# Application of DOA Estimation Algorithms in Smart Antennas Systems

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**Abstract:** Concept of wireless communication systems which use smart antennas is based on digital signal processing algorithms. Thus, the smart antennas systems become capable to locate and track signals by both: users and interferers and dynamically adapts the antenna pattern to enhance the reception in Signal-Of-Interest direction and minimizing interference in Signal-Of-Not-Interest direction. Hence, Space Division Multiple Access system, which uses smart antennas, is being used more often in wireless communications, because it shows improvement in channel capacity and co-channel interference. However, performance of smart antenna system greatly depends on efficiency of digital signal processing algorithms. The algorithm uses the Direction of Arrival (DOA) algorithms to estimate the number of incidents plane waves on the antenna array and their angle of incidence. This paper investigates performance of the DOA algorithms such as MUSIC, ESPRIT and ROOT MUSIC on the uniform linear array in the presence of white noise. The simulation results show that MUSIC algorithm is the best. The resolution of the DOA techniques improves as number of snapshots, number of array elements and signal-to-noise ratio increases.

**Keywords:** smart antenna, DOA, MUSIC, ESPRIT, ROOTMUSIC, SDMA, adaptive beamforming.

## 1. Introduction

The high demand on the usage of the wireless communication system calls for higher system capacities. The system capacity can be improved either by enlarging its frequency bandwidth or allocating new portion of frequency spectrum to wireless services. But since the electromagnetic spectrum is a limited resource, it is not easy to get new spectrum allocation without the international coordination on the global level. One of the approaches is to use existing spectrum more efficiently, which is a challenging task. Efficient source and channel coding as well as reduction in transmission power or transmission bandwidth or both are possible solutions to the challenging issue. With the advances in digital techniques, the frequency efficiency can be improved by multiple access technique (MAT), which gives mobile users access to scarce resource (base station) and hence improves the system's capacity [1]. Family of existing Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA) can be enlarged by adding a new parameter 'space' or 'angle' [2], which results in MAT known as 'Space Division Multiple Access' (SDMA). At the receiver's side, the transmitted signal is received with its multipath components plus interferers' signal, as well as with present noise. Thus, detection of the desired signal is a challenging task. The Smart Antenna System (SAS) employs the antenna elements and the digital signal processing which enables it to form a beam to a desired direction taking into account the multipath signal components. In this way, Signal-to-Interference-and-Noise Ratio (SINR) improves by producing nulls towards the interferers in the direction of Signal-Of–Not-Interest (SONI) [3].The performance of SAS greatly depends on the performance on DOA estimation.

The subspace based DOA estimation algorithms MUSIC, ESPRIT and ROOTMUSIC provide high resolution, are more accurate and are not limited to physical size of array aperture [2][5]. In this paper we are investigating the performance of MUSIC, ESPRIT and ROOT MUSIC algorithms. The performance of these algorithms is analysed by considering parameters like number of array elements, user space distribution, number of snapshots, signal to noise ratio, Mean Square Error (MSE), which results in optimum array design in SAS.

The conclusions of the work are drawn from the simulated results using MATLAB.

## 2. SAS and SDMA

SAS is a self organising system, which can locate and track signals, dynamically adjust the antenna pattern to enhance a reception, while minimizing interference using signal processing algorithms [3]. SDMA with SAS can generate multiple beampatterns: each is assigned to one user, improving frequency reuse capability and increase in channel capacity.

In digital beamforming antenna systems, the signals are detected and digitized at element level. RF signals at each antenna element are downconverted to baseband signals of two components: I (amplitude) and Q (phase) information. It locates the Signal-Of-Interest (SOI) using DOA algorithm. DOAs of all the signals are computed by calculating time delays between antenna elements. In the next step, it is fed to adaptive algorithm which uses cost (error) function for calculating the optimum filter weights that generate an array factor for an optimal signal-to-interference ratio (SIR).

Specifically, this results in an array pattern where ideally the maximum of the pattern is placed toward the source or SOI, while nulling interferers of SNOIs,[1][3].

Estimation of DOA entirely depends on the performance of selected DOA algorithm.

# 3. DOA Estimation Algorithm

The DOA algorithms are classified as quadratic type and subspace type [4]. The Barltett and Capon (Minimum Variance Distortionless Response) [4] are quadratic type algorithms. Bartlett method is an extension of classical Fourier Transform based spectrum analysis. It maximises the power of beam forming output for a given input signal.

Capon‘s method attempts to minimize the power contributed by noise and any signals coming from other direction than desired. The Capon’s perform well with respect to Barlett [4]. The both methods involve evaluation of spectrum and then finding the local maxima which gives the estimation of DOA. The both methods are highly dependent on physical size of array aperature, which results in poor resolution and accuracy, [3] [5] [8] [10].

Subspace based DOA estimation method is based on the eigendecomposition [11] [12] [13]. The observed covariance matrix is decomposed into two orthogonal spaces: signal and noise. The DOA estimation is calculated from any one of the subspaces [6]. The subspace based DOA estimation algorithm MUSIC and ESPRIT provide high resolution, they are more accurate and not limited to physical size of array aperture [2] [5].

The various DOA algorithm performance is analysed based on number of snapshots, number of users, user space distribution, number of array elements, SNR and MSE.

**Assumptions:** In order to make DOA problem practically traceable, the transmission medium is assumed to be isotropic and non-dispersive. The sources are in the far-field of the array. The resulting radiation incident wave to the array is in the form of a sum of plane waves. Location parameter space is reduced to a single dimensional subset from $\theta_i \in (-\pi, \pi)$. The Uniform Linear Array (ULA) is considered with number of signals, D, number of array elements, M, and, wavelength, λ. The number of signals is smaller than number of array elements. The each channel noise, which is a white noise, is non-coherent with respect to each signal and narrowband with the same known center frequency, $f_0$.

## 3.1 MUSIC

MUSIC stands for **MU**ltiple **SI**gnal **C**lassification, one of the high resolution subspace DOA algorithms, which gives the estimation of number of signals arrived, hence their direction of arrival [4]. MUSIC deals with the decomposition of covariance matrix into two orthogonal matrices, i.e., signal-subspace and noise-subspace. Estimation of DOA is performed from one of these subspaces, assuming that noise in each channel is highly uncorrelated. This makes the covariance matrix diagonal. The covariance matrix is given by:

$$S_x = F(\theta) S_S\, F(\theta)^H + \sigma_w^2\, \mathrm{I} \tag{1}$$

where $F(\theta) = [f(\theta_1) : f(\theta_1) : \ldots : f(\theta_D)]$ is a MxD array steering matrix, $\sigma_w^2$ is noise variance and I is an identity matrix of size MxM.

Writing the spatial covariance matrix in terms of eigenvalues and eigenvectors [1] gives:

$$S_x = \sum_{i=1}^{M} P_i \phi_i \phi_i^H \tag{2}$$

The noise subspace eigenvalues and eigenvectors are:

$$p_i = \quad i = D+1, D+2, ..., M \tag{3}$$

$$\phi i = \quad i = D+1, D+2, ..., M \tag{4}$$

The noise subspaces can be written in the form of $Mx(M-D)$ matrix:

$$\cup_N = [\phi_{D+1,}\phi_{D+2,}...,\phi_M] \tag{5}$$

Equation (5) indicates that we can find out the desired value DOA of $\theta_1$, $\theta_2$,…, $\theta_D$ by finding a set of vectors that span $\cup_N$ and projecting arraymanifold matrix $f(\theta)$ onto $\cup_N$ for all values of $\theta$ and evaluating the D values of $\theta$, where the projection is zero:

$$\left\| f_i^H \cup_N \right\|^2 = 0 \quad i = 0, 1, ..., D \tag{6}$$

The MUSIC Pseudospectrum is given as:

$$P_{mu}(\theta) = \frac{1}{abs[F(\theta)^H \cup_N \cup_N^H F(\theta)]} \tag{7}$$

### 3.2 ESPRIT

Its acronym stands for **Estimation of Signal Paramter via Rotational Invariance Technique.** This algorithm is more robust with respect to array imperfections than MUSIC [9], [14], [15]. Computation complexity and storage requirements are lower than MUSIC as it does not involve extensive search throughout all possible steering vectors. But, it explores the rotational invariance property in the signal subspace created by two subarrays derived from original array with a translation invariance structure. It is based on the array elements placed in identical displacement forming matched pairs, with M array elements, resulting in m=M/2 array pairs called “doublets”.

Computation of signal subspace for the two subarrays, $P_1$ and $P_2$, results in two vectors $V_1$and $V_2$, such that Range [S] = Range[$B$]. Also, there should exist a non-singular matrix T of DxD such that $V_S = \overline{B}T$, where $V_S$ can be decomposed into $V_1$and $V_2$:

$$V_1 = BT, V2 = B\varphi T \tag{8}$$

$$\varphi = diag[e^{jkd\sin(\theta_1)}, e^{jkd\sin(\theta_1)}, ..., e^{jkd\sin(\theta_D)}] \tag{9}$$

DxD is diagonal, unitary matrix with phase shifts betwen doublets for each DOA, there exists a unique rank D matrix F∈C such that:

$$[V_1|V_2]F = V_1W_1 + V_2W_2 = BTW_1 + B\varphi TW_1 = 0 \tag{10}$$

Rearranging equation (10), we get:

$$BT\Psi = B\varphi T \tag{11}$$

where $\psi = -F_1F_2^{-1}$

With B as full rank and sources having distinct DOA, then $\Psi$ becomes:

$$\Psi = T^1\varphi T \tag{12}$$

Equation (11) indicates that if we are able to find out eigenvalues of $\Psi$, which are diagonal elements of $\varphi$, we can estimate DOA as $\varphi = (a_1, a_2, ..., a_D)$ where

$$a_i = e^{jkd\sin(\theta_i)} \quad i = 1, 2, ..., D \tag{13}$$

DOA can be calculated by:

$$\theta_i = \sin^{-1}\left[\frac{\arg(a_i)}{kd}\right] \tag{14}$$

### 3.3 ROOTMUSIC

The MUSIC spectrum is an all pole function of the form

$$P_{mu}(\theta) = \frac{1}{abs[F(\theta)^H \cup_N \cup_N^H F(\theta)]} \tag{15}$$

Let $C = \cup_N \cup_N^H$ using equation (15) may be written as:

$$p_{mu}^{-1} = \sum_{m=1}^{M}\sum_{n=1}^{M} \exp^{(j(m-1)2\pi d\sin(\theta_b)/\lambda)} C_{mn}A \tag{16}$$

where $A = \exp^{(-j(m-1)2\pi d\sin(\theta_b)/\lambda)}$, and $C_{mn}$ is the entry in the m^th^ row and n^th^ column of C. Combination of two sums into one gives equation (17):

$$p_{mu}^{-1} = \sum_{n=1}^{M} C_l \exp^{(-j2\pi dl\sin(\theta_b)/\lambda)} \tag{17}$$

where $C_l = \sum_{m-n=l} C_{mn}$ is the sum of the entries of C. Along the $l^{\text{th}}$ diagonal polynomial representation D(z) will be:

$$D(z) = \sum_{l=-M+1}^{M+1} C_l z^{-1} \tag{18}$$

If the eigendecomposition corresponds to the true spectral matrix, then MUSIC spectrum $P_{mu}(\theta)$ becomes equivalent to the polynomial D(z) on the unit circle and peaks in the MUSIC spectrum exists as ROOTs of the D(z) lie close to the unit circle [4]. A pole of D(z) at $z=z_l=|z_l| \exp(j\arg(z_l))$ will result in a peak in the MUSIC spectrum at

$$\theta = \sin^{-1}(\{\lambda / 2\pi d\}\arg[z_l]) \tag{19}$$

# 4. Simulation Results

The MUSIC, ROOTMUSIC & ESPRIT techniques for DOA estimations are simulated using MATLAB. Performance of the algorithm has been analyzed by considering Mean Squared Error (MSE) for 50 trials as a function of array elements, as a function of SNR and as a function of snapshots. The simulation has been run for four signals coming from different angles $14^0$, $24^0$, $35^0$, $55^0$, with 500 snapshots (n), with SNR of 10dB, and with array size of 16 (M).

## 4.1 MUSIC spectrum for varying number of array elements

MUSIC spectrum for varying number of array elements is shown in Figure 1. It indicates that as array size increases the peaks of the spectrum become sharper and hence increases resolution capability of MUSIC.

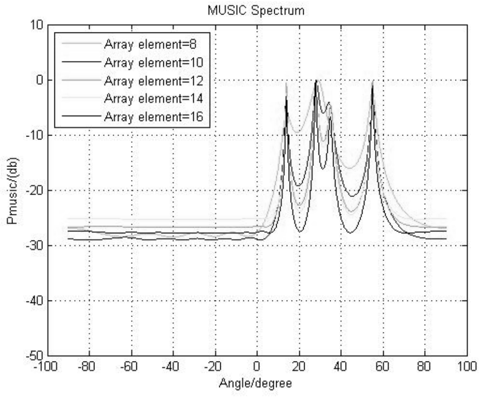

**Figure 1.** MUSIC spectrum for varying number of array elements

## 4.2 MUSIC spectrum for varying SNR

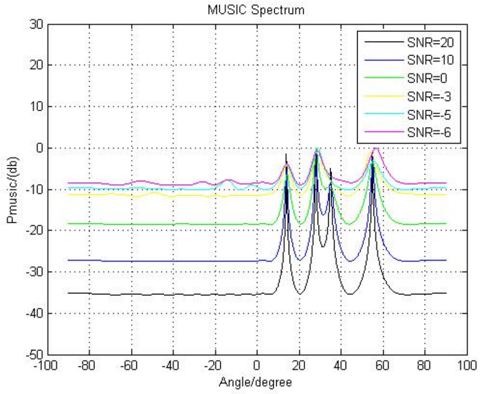

**Figure 2.** MUSIC spectrum for varying SNR

Figure 2 indicates that as SNR value decreases, peaks in spectrum start to disappear and hence decreases resolution capability of MUSIC for closely spaced signals like $28^0$ and $35^0$.

## 4.3 MUSIC spectrum for varying number of snapshots

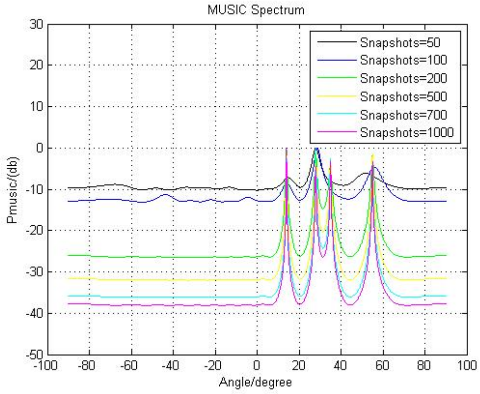

**Figure 3.** MUSIC spectrum for varying number of snapshots

Figure 3 indicates the ability of MUSIC to resolve closely spaced signals $28^0$ and $35^0$ as a function of number of snapshots. As snapshots increased from 50 to 200 resolution capability of MUSIC increases, we can clearly identify these two signals. Peaks in the spectrum become further sharper for snapshots 500, 700 and 1000.

### 4.4 DOA Estimation for varying Number of snapshots (M=8, SNR=10 dB, four signals: $14^0$, $28^0$, $35^0$, $55^0$)

Table 1 indicates that at 200 snapshots MUSIC cannot resolve the closely spaced signals $28^0$ and $35^0$.

**Table 1.** DOA estimation for array size 8.

| **Snapshots** | **MUSIC** |
|---|---|
| 200 | $-48^0$ , $14.6^0$, $27.6^0$, $54.1^0$ |
| 1000 | $14.2^0$, $27.9^0$, $34.6^0$, $54.9^0$ |
| **Snapshots** | **ESPRIT** |
| 200 | $13.98^0$, $29.90^0$, $37.28^0$, $56.42^0$ |
| 600 | $14.00^0$, $28.20^0$, $35.82\ ^0$, $55.27^0$ |
| **Snapshots** | **ROOTMUSIC** |
| 200 | $-19.46^0$, $16.70^0$, $28.24^0$, $58.89^0$ |
| 2000 | $-20.59^0$, $13.97^0$, $30.71^0$, $55.28^0$ |

It requires 1000 snapshots to resolve these two signals with array size of 8. ESPRIT requires 600 snapshots to resolve $28^0$ and $35^0$ signals accurately and ROOTMUSIC takes more than 2000 snapshots to resolve these two signals correctly.

### 4.5 Performance Analysis of MUSIC, ESPRIT and ROOT MUSIC: DOA estimation for varying number snapshots (SNR=10dB, array size=16)

Tables 2 to 4 indicate that for 200 snapshots MUSIC gives an accurate estimation for four signals. If the number of snapshots increases to 1000, peaks in the spectrum become sharper and deeper as shown in Fig. 4, and thus means improvement of the resolution capability of MUSIC. ESPRIT and ROOT MUSIC identify the four signals, but the MSE is close, but not exactly zero. For snapshot value of 100, both MUSIC and ESPRIT fail to detect closely spaced signals $28^0$ and $35^0$ , but ROOT MUSIC identifies them very well: typical value is $27.13^0$ and $35.28^0$ respectively. Table 4 reveals that MSE by MUSIC for varying number of snapshots from 200 to 1000 is zero for $14^0$ and $28^0$, and almost zero for $35^0$ and $55^0$ . For closely spaced signals $28^0$ and $35^0$ MUSIC gives MSE zero at snapshots 700 and 1000 compared to other two techniques (Figures 4 to 6).

**Table 2.** DOA estimation by MUSIC.

| **DOA** | **n=200** | **n=500** | **n=700** | **n=1000** |
|---|---|---|---|---|
| 14 | 14 | 14 | 14 | 14 |
| 28 | 28 | 28 | 28 | 28 |
| 35 | 35 | 34.9 | 35 | 35 |
| 55 | 55 | 55 | 55.1 | 55 |

**Table 3.** DOA estimation by ESPRIT.

| **DOA** | **n=200** | **n=500** | **n=700** | **n=1000** |
|---|---|---|---|---|
| 14 | 14.01 | 14.00 | 14.04 | 14.03 |
| 28 | 27.85 | 27.97 | 27.95 | 28.00 |
| 35 | 34.60 | 34.96 | 34.95 | 34.93 |
| 55 | 55.21 | 55.02 | 55.06 | 55.03 |

**Table 4.** DOA estimation by ROOT MUSIC.

| **DOA** | **n=200** | **n=500** | **n=700** | **n=1000** |
|---|---|---|---|---|
| 14 | 14.06 | 14.01 | 14.04 | 13.99 |
| 28 | 27.97 | 28.09 | 28.01 | 28.05 |
| 35 | 35.1 | 34.99 | 34.92 | 34.93 |
| 55 | 55.02 | 54.91 | 54.99 | 55.03 |

### 4.6 MSE for varying number snapshots

Tables 5 to 7 reveal that MSE for MUSIC for varying number of snapshots from 200 to 1000 is zero for $14^0$ and $28^0$. For closely spaced signals $28^0$ and $35^0$ MUSIC gives MSE zero at snapshots 700 and 1000 compared to other two techniques. Figures 4 to 6 indicate this fact.

**Table 5.** MSE for DOA Estimation by MUSIC.

| **DOA** | **n=200** | **n=500** | **n=700** | **n=1000** |
|---|---|---|---|---|
| 14 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 28 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 35 | 0.0000 | 0.0002 | 0.0000 | 0.0000 |
| 55 | 0.0000 | 0.0000 | 0.0002 | 0.0000 |

**Table 6.** MSE for DOA Estimation by ESPRIT.

| **DOA** | **n=200** | **n=500** | **n=700** | **n=1000** |
|---|---|---|---|---|
| 14 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 28 | 0.0004 | 0.0000 | 0.0000 | 0.0000 |
| 35 | 0.0031 | 0.0000 | 0.0000 | 0.0001 |
| 55 | 0.0010 | 0.0000 | 0.0001 | 0.0000 |

**Table 7.** MSE for DOA Estimation by ROOT MUSIC.

| DOA | n=200 | n=500 | n=700 | n=1000 |
|---|---|---|---|---|
| 14 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 28 | 0.0000 | 0.0002 | 0.0000 | 0.0001 |
| 35 | 0.0002 | 0.0000 | 0.0001 | 0.0001 |
| 55 | 0.0000 | 0.0001 | 0.0000 | 0.0000 |

## 4.7 DOA estimation for varying number of array elements (SNR=10dB, snapshots=200)

Tables 8 to 10 indicate that the MUSIC can identify closely spaced signals at array size of 14. ESPRIT identifies $28^0$ and $35^0$ at array size 16. ROOTMUSIC also identifies at the same array size of 16 as for these values MSE is less.

**Table 8.** DOA estimation by MUSIC.

| DOA | M=10 | M=12 | M=14 | M=16 |
|---|---|---|---|---|
| 14 | 13.6 | 14 | 14 | 14 |
| 28 | 28.3 | 27.8 | 28 | 28 |
| 35 | 33.6 | 34.1 | 35 | 35 |
| 55 | 54.9 | 55.1 | 55.1 | 55.1 |

**Table 9.** DOA estimation by ESPRIT.

| DOA | M=10 | M=12 | M=14 | M=16 |
|---|---|---|---|---|
| 14 | 14.08 | 14.17 | 14.06 | 13.76 |
| 28 | 28.46 | 27.70 | 27.76 | 28.04 |
| 35 | 36.17 | 34.71 | 34.98 | 34.68 |
| 55 | 54.56 | 54.95 | 55.13 | 55.09 |

**Table 10.** DOA estimation by ROOT MUSIC.

| DOA | M=10 | M=12 | M=14 | M=16 |
|---|---|---|---|---|
| 14 | 13.75 | 13.82 | 14.00 | 13.97 |
| 28 | 28.42 | 27.82 | 27.93 | 27.89 |
| 35 | 34.51 | 35.36 | 34.83 | 35.10 |
| 55 | 55.45 | 55.19 | 55.76 | 54.75 |

## 4.8 MSE varying number of array elements

Table 11 to 13 gives idea about MSE for three algorithms. For closely spaced signals coming at $28^0$ and $35^0$, MSE by MUSIC is zero with array size 14 and 16. To identify DOA $28^0$ and $35^0$ with array size 16, ESPRIT gives MSE as 0 and 0.0019, respectively. With array size 16, ROOTMUSIC gives MSE as 0.0002 to identify DOA $28^0$ and $35^0$. Figs. 8. to 10. clearly reveal the above fact.

**Table 11**. MSE by MUSIC.

| DOA | M=10 | M=12 | M=14 | M=16 |
|---|---|---|---|---|
| 14 | 0.0032 | 0.0000 | 0.0000 | 0.0000 |
| 28 | 0.0018 | 0.0008 | 0.0000 | 0.0000 |
| 35 | 0.0392 | 0.0162 | 0.0000 | 0.0000 |
| 55 | 0.0002 | 0.0002 | 0.0002 | 0.0002 |

**Table 12.** MSE by ESPRIT.

| DOA | M=10 | M=12 | M=14 | M=16 |
|---|---|---|---|---|
| 14 | 0.0002 | 0.0006 | 0.0001 | 0.0011 |
| 28 | 0.0044 | 0.0018 | 0.0011 | 0.0000 |
| 35 | 0.0278 | 0.0017 | 0.0000 | 0.0019 |
| 55 | 0.0038 | 0.0000 | 0.0004 | 0.0002 |

**Table 13.** MSE by ROOTMUSIC.

| DOA | M=10 | M=12 | M=14 | M=16 |
|---|---|---|---|---|
| 14 | 0.0012 | 0.0006 | 0.0000 | 0.0000 |
| 28 | 0.0036 | 0.0006 | 0.0001 | 0.0002 |
| 35 | 0.0048 | 0.0027 | 0.0005 | 0.0002 |
| 55 | 0.0041 | 0.0007 | 0.0011 | 0.0012 |

## 4.9 DOA estimation for varying number of SNR

Table 14 reflects the performance of algorithms for different values of SNR. As SNR decreases, the resolution capability of algorithm decreases, as well. MUSIC performs well in identifying signal even though SNR value is poor (-6dB).

**Table 14.** DOA estimation by MUSIC,ESPRIT and ROOT MUSIC for $35^0$.

| TYPE | SNR= 10dB | SNR= 0 dB | SNR= -6dB | SNR= -10dB |
|---|---|---|---|---|
| MUSIC | 35.00 | 35.00 | 34.10 | 34 |
| ESPRIT | 35.11 | 35.74 | 34.99 | 31.56 |
| ROOT MUSIC | 34.97 | 35.26 | 35.32 | 36.6 |

ESPRIT identifies the signal (for SNR value of -6dB), but still error is present. ROOTMUSIC identifies the signal well (SNR value -6dB) compared to ESPRIT. This reflects that if SNR decreases further, the MUSIC performs better than ESPRIT and ROOTMUSIC.

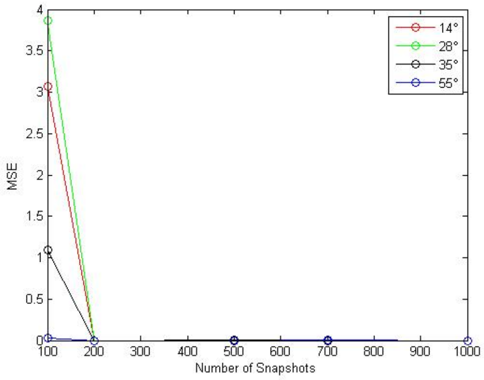

**Figure4.** MSE by MUSIC as a function of snapshots

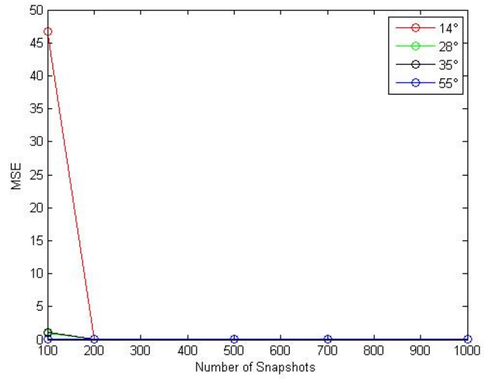

**Figure 5.** MSE by ESPRIT as a function of snapshots

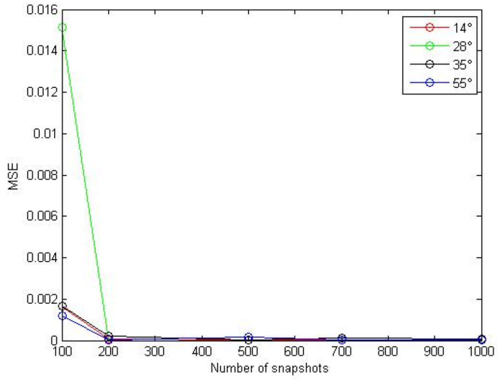

**Figure 6.** MSE by ROOT MUSIC as a function of snapshots

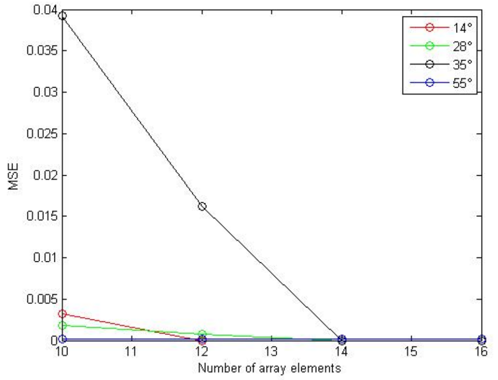

**Figure 7.** MSE by MUSIC as a function of array elements.

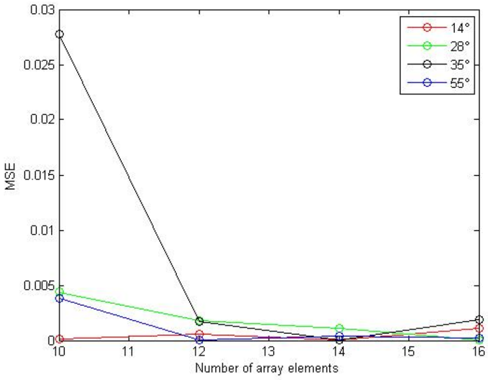

**Figure 8.** MSE by ESPRIT as a function of array elements

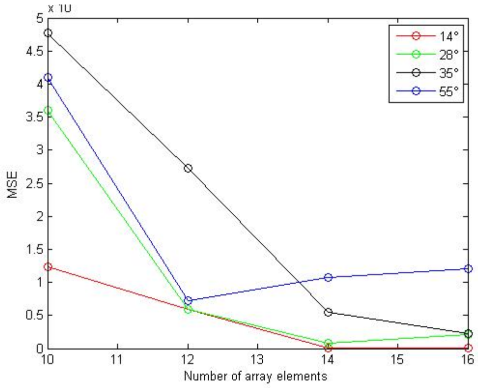

**Figure 9.** MSE by ROOTMUSIC as a function of array elements

## 5. Conclusions

This paper presents the results of direction of arrival estimation using MUSIC, ESPRIT and ROOTMUSIC algorithms. These three methods have greater resolution and accuracy and hence these are investigated much in detail. The simulation results show that performance of MUSIC, ESPRIT and ROOTMUSIC improves with more elements in the array, with higher number of snapshots of signals and greater angular separation between the signals. These improvements are analysed in the form of sharper peaks in MUSIC spectrum and smaller errors in angle detection. Tables 5 to 7 indicate that as number of snapshots increases, the MSE decreases which results in an accurate detection of closely spaced signals. For MUSIC, the ideal value of snapshot is 700 which give MSE as zero. Table 14 reflects the degradation of performance of ESPRIT and ROOT MUSIC as SNR values decrease, which gives higher MSE as compared to MUSIC.

Clearly, MUSIC is more stable and accurate and provides high resolution even at lower value of SNR. This adds new possibility of user separation through SDMA and can be widely used in the design of smart antennas systems.

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