ ) on sub-path  $k$  ( $Sp_{thk}$ ) can be computed by using the following formulas :

$$Y_{Sp_{thk}} = Y_{c_{Sp_{thk}}} + Y_{t_{Sp_{thk}}} \quad (1)$$

with:

$$\Psi_{c_{Sp_{thk}}} = \sum_{p_i \in Sp_{thk} \cap (R_N \cup Trans_{NC})} (q_{ie} - L_i) \quad (2)$$

$$\Psi_{t_{Sp_{thk}}} = \min_{p_i \notin Sp_{thk}} (H_i - q_{ie}) \quad (3)$$

$$p_i^\circ = OUT(Sp_{thk})$$

The compensable and transmissible margins on the mono-synchronized subpath  $Sp_{thk}$  allow

to compensate the accrued delays and to keep the quality and safety of the railway traffic by preventing the disruptions occurring during the static intervals associated to the places of the P-TPN model (M). In the railway network under review, passive robustness preserves the railway network's operating frequency since it will be possible to control residual disturbances that arise as minor disruptions not affecting the railway traffic stability. The ultimate objective is to reduce the service quality degradation (traffic disruption, cumulative delay, passenger complaints etc.) and to preserve the railway network stability and safety.

The proposed approach for the passive robustness computation for a Railway Transport System: delay case.

### A/ Algorithm for computing the passive robustness margin

The following is an algorithm for computing a lower bound of the passive robustness margin of the analysed transport system at a node  $n$ .

#### Algorithm.

Let:  $S = \cup Sp_{thk}$  : union of the mono-synchronized subpaths,

$\Psi_{c_{Sp_{thk}}}$ : the compensable delay margin on the mono-synchronized subpath  $Sp_{thk}$

$Res(S, Sp_{thk})$ : Maximum transmissible disturbance residue in  $OUT(Sp_{thk})$  (disturbance spreading on a parallel path)

$M_0$ : Initial marking

$\varphi$ : the set of mono-synchronized subpaths

$n$ : the node whose temporal disturbance has appeared

$F$ : Set of elementary mono-synchronized subpaths whose the temporal disturbance residue spreads.

$$E(S, Sp_{thk}) = \{ S / (OUT(S) = OUT(Sp_{thk})) \wedge (\sum_{p \in S} M_0(p) = \sum_{p \in Sp_{thk}} M_0(p)) \wedge (\Omega \in S \wedge \Omega = IN(S)) \}$$

$$\varphi = \{ Sp_{thk} / (n^\circ = IN(Sp_{thk})) \wedge (Sp_{thk} \in C_{ms}) \wedge (Sp_{thk} \in M) \}$$

$$Margin \leftarrow \min_k [\Psi_{c_k} + F(M \setminus Sp_{thk}, OUT(Sp_{thk})^\circ, \min(H_i - q_{ie}) + \Omega_i)]$$

$$p_i^\circ = OUT(Sp_{thk}) \quad \text{Max}(Res(S, Sp_{thk}))$$

$$p_i^\circ \notin Sp_{thk} \quad S / (p_i \notin S) \wedge (p_i^\circ = OUT(Sp_{thk}))$$

$F(M^*, p^*, \Psi_t)$

{

$$\varphi^* = \{ Sp_{thk} / (p^* \in Sp_{thk}) \wedge (Sp_{thk} \in C_{se}) \wedge (Sp_{thk} \in M^*) \}$$

If  $(\varphi^* = \Phi$  or  $\Psi_t = 0)$  thus  $(F \leftarrow \Psi_t)$

Else

{

$$F \leftarrow \min_j \{ \min[\Psi_t, (\Psi_{c_k} + F(M^* \setminus Sp_{thk}, OUT(Sp_{thk})^\circ, \min(H_i - q_{ie}) + \Omega_i))] \}$$

$$p_i^\circ = OUT(Sp_{thk}) \quad \text{Max}(Res(S, Sp_{thk}))$$

$$p_i^\circ \notin Sp_{thk} \quad p_i \notin S \quad p_i^\circ = OUT(Sp_{thk})$$

}

}

## B/ Application

Let  $\Omega$  be a time disturbance in  $P_{77}$  observed in  $T_{77}$  as it can be seen in Figure 3. The disturbance  $\Omega$  is transmitted to the two paths  $Sp_{th12} = (P_{77}, T_{77}, P_{78}, T_{78})$  and  $Sp_{th13} = (P_{79}, T_{79}, P_{80}, T_{80}, P_{45}, T_{45})$  through the place  $P_{79}$  ( $S = Sp_{th12} \cup Sp_{th13}$ ). Table 1 gives the local passive robustness margins associated to each mono-synchronized subpath  $Sp_{thk}$ .

### Example

The delay passive robustness margin at  $T_{77}$  shall be computed.

According to Figure 3, and referring to the following algorithm:

$$\phi = \{Sp_{th12}; Sp_{th14}\}$$

$$\text{Margin} \leftarrow \min_k [\Psi c_k + F(M \setminus Sp_{thk}, \text{OUT}(Sp_{thk}^o, \min(H_i - q_i) + \Omega_i))] \\ p_i^o = \text{OUT}(Sp_{thk}^o) \quad \text{Max}(\text{Res}(S, Sp_{thk})) \\ p_i^o \notin Sp_{thk} \quad S / (p_i \notin S) \wedge (p_i^o = \text{OUT}(Sp_{thk}^o))$$

$$\text{Margin} \leftarrow \begin{cases} \min[3 + F(M \setminus Sp_{th12}, P_{79}, \min(H_{204} - q_{204e}) + \Omega_1)] \\ \min[4 + F(M \setminus Sp_{th14}, P_{77}, \min(H_{76} - q_{76e}) + \Omega_2)] \end{cases}$$

$$\text{Margin} \leftarrow \begin{cases} \min[3 + F(M \setminus Sp_{th12}, P_{79}, +\infty)] \\ \min[10 + F(M \setminus Sp_{th14}, P_{77}, 9 + \Omega_2)] \end{cases}$$

### Step 1: Computing of $F(M \setminus Sp_{th12}, p_{79}, +\infty)$

$$\phi^* = \{Sp_{th13} = (P_{79}, T_{79}, P_{80}, T_{80}, P_{45}, T_{45}), Sp_{th15} = (P_{79}, T_{79}, P_{80}, T_{80}, P_{204}, T_{78})\}$$

$$F \leftarrow \min \begin{cases} \min[+\infty, 18 + F(M \setminus Sp_{th12} \setminus Sp_{th13}, P_{46}, +\infty)] = +\infty \\ \min[+\infty, 13 + F(M \setminus Sp_{th12} \setminus Sp_{th15}, P_{79}, 17 + \Omega_3)] \end{cases}$$

### Step 1.1: Calculation of $F(M \setminus Sp_{th12} \setminus Sp_{th15}, P_{79}, 17 + \Omega_3)$

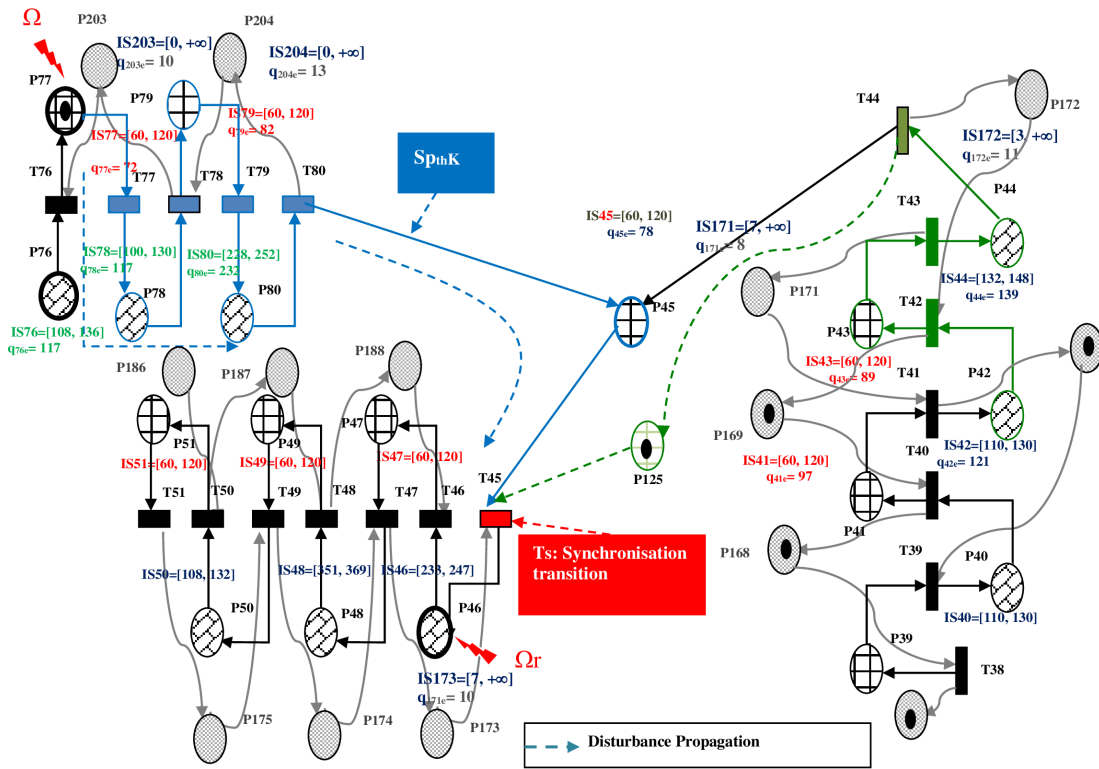


Figure 3. Disturbance propagation on the path (Sousse - Monastir)

Table 1. Local passive robustness margins

Elementary mono-synchronized subpath	PRd( $Sp_{thk}$ )	$\Psi t_{Sp_{thk}}$
$Sp_{th12} = (P_{77}, T_{77}, P_{78}, T_{78})$	3	$+\infty$
$Sp_{th13} = (P_{79}, T_{79}, P_{80}, T_{80}, P_{45}, T_{45})$	18	$+\infty$
$Sp_{th14} = (P_{77}, T_{77}, P_{78}, T_{78}, P_{203}, T_{76})$	10	9
$Sp_{th15} = (P_{79}, T_{79}, P_{80}, T_{80}, P_{204}, T_{78})$	13	17
$Sp_{thL} = (P_{42}, T_{42}, P_{43}, T_{43}, P_{44}, T_{44}, P_{125}, T_{45})$	18	$+\infty$
$Sp_{th16} = (P_{42}, P_{42}, P_{43}, T_{43}, P_{171}, T_{41})$	1	37
$Sp_{th17} = (P_{44}, T_{44}, P_{112}, T_{42})$	8	9

Therefore, the delay passive robustness margin at  $T_{77}$ :  $\text{Margin} \leftarrow 17 + \Omega_{c3}$ ;  $\text{Margin} \leftarrow 34$  as  $\Omega_2 \leq 9$ .

### Physical interpretation

It is easy to check if the computed passive robustness margin corresponds to the case when no changes are required to preserve the transport system specifications against disturbances.

In fact, a disturbance related to the computed passive robustness margin cannot lead to a degraded service (traffic perturbation, cumulative delay, travelers filling claims etc.) and will not influence the stability and the security of the selected networks. In the analysed case, it is possible to get on the metro leaving from Monastir station on time by minimizing the time when the metro is stopped at different stations.

### Remarks

- The margin calculated by the employed algorithm is a lower bound of the effective passive robustness margin. Holding a lower bound allows the detection of time constraint violations, however, a failure cannot be proven when the disturbance exceeds the lower bound;
- The computational algorithm builds a passive robustness lower bound at a node  $n$ . The ability of the algorithm to provide an upper limit would prevent false alarms (by computing the effective maximal margin).

## REFERENCES

- Bosso, N., Magelli, M. & Zampieri, N. (2022) Monitoring Systems for Railways Freight Vehicles. In: *Proceedings of the 18<sup>th</sup> International Conference on Railway Engineering Design & Operation, COMPRAIL 2022, 21–23 September, 2022, Valencia, Spain*.
- Cats, O. & Krishnakumari, P. (2020) Metropolitan rail network robustness. *Physica A: Statistical Mechanics and its Applications*. 549, 124317. doi: 10.1016/j.physa.2020.124317.
- Cats, O., Krishnakumari, P. & Tundulyasaree, K. (2019) Rail network robustness: The role of rapid development and a polycentric structure in withstanding random and targeted attacks. In: *Proceedings of the Transportation Research Board (TRB) 98th Annual Meeting, 13-17 January, 2019, Washington, DC, USA*. Transportation Research Board (TRB). pp. 13-17.
- Ge, L., Voß, S. & Xie, L. (2022) Robustness and disturbances in public transport. *Public Transport*. 14(1), 191-261. doi: 10.1007/s12469-022-00301-8.
- International Organization for Standardization (ISO). (2013) *Enterprise-control systems integration, Part 2: Objects and attributes for enterprise-control system integration*. <https://www.iso.org/fr/standard/57310.html> [Accessed 4th May 2023].
- Khansa, W., Denat, J. P. & Collart-Dutilleul, S. (1996) P-Time Petri Nets for Manufacturing Systems. In: *Proceedings of the IEEE Workshop on Discrete Event*

## 5. Conclusion

This paper proposed a local semantic interpretation of the classical model of a manufacturing site with maximum time constraints. It allows using the definition of the passive robustness in relation to the input of a path presented in previous works for a metro line. To compute a lower bound of the passive robustness margin, an algorithm was proposed. Holding a lower bound provided the ability to filter false alarms when the disturbance did not reach the lower bound, or to transmit the information that a time disturbance may be critical to the transport management.

The computed passive robustness lower bound increases a system's ability to sustain its specified qualities without any implemented action (no behavior adaptation is needed). The disadvantage of the implemented algorithm is that it doesn't permit to define a value corresponding to a non-acceptable time disturbance, as only a lower bound is used.

A validation for a real railway transport system shows the efficiency of these algorithms for computing the passive and active robustness margins.

In the future, a comparative study based on several similar cases could be developed. The integration of stochastic time durations will be considered.

Moreover, future works could target the integration of the maintenance layer into the global control and supervision of a time-critical railway network, integrating both the dedicated railway approach (Nicola et al., 2022) and the global normative framework of the concepts of industry 4.0.

- Systems, WODES'96, 19-21 August, 1996, Edinburgh, Scotland, UK*. Institute of Electrical Engineers, Computing and Control Division. pp. 94-102.
- Long, J. & Descotes-Genon, B. (1993) Flow optimization method for control synthesis of flexible manufacturing systems modeled by controlled timed Petri nets. In: *Proceedings of the IEEE International Conference on Robotics and Automation, 2-6 May, 1993, Atlanta, GA, USA*. pp. 598-603.
- Mhalla, A., Dutilleul, S. C. & Zhang, H. (2016) Robust control under uncertainty for seaport handling equipments. *Transportation Research Procedia*. 14, 203-212. doi:10.1016/j.trpro.2016.05.056.
- Mhalla, A. & Gaied, M. (2018) Modeling and robustness study of railway transport networks using P-timed Petri nets. *Journal of Engineering*. 2018(1), 1-12. doi:10.1155/2018/2083576.
- Müller-Hannemann, M., Rückert, R., Schiewe, A. & Schöbel, A. (2022) Estimating the robustness of public transport schedules using machine learning. *Transportation Research Part C: Emerging Technologies*. 137, 103566. doi:10.1016/j.trc.2022.103566.
- Restel, F. & Haładyn, S. M. (2022) The Railway Timetable Evaluation Method in Terms of Operational Robustness against Overloads of the Power Supply System. *Energies*. 15(17), 6458. doi:10.3390/en15176458.
- Wang, Z., Yan, H., Huang, Y. & Li, Y. (2019) Robustness assessment of China's high-speed railway transportation network. In: H. Fujita (ed.) *Abstracts of the ICA, 1 (29th International Cartographic Conference, ICC 2019, 15-20 July 2019, Tokyo, Japan*. Göttingen, Germany, Copernicus Publications.
- Zhang, Y. & Ng, S. T. (2022) Robustness of urban railway networks against the cascading failures induced by the fluctuation of passenger flow. *Reliability Engineering & System Safety*. 219, 108227. doi:10.1016/j.res.2021.108227.