State Estimation via Observers with Unknown Inputs: Application to a Particular Class of Uncertain Takagi-Sugeno Systems

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Abstract: This paper deals with the design of a multiple observer allowing estimating the state vector of a nonlinear system described by a Takagi-Sugeno multiple model subject to modelling and input uncertainties which are considered as unknown inputs. The main contribution of the paper is the conception of a multiple observer based on the elimination of these unknown inputs. Convergence conditions are established in order to guarantee the convergence of the state estimation error. These conditions are expressed in Linear Matrix Inequality (LMI) formulation. An example of simulation is given to illustrate the proposed method.

Keywords: Multiple model approach, Takagi-Sugeno models, multiple observer, unknown inputs, state estimate, modelling and input uncertainties.

1. Introduction

State estimation plays a significant role in the context of monitoring and/or of diagnosis of systems. It is an analytical source of redundancy used to generate failure symptoms of the system by making a comparison between the real behaviour signals of the system and the estimated signals. A non desired variation between these signals indicates the possible presence of faults affecting the system. These faults indicators are named residues. Their generation is based on the use of the state observers.

A state observer is a dynamical system allowing the state reconstruction from the system model and the measurements of its inputs and outputs [20]. In fact, the observer controlled by the same inputs applied to the system is able to provide the same output signals provided that the model employed reproduces with precision the behaviour of the system to be supervised. The observer design can be delicate according to the type and the complexity of the considered model.

Two types of models are distinguished according to the linear or non linear character of the system. Linear models have simple structures. They are the base of several applications and research works.

In the case of linear systems, observers can be designed for uncertain systems with time-delay perturbations [8] and unknown input systems [7]. Several researches were achieved concerning the state estimation in the presence of both known and unknown inputs [26], [29], [7]. These works can be gathered into two categories. The first one supposes an a priori knowledge of information on these non measurable inputs; in particular, Johnson [17] proposes a polynomial approach and Meditch [21] suggests approximating the unknown inputs by the response of a known dynamic system. The second category proceeds either by estimation of the unknown inputs [18], [19], or by their complete elimination from the system equations [11].

However, in the majority of real cases the nonlinear nature of the process cannot be neglected. The assumption of linearity is checked only locally around an operating point. Real physical processes present complex behaviours with nonlinear laws. As, it is delicate to synthesize an observer for a nonlinear system, the multiple model approach constitutes a tool which is largely used in the modelling of nonlinear systems [22], [6].

The principle of the multiple model approach is based on the reduction of the system complexity by the decomposition of its operating space in a finite number of operating zones. Each zone is characterized by a local model (named also, sub-model). Each sub-model is a simple and linear system around an operating point. The relative contribution of each sub-model is quantified with the help of a weighting function. The global behaviour of the nonlinear system is obtained by the sum of the local models balanced by the weighting functions.

Various studies dealing with the presence of unknown inputs acting on the nonlinear system were published [3], [4], [18], [19]. The problem of state estimation of nonlinear systems submitted to uncertainties has received considerable attention [13], [1], [24], [28]. In practice, there are many situations where some of the system inputs are inaccessible. The recourse to the use of an unknown input observer is then necessary in order to be able to estimate the state of the considered system. For state estimation, the suggested technique consists in associating to each local model a local unknown input observer. The multiple observer or global observer is the sum of the local observers weighted by the weighting functions associated to the local models [25].

In this paper, the problem of state estimation of an uncertain Takagi-Sugeno multiple model is addressed. The purpose of this work is to extend the principle of the design of observers with unknown inputs to uncertain system case. Only model's and input uncertainties are considered in this paper.

Others works dealing with uncertain systems choose to estimate the sate using different kinds of observers, such as sliding mode observer [1], Proportional and Proportional Integral observer [23]. The main contribution in this paper is the development of an unknown input multiple observer for uncertain nonlinear systems modelled by Takagi-Sugeno models. The convergence conditions of the sate estimation error are expressed in terms of linear matrix inequalities (LMI).

The paper is organized as follows. Section 2 recalls the multiple model approach. In section 3, the multiple observer of a system with unknown inputs is presented. Section 4 presents the main results concerning the synthesis of a multiple observer to estimate the state of nonlinear system submitted to modelling and inputs uncertainties. Finally, in section 5, a numerical example is given to show the validity of the proposed methodology.

2. Elementary Background on the Multiple Model Approach

The idea of the multiple model approach is to apprehend the total behaviour of a system by a set of local models, each of them can be for example a linear time invariant system valid in a particular operating zone of the system. The local models are then aggregated by means of an interpolation mechanism.

It should be noted that various realisations of multiple models can be employed in order to generate the global output of the multiple model [9]. Two main structures of multiple models can be distinguished according to the nature from the coupling between local models. In the first case, the submodels share the same state-space and consequently the multiple model is composed of homogeneous submodels. In the second one, decoupled multiple structure, the submodels do not have the same state-space and the multiple model uses heterogeneous submodels. The first structure, known as Takagi-Sugeno multiple model, is the most used in multiple model class representation [15]. The association of their local models is performed in the dynamic equation of the multiple model using a common state vector. This model has been initially proposed, in a fuzzy modelling framework, by Takagi and Sugeno [27] and in a multiple model modelling framework by Johansen and Foss [16]. This model has been largely considered for analysis, modelling, control and state estimation of nonlinear systems.

The main advantage of Takagi-Sugeno structure is its simplicity because it originates from the interpolation between linear systems. Thus, analysis and design methods developed for linear systems can be generalized to nonlinear systems [12].

The Takagi-Sugeno model representation is given by:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + D_i) \\ y(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(C_i x(t) + E_i u(t) + N_i) \end{cases} \quad (1)$$

where $\mu_i(\xi(t))$ are the activation functions and $\xi(t)$ is the decision vector which is a real time accessible variable. It may depend on the known input, output and/or the measured state variables.

If $E_i = N_i = 0$ and the output $y(t)$ is linear, i.e. $C_1 = C_2 = ... = C_M = C$, the structure of the Takagi-Sugeno multiple model becomes:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + D_i) \\ y(t) = Cx(t) \end{cases} \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^p$ represents the measured output. $A_i \in \mathbb{R}^{n*n}$ is the state matrix, $B_i \in \mathbb{R}^{n*m}$ is the matrix of input and $C \in \mathbb{R}^{p*n}$ is the output matrix of the system. M is the number of local models. It depends on the modelling precision, the nonlinear system complexity and the choice of the weighting functions structure.

Matrices A_i, B_i, D_i and C can be obtained by using the direct linearization of the nonlinear model around several operating points, or alternatively by using an identification procedure [5], [14], [10].

The weighting functions $\mu_i(\xi(t))$ quantify the relative contribution of each sub-model to the global model according to the current operating point of the system. They are nonlinear in $\xi(t)$. These weighting functions must satisfy the following convex sum properties:

$$0 \le \mu_i(\xi(t)) \le 1 \quad \text{and} \quad \sum_{i=1}^{M} \mu_i(\xi(t)) = 1 \quad (3)$$

The weighting functions can be obtained from Gaussian functions:

$$\mu_i(\xi(t)) = \frac{\omega_i(\xi(t))}{\sum_{j=1}^{M} \omega_j(\xi(t))} \quad (4)$$

where:

$$\omega_i(\xi(t)) = e^{\left(\frac{-\xi(t)-(\xi(t))^{(i)})^2}{\sigma^2}\right)} \quad (5)$$

The variable of decision $\xi(t)$ is accessible in real time and it depends of measurable variables like system inputs or outputs.

3. State Estimation Using Observers with Unknown Inputs

In this part, one considers the state estimation of a nonlinear system perturbed by unknown inputs. The structure of that observer results of the aggregation of local observers [6]. The design of this multiple observer is based on the elimination of the unknown inputs.

Multiple observer design

The following Takagi-Sugeno multiple model representing a nonlinear system with unknown inputs is considered:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + R\overline{u}(t) + D_i) \\ y(t) = Cx(t) \end{cases} \quad (6)$$

where $x(t) \in \mathbb{R}^n$ represents the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $\overline{u}(t) \in \mathbb{R}^q, q < n$ represents the vector of unknown inputs, $y(t) \in \mathbb{R}^p$ is the measured output. A_i, B_i, D_i and C are known constant matrices with appropriate dimensions. R is the matrix of influence of the unknown inputs which is assumed to be known.

Consider the global multiple observer described as follows [3]:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(N_i z(t) + G_{i1} u(t) + G_{i2} + L_i y(t)) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases} \quad (7)$$

$N_i \in \mathbb{R}^{n*n}$, $G_{i1} \in \mathbb{R}^{n*m}$, $L_i \in \mathbb{R}^{n*p}$ is the gain of the i^{th} local observer, $G_{i2} \in \mathbb{R}^n$ is a constant vector and E is a matrix transformation. All these matrices or vectors have to be determined in order to guarantee the asymptotic convergence of $\hat{x}(t)$ towards $x(t)$.

The state estimation error is given as follows:

$$\begin{aligned} e(t) &= x(t) - \hat{x}(t) \\ &= (I + EC)x(t) - z(t) \end{aligned} \quad (8)$$

By direct time derivative, the dynamic evolution of $e(t)$ is given by (9):

$$\begin{aligned} \dot{e}(t) = &\sum_{i=1}^{M} \mu_i(\xi(t))(P(A_i x(t) + B_i u(t) + R\overline{u}(t) + D_i) \\ &-\sum_{i=1}^{M} \mu_i(\xi(t))(N_i z(t) + G_{i1} u(t) + G_{i2} + L_i y(t)) \end{aligned} \quad (9)$$

where $P=(I+EC)$

Replacing $y(t)$ and $z(t)$ by their expressions, (9) becomes:

$$\dot{e}(t)=\sum_{i=1}^{M}\mu_i(\xi(t))(N_ie(t)+(PA_i-N_i-K_iC)x(t)) \\ +\sum_{i=1}^{M}\mu_i(\xi(t))((PB_i-G_{i1})u(t)+PR\bar{u}(t)+PD_i-G_{i2}) \quad (10)$$

with $K_i=L_i+N_iE$.

If the following conditions are fulfilled [2]:

$$PR=0 \quad (11.a)$$

$$P=I+EC \quad (11.b)$$

$$N_i=PA_i-K_iC \quad (11.c)$$

$$L_i=K_i-N_iE \quad (11.d)$$

$$G_{i1}=PB_i \quad (11.e)$$

$$G_{i2}=PD_i \quad (11.f)$$

$$\sum_{i=1}^{M}\mu_i(\xi(t))N_i \text{ Stable} \quad (11.g)$$

The equation (10) reduces to:

$$\dot{e}(t)=\sum_{i=1}^{M}\mu_i(\xi(t))N_ie(t) \quad (12)$$

Let us consider the following Lyapunov function $V(t)$:

$$V(t)=e(t)^TXe(t) \quad (13)$$

where X denotes a positive definite symmetric matrix.

Its derivative with regard to time is given by:

$$\dot{V}(t)=\dot{e}(t)^TXe(t)+e(t)^TX\dot{e}(t) \quad (14)$$

By substituting $\dot{e}(t)$ given by (12) in (14), one obtains:

$$\dot{V}(t)=\sum_{i=1}^{M}\mu_i(\xi(t))e(t)^T(N_i^TX+XN_i)e(t) \quad (15)$$

Thus, the asymptotic convergence of the multiple observer is guaranteed and the state estimation error $e(t)$ converges towards zero, if the conditions (11) are verified and $\dot{V}(t)<0$, that is $N_i^TX+XN_i<0$.

Global convergence conditions of the multiple observer

The state estimation error between the multiple model (6) and the unknown input multiple observer (7) converges towards zero, if all the pairs (A_i,C) are observables and if the following conditions are checked $\forall\ i\in\{1,...,M\}$ [3]:

$$N_i^TX+XN_i<0 \quad (16.a)$$

$$N_i=PA_i-K_iC \quad (16.b)$$

$$P=I+EC \quad (16.c)$$

$$PR=0 \quad (16.d)$$

$$L_i=K_i-N_iE \quad (16.e)$$

$$G_{i1}=PB_i \quad (16.f)$$

$$G_{i2}=PD_i \quad (16.g)$$

where $X\in\mathbb{R}^{n*n}$ is a positive definite symmetric matrix.

Using the equation given by (16.b), the expression (16.a) can be rewritten:

$$(PA_i-K_iC)^TX+X(PA_i-K_iC)<0,\forall i\in\{1,...,M\} \quad (17)$$

It is noted that the inequalities (17) are bilinear compared to variables X and K_i. To be reduced to the case of a linear problem, changes of variables are used.

Method of resolution

In order to resolve the system (16), three steps are needed:

1. The matrix E is given by using the expression (16.d), where $(CR)^-$ is the pseudo-inverse of CR:

$$E=-R(CR)^- \quad (18)$$

Proof:

$PR=0;\ P=I+EC$

$(I+EC)R=0$

$R+ECR=0$

$ECR=\text{-}R$

$E=\text{-}R\ (CR)^{(-)}$

□

By determining the matrix E, the matrix P is deduced from (16.c):

$$P=I+EC=I-R(CR)^-C \quad (19)$$

2. To linearize the inequalities (17), the following change of variables is used:

$$W_i=XK_i \quad (20)$$

One obtains linear matrix inequalities (21) that can easily solve by the means of LMI tools:

$$(PA_i)^T X + X(PA_i) - C^T W_i^T - W_i C < 0, \forall i \in \{1,...,M\} \quad (21)$$

Finally, the matrices K_i are derived from:

$$K_i = X^{-1} W_i \quad (22)$$

3. The other matrices defining the multiple observer are deduced knowing E, P and K_i:

$$\begin{aligned} N_i &= PA_i - K_i C \\ L_i &= K_i - N_i E \\ G_{i1} &= PB_i \\ G_{i2} &= PD_i \end{aligned} \quad (23)$$

4. Proposed Extension to Uncertain Systems

Multiple model representation of an uncertain system

The representation of a nonlinear system subject to modelling and input uncertainties is given by the following equations:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))((A_i \pm \Delta A_i)x(t) \\ \qquad + (B_i \pm \Delta B_i)u(t) + D_i) \\ y(t) = Cx(t) \end{cases} \quad (24)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ the input vector and $y(t) \in \mathbb{R}^p$ the measured outputs. Matrices $A_i \in \mathbb{R}^{n*n}$, $B_i \in \mathbb{R}^{n*m}$ denote respectively the state matrix and the input matrix associated with the ith local model. D_i is introduced to take into account the operating point of the system. ΔA_i are the matrices of modelling uncertainties and ΔB_i represent the input uncertainties of the system. The weighting functions must satisfy the conditions (3).
By developing the expression given by the equation (24), one obtains:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))((A_i x(t) + B_i u(t) + D_i) \\ \qquad + (\pm \Delta A_i x(t) \pm \Delta B_i u(t))) \\ y(t) = Cx(t) \end{cases} \quad (25)$$

The aim in this section consists in estimating the state of the system described by an uncertain Takagi-Sugeno multiple model structure (24). A solution suggested with this problem consists in taking account of the modelling and input errors as unknown inputs what makes it possible to apply the results obtained in Section 3.

Noting $\bar{u}(t) = (\pm \Delta A_i x(t) \pm \Delta B_i u(t))$, the system (25) becomes:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))((A_i x(t) + B_i u(t) + \bar{u}(t) + D_i) \\ y(t) = Cx(t) \end{cases} \quad (26)$$

By comparing the equation (26) with the equation (6), one notices that the two equations are almost identical. The only difference is that the matrix R is replaced by the matrix identity. It is possible under these conditions to adapt the results of Section 3 for the design of a multiple observer in the presence of modelling and input uncertainties.

Multiple observer design

The conception of the multiple observer is based on the elimination of uncertainties affecting the nonlinear system represented by the Takagi-Sugeno model given by the equation (24). These uncertainties are considered as unknown inputs.
The structure of the multiple observer is as follows:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(N_i z(t) + G_{i1} u(t) + G_{i2} + L_i y(t)) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases} \quad (27)$$

where $\hat{x}(t)$ is the state estimation. $N_i \in \mathbb{R}^{n*n}$, $G_{i1} \in \mathbb{R}^{n*m}$, $L_i \in \mathbb{R}^{n*p}$ is the gain of the local observer, $G_{i2} \in \mathbb{R}^n$ is a constant vector and E a matrix of transformation.

The method of resolution allowing determining the gains of local observers is that given in the part (3-2). The deduction of the matrices N_i and gains G_{i1} and G_{i2} are given respectively by the equations (16.b), (16.f) and (16.g).

Second approach

Consider the system described by the equation (24), an observer able to estimate this system state is given by the equation (27). If the conditions (11.b-11.g) are fulfilled, the state estimation error is given by:

$$\dot{e}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(N_i e(t) + P\bar{u}(t)) \quad (28)$$

where $\bar{u}(t) = (\pm \Delta A_i x(t) \pm \Delta B_i u(t))$.

Consider the same Lyapunov function $V(t)$ given by (13).

The problem of robust state and faults estimation is reduced to find the gains of the observer to ensure an asymptotic convergence of $e(t)$ towards zero if $\bar{u}(t) = 0$ and to ensure a bounded error in the case where $\bar{u}(t) \neq 0$, i.e.:

$$\begin{aligned} &\lim_{t\to\infty} e(t) = 0 && \text{for } \bar{u}(t) = 0 \\ &\|e(t)\|_{Q_e} \le \lambda \|\bar{u}(t)\|_{Q_u} && \text{for } \bar{u}(t) \neq 0 \text{ and } e(0) = 0 \end{aligned} \quad (29)$$

where $\lambda > 0$ is the attenuation level. To satisfy the constraints (13), it is sufficient to find a Lyapunov function $V(t)$ such that:

$$\dot{V}(t) + e(t)^T Q_e e(t) - \lambda^2 \bar{u}(t)^T Q_u \bar{u}(t) < 0 \quad (30)$$

where Q_e and Q_u are two positive definite matrices.

The inequality (30) can also be written as:

$$\psi(t)^T \Omega \psi(t) < 0 \quad (31)$$

with:

$$\psi = \begin{bmatrix} e \\ \bar{u} \end{bmatrix}, \text{ and}$$
$$\Omega = \begin{bmatrix} N_i^T X + X N_i + Q_e & XP \\ P^T X & -\lambda^2 Q_u \end{bmatrix} \quad (32)$$

The quadratic form in (31) is negative if $\Omega < 0$.

The method of resolution of (31) is given in [18].

5. Simulation Example

In this part, only the first approach is considered for simulation.

Let us consider the multiple model, made up of two local models and involving two states and two outputs.

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i(\xi(t))((A_i \pm \Delta A_i)x(t) \\ \qquad + (B_i \pm \Delta B_i)u(t) + D_i) \\ y(t) = Cx(t) \end{cases} \quad (33)$$

Posing $\bar{u}(t) = (\pm \Delta A_i x(t) \pm \Delta B_i u(t))$:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i(\xi(t))((A_i x(t) + B_i u(t) + \bar{u}(t) + D_i) \\ y(t) = Cx(t) \end{cases}$$
(34)

The numerical values of matrices are:

$$A_1 = \begin{bmatrix} -2 & 0 \\ -2 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} -1 & -2 \\ 2 & -4 \end{bmatrix},\ D_1 = D_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ B_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

One takes $\Delta A_i = 0.1 * A_i$ and $\Delta B_i = 0.1 * B_i$

During simulation, one distinguishes the 4 borderline cases: $+\Delta A_i x(t) + \Delta B_i u(t)$, $+\Delta A_i x(t) - \Delta B_i u(t)$, $-\Delta A_i x(t) + \Delta B_i u(t)$ and $-\Delta A_i x(t) - \Delta B_i u(t)$. The study of these cases makes it possible to give an idea on the effectiveness of the method.

The decision vector is depending on the system input. The system (34) was simulated using Gaussian functions for the weighting functions $\mu_i(u(t))$ obtaining from the equations (4) and (5), with $\xi^1 = u^1 = 0.25$, $\xi^2 = u^2 = 0.25$ and $\sigma = 0.15$. These functions are shown on Figure 1.

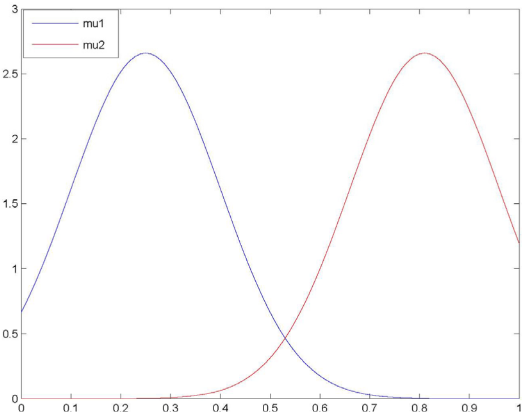

Figure 1. The weighting functions

The known system input $u(t)$ is represented in Figure 2.

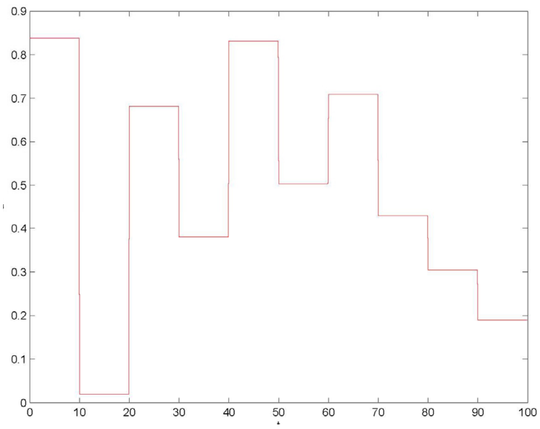

Figure 2. The known input $u(t)$

The structure of the multiple observer is:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{2} \mu_i(u(t))(N_i z(t) + G_{i1}u(t) + G_{i2} + L_i y(t)) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases}$$

Its matrices are:

$$N_1 = N_2 = \begin{bmatrix} -5.5 & 0 \\ -2 & -5.5 \end{bmatrix}, E = \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}$$

L_1, L_2, G_{11} and G_{21} are null matrices.

The simulation results are shown in Figures 3, 4, 5 and 6.

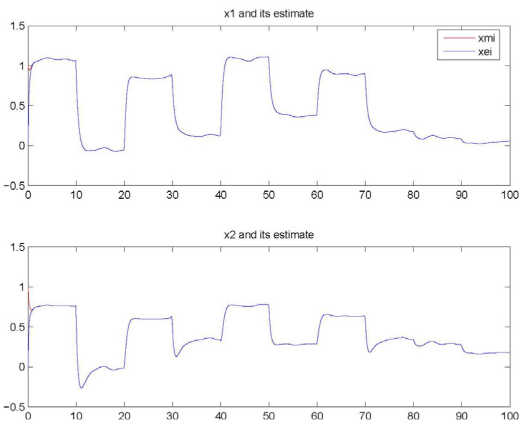

Figure 3. States and their etimates associated for (+deltaA+deltaB)

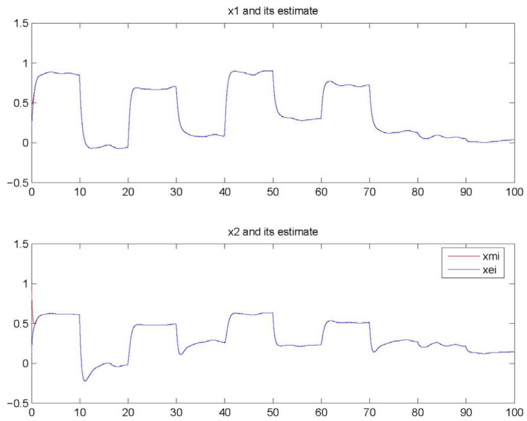

Figure 4. States and their etimates associated for (+deltaA-deltaB)

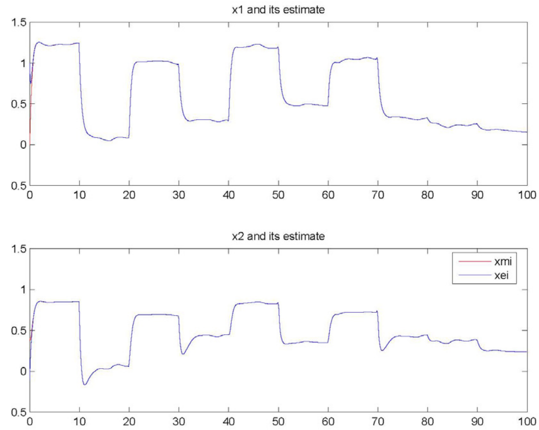

Figure 5. States and their etimates associated for (-deltaA+deltaB)

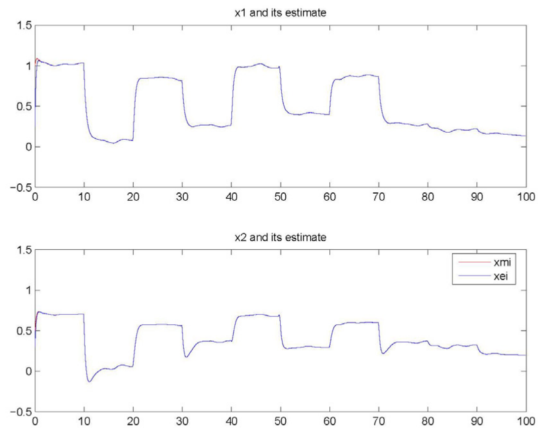

Figure 6. States and their etimates associated for (-deltaA-deltaB)

Thus, one succeeds in estimating the system state for nonlinear systems described by Takagi-Sugeno models, in the presence of modelling and input uncertainties. Figures 3 to 6 show the time evolution of the system states and their estimates for the for borderline cases: $+\Delta A_i x(t) + \Delta B_i u(t)$, $+\Delta A_i x(t) - \Delta B_i u(t)$, $-\Delta A_i x(t) + \Delta B_i u(t)$ and $-\Delta A_i x(t) - \Delta B_i u(t)$.

The actual state and the estimated state are superimposed except in the vicinity of the origin. This disparity is due to the choice of the initial conditions of the multiple observer. A model uncertainty equal to 10% of the system matrices does not influence the estimate results.

Figure 7 presents the evolution of the state error estimation. It is shown that the four errors converge towards zero. It can be concluded that the proposed method allows estimating well the system state even in the case of modelling and input uncertainties.

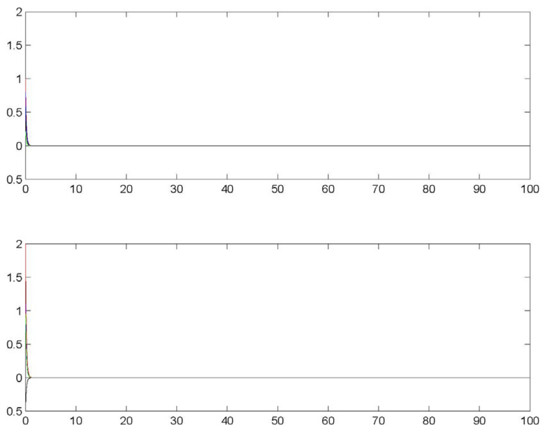

Figure 7. State estimation error

6. Conclusion

This paper addresses a new method to design multiple observer for uncertain nonlinear systems represented by Takagi-Sugeno models. Models' and input uncertainties considered in this work are assumed to be as unknown inputs. The convergence of the estimation error is studied using the second method of Lyapunov and the synthesis conditions of the observer are expressed in LMI terms. The calculation of the gains of the multiple observer is based on the calculation of gains of local observers. The simulation results show that the estimation of the state is very satisfactory. Future works will deal with the extension of the proposed method in order to take into consideration the modelling and outputs uncertainties.

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