An ACO Algorithm for Optimal Capacitor Banks Placement in Power Distribution Networks

Dinu Călin Secui[1], Simona Dziţac[1], Gabriel Valentin Bendea[1], Ioan Dziţac[2]

[1]University of Oradea, Romania

E-mail: csecui@uoradea.ro, simona.dzitac@gmail.com, gbendea@uoradea.ro

[2]Aurel Vlaicu University of Arad & Cercetare Dezvoltare Agora (R&D Agora), Oradea, Romania

E-mail: idzitac@rdsor.ro

Abstract: This paper aims to present and apply an algorithm based on Ant Colony Optimization (ACO) for optimal allocation of capacitor banks in electric power distribution networks. A nonlinear function based on costs is used as a criterion of the mathematical optimization model. Also the model imposes equality constraints described by the network operating equations and inequality constraints required to maintain within admissible limits the parameters characterizing the system state. The algorithm is applied for a test-network having 35 nodes, the results indicating its validity and efficiency.

Keywords: Ant colony optimization algorithm (ACO), capacitor banks placement, power loss reduction

Dinu C. Secui received his M.Sc. in Power Engineering (1993) from Technical University of Timisoara and Ph.D. in Energy Engineering (2003) from University of Oradea. Now he is reader at Energy Engineering Faculty, University of Oradea, Romania. His current research interests include different aspects of Reliability in Power Systems and Optimization Techniques in Power Engineering. He has authored 3 books and more than 60 scientific papers in journals and conferences proceedings. He participated at 11 research grants and projects.

Simona Dziţac received B.Sc. (2000) and M. Sc. (2001) in Mathematics-Physics, B.Sc. (2005) and M. Sc. (2007) and Ph.D. (2008) in Energy Engineering from University of Oradea and B.Sc. in Economic Informatics (2007) from University of Craiova, Romania. Her current research interests include   Reliability, Applied Mathematics and Computer   Science in Engineering fields. She (co-)authored 8 books and 65 scientific papers in journals and conferences proceedings.

Gabriel V. Bendea received his M.Sc. in Power Engineering (1992) from Technical University of Timisoara and Ph.D. in Energy Engineering (2001) from University of Oradea. Now he is reader at Energy Engineering Faculty, University of Oradea, Romania. His current research interests include different aspects of Power System Reliability and Power Generation. He has authored 6 books and more than 55 scientific papers in journals and conferences proceedings. He participated at 22 research grants and projects.

Ioan Dziţac received his M.Sc. in Mathematics (1977) and Ph. D in Information Sciences (2002) from "Babes-Bolyai" University of Cluj-Napoca. Now he is professor at Mathematics-Informatics Department, Aurel Vlaicu University of Arad, Romania and director of R&D Agora, Oradea, Romania. His current research interests include different aspects of Artificial intelligence and Distributed systems. He has (co-)authored 18 books and more than 70 papers, more than 50 de conferences participation, member in International Program Committee of 40 conference and workshops.

1. Introduction

Power distribution networks (PDN) are important structures within the power system, and therefore the improvement of their performances is set as main target of electricity companies, in the context of market liberalization. To achieve this goal, the optimal placement of capacitor banks in PDN is one of the possible solutions, having multiple positive effects, such as: voltage level and power factor improvement in the network, feeders' capacity increment, active power losses reducing [1, 2]. All these effects imply lower operating costs for companies. In order the benefits of capacitor banks placement to be as big, they must be positioned and sized correctly using appropriate mathematical models and methods [3, 4, 5].

Most mathematical models used in literature have as objective function minimizing the costs with technical and economic restrictions of inequality and equality, but the solving techniques have diversified over time following the development of computing techniques and algorithms. Regarding the capacitors placement problem within the PDN, the classic techniques are based upon integer programming [6], nonlinear programming (gradient method) [7], but the solutions obtained do not guarantee achieving the optimum. The problem has been addressed successfully by using the techniques of search and optimization based on simulated annealing [8], genetic algorithm [3,9], immune algorithm [10], Particle Swarm

Optimization [11, 12, 13], Ant Colony Search Algorithm [1] or hybrid solutions [14, 15].

Capacitor banks placement in PDN is a complex combinatory problem which can be solved using the ability of ant system algorithms [16, 17, 18] or later developed varieties [19, 20], applied in various fields [21, 22, 23, 24].

In this paper, an algorithm based on Ant Colony Optimization (ACO) is presented in order to solve the problem of optimal capacitor banks placement in PDN, considering their layout on the low voltage nodes of the network. Objective function is based on a nonlinear cost model with equality restrictions, described by the equations of the network functioning, and inequality constraints related to the voltage level, voltage drops and capacitors' limits.

2. Optimization Problem Formulation

A medium and low voltage radial power distribution network of n nodes is considered. Figure 1 shows the medium voltage nodes n_M and the low voltage ones n_J. The network is fed from a high/medium voltage power station nodes and has got n_S line sections (S) and n_T transformers (T) of medium/low voltage.

- transformers longitudinal electricity parameters, represented by the impedance of a (j,j') transformer:

$$\underline{Z}^T_{i,j} = R^T_{j,j'} + i \cdot X^T_{j,j'} \tag{2}$$

- transformers cross electricity parameters, represented by the admittance of a (j,j') transformer:

$$\underline{Y}^T_{j,j'} = G^T_{j,j'} - i \cdot B^T_{j,j'} \tag{3}$$

- active power (P_{Lj}) and reactive power (Q_{Lj}) consumed in every j node of the network.

The following notation that have been used: $(R^S_{i,j}, X^S_{i,j})$, $(R^T_{j,j'}, X^T_{j,j'})$ are the resistance and reactance of the (i,j) line section, respectively the resistance and reactance of a transformer that are positioned between the (j,j') nodes, and $(G^T_{j,j'}, B^T_{j,j'})$ is the conductance and susceptance of (j,j') transformer.

The optimal placement problem of capacitor banks in power distribution networks consists in determination of layout and dimension of capacitor banks, so that the costs due to active power losses in all network and the costs of acquisition and installation of capacitor banks to be minimal, but complying with technical restrictions.

The mathematical model of the optimal allocation of capacitor banks in a power distribution network comprises of:
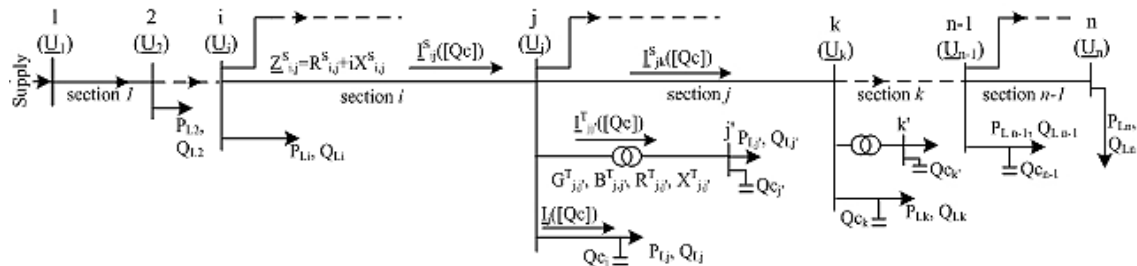

Figure 1. Radial distribution power network having branches

In order to solve the problem, the following are considered known:

- power network configuration;

- power lines longitudinal electricity parameters, that are represented as complex numbers by the impedance of a section (i,j):

$$\underline{Z}^S_{i,j} = R^S_{i,j} + i \cdot X^S_{i,j} \tag{1}$$

a). Discreet variables that have to be optimized, and are represented by the reactive power vector $Qc=[Qc_1, Qc_2,...,Qc_n]$ of capacitor banks that must be installed in power network nodes.

b). Objective function F, that consists of the cost of active power losses in all network and the acquisition costs of capacitor banks, and have the following expression:

$$\min\{F\} = \min\Big\{Cp \cdot (\Delta P^S([Qc]) + \Delta P^T([Qc])) + Cq \cdot \sum_{i=1}^{n} Qc_i\Big\} \quad (4)$$

where:

ΔP^S, ΔP^T are losses of active power in all power line sections (S) of network having n nodes, respectively in all power transformers (T) installed in the network;

Cp is the cost of the active power losses, in [m.u./kW year];

Cq is the annual cost unitary installation of the capacitor banks, in [m.u./kVAr];

$m.u.$ – monetary units [EUR, USD etc.].

The two categories of losses ΔP^S, ΔP^T may be calculated using the following relations:

$$\Delta P^S = \sum_{(i,j)\in ML} \Delta P^S_{i,j}$$
$$= \sum_{(i,j)\in ML} 3 \cdot R^S_{i,j} \cdot \left(I^S_{i,j}([Qc])\right)^2 \quad (5)$$

$$\Delta P^L = \sum_{(j,j')\in MT} \Delta P^T_{j,j'}$$
$$= \sum_{(j,j')\in MT} U_j([Qc])^2 \cdot G^T_{j,j'} \quad (6)$$
$$+ \sum_{(j,j')\in MT} 3 \cdot R^T_{j,j'} \cdot \left(I^T_{j,j'}([Qc])\right)^2$$

where:

$U_j([Qc])$ is the voltage of j node, depending to the vector of variables $[Qc]$;

$\Delta P^S_{i,j}$, $\Delta P^T_{j,j'}$ are the active power losses for a random (i, j) section of line, respectively for a (j, j') transformer;

$I^S_{i,j}([Qc])$, $I^T_{j,j'}([Qc])$ are the currents through line type sides and through transformers;

ML and MT sides set of power line type, respectively of transformers type.

Currents running through transformers and line type sides depend to $[Qc]$ variables. $I^S_{i,j}([Qc])$, $I^T_{(j,j')}([Qc])$ and $U_j([Qc])$ characterize the network status by using a system of equations that represent the equality restrictions of the problem. These values are determined by calculating the power circulation for permanent nominal operation of the analyzed network.

c). The restriction relations are defined by a set of equations and inequations that guarantee network operation and keeping in admitted limits the parameters characterizing the system state. The following restrictions are considered:

c1). Equality restrictions between currents that circulate through network sides $R^I_{(i,j)}$, respectively between voltages of two consecutive nodes ($R^U_{(i,j)}$ and $R^U_{(j,j')}$ type), relations that insure the power network is normally operating in each moment:

$$R^I_{(i,j)} = \underline{I}^L_{i,j} - \underline{I}_j - k_T \cdot \underline{I}^T_{j,j'} - \sum_{k\in S_j} \underline{I}^L_{j,k} = 0 \quad (7)$$

$$R^U_{(i,j)} = \underline{U}_i - \underline{U}_j - \sqrt{3} \cdot \underline{Z}^L_{i,j} \cdot \underline{I}^L_{i,j} = 0 \quad (8)$$
for power lines

$$R^U_{(j,j')} = \underline{U}_j \cdot k_T - \underline{U}_{j'} - \sqrt{3} \cdot \underline{Z}^T_{j,j'} \cdot \underline{I}^T_{j,j'} = 0 \quad (9)$$
for transformers

where $\underline{I}_j = \dfrac{\underline{S}^*_{Lj}}{\sqrt{3}\cdot \underline{U}^*_j} = \dfrac{(P_{Lj}+i\cdot Q_{Lj})^*}{\sqrt{3}\cdot \underline{U}^*_j}$ \quad (10)

S_j represents the nodes set connected to j node.

Number of restrictions of $R^I_{(i,j)}$ type is equal to (n_S - 1), the one of restrictions of $R^U_{(i,j)}$ type is n_S and of $R^U_{(j,j')}$ type is n_T.

c2) Inequality restrictions (as many as $2n$) refer to keeping voltage module into the admissible limits:

$$U_i^{min} \le U_i([Qc]) \le U_i^{max} , \quad i = 1...n \quad (11)$$

c3) Inequality restrictions (as many as $2n$) referring to limits of capacitor banks reactive power allocated in network nodes:

$$Qc_i^{min} \le Qc_i \le Qc_i^{max} , \quad i = 1...n \quad (12)$$

c4) Inequality restrictions referring to keeping voltage drops into the admissible limits for all sided sections of the network (voltage drops $\Delta U_{l,e}([Qc])$ that are included between feeding (supply) point 1and extreme consumption nodes of the network):

$$\Delta U_{1,e}([Qc]) \le \Delta U_{adm}^{max} \quad (13)$$

where e is the number of extreme consumption nodes of the network.

3. Algorithm Based on ACO for Optimal Placement of Capacitor Banks in PDN

3.1 Description and mathematical formulation of ACO algorithm

ACO algorithm was first proposed by M. Dorigo [25] and is inspired by the actual behavior of ants, which are able to find the shortest path between nest and food sources. In searching for food the ants deposit a substance called "pheromone", and the critical communication mean of the ants in order to generate the shortest path finding behavior is the amount of pheromone deposited on the routes covered by ants. If the path is used by many ants, the amount of pheromone on this route increases, and also increases the possibility of this path to become the solution. In the same time, the route which is not covered by ants is poor of pheromone and its possibility of becoming the goal path decreases.

The algorithm begins by setting parameters and initialization of pheromone on each path, and then, in an iterative cycle, each ant finds a solution, generating a complete tour by choosing the locations covered in accordance to a probabilistic rule. Mathematically, the probability ($p^{(h)}_{(i,j)}$) with which an ant h chooses to move from a location i to another one j is:

$$p^{(h)}(i,j) = \begin{cases} \dfrac{\left(\tau(i,j)\right)^\alpha \cdot \left(\eta(i,j)\right)^\beta}{\displaystyle\sum_{r \in N_h(i)} \left(\tau(i,r)\right)^\alpha \cdot \left(\eta(i,r)\right)^\beta}, & if \ j \in N_h(i) \\ 0, & otherwise \end{cases}$$

(14)

where: $\tau(i,j)$ is the pheromone amount stored on the edge between the locations i and j;

$N_h(i)$ represents the set of nodes which remain to be visited by ant k situated in location i;

$\eta(i,j)$ is a parameter which offers additional heuristic information to determine more quickly the solution;

α, β are coefficients indicating the importance of $\tau(i,j)$, respective $\eta(i,j)$ parameter.

The amount of pheromone deposited on edge (i,j) is permanently altered. If $\tau(i,j)$ is the intensity of pheromone trail on the edge (i,j) at time t, then after n periods of time ($p^{(h)}_{(i,j)}$) the pheromone trail updating is done in accordance to the relation:

$$\tau_{ij}(t+n) = \rho \cdot \tau_{ij}(t) + (1-\rho) \cdot \sum_{h=1}^{m} \Delta\tau_{ij}^{h}$$

(15)

where: ρ is the pheromone evaporation rate, having values within the [0,1] interval;

m is the number of ants;

$\Delta\tau_{ij}^{h}$ represents the amount of pheromone stored on the edge (i,j) due to ant h contribution in a tour, contribution determined based on the relation:

$$\Delta\tau_{ij}^{h} = \begin{cases} Q/L_h, & if \ (i,j) \in S_h \\ 0, & otherwise \end{cases}$$

(16)

where: Q is a constant;

L_h is the tour length of the ant h;

S_h is the set of edges (i,j) forming ant h tour.

The algorithm runs iteratively until the stopping condition is met, such as imposing a maximum number of iterations admitted or when all the ants follow the same path.

3.2 ACO applied to optimize the capacitor banks placement

The algorithm designed for solving the optimization model is based on ACO metaheuristic. The calculation of currents flowing through branches and the nodes voltage of the power distribution network is required in order to apply and implement ACO algorithm in a programming language.

a) Calculation of the currents flowing through the network branches $\underline{I}^{S}_{i,j}([Qc])$, $\underline{I}^{T}_{j,j'}([Qc])$ and the voltage in the network nodes $\underline{U}_j([Qc])$ was done using an iterative method (based on relations (7)÷(10)), which is specific to radial networks, known in literature as backward/forward sweep. In the source node 1 the voltage is considered constant. For each solution generated by the ACO algorithm we have calculated the flow of currents and the nodes voltage, using a Mathcad computation program (PowerFlow) designed by the authors. Then the imposed restrictions are verified, based upon the (11) ÷ (13) inequations and the power losses in the network are computed, using the relations (5) and (6).

b) ACO algorithm is based on iterative application of relations (14)÷(17) and on calling PowerFlow computation program for each solution found by an ant. To calculate $\eta(i,j)$ parameter the authors have proposed the relation:

$$\eta(i, j) = \left(\left| Qc_i^{opt} - Qc_0(i-1) \right| + \varepsilon \right)^{-1} \qquad (17)$$

where: Qc_i^{opt} is the best solution determined by the current iterative step;

Qc_0 is a constant representing unit of power selected for a capacitor bank installed in a network node.

The coefficient ε was computed experimentally by the authors, for the specific data of this problem.

c) The main steps in implementing the algorithm based on ACO to optimize the capacitor banks placement are listed below:

Step 1. *Parameters Setting.* The parameters of ACO algorithm are set: the number of ants forming the colony (*m*), the maximum number of iterations set N^{max}, the coefficients α, β and ε, the pheromone evaporation rate ρ, the values of the constants Q and Qc_0.

Step 2. *Data Input.* The network structure is read, the parameters of the power network ($\underline{Z}^S_{i,j}$, $\underline{Z}^T_{i,j}$, $\underline{Y}^T_{j,j'}$), active and reactive loads of nodes (P_{Lj}, Q_{Lj}) are introduced, and also the constants Cp and Cq.

Step 3. *Calculation of objective function for the initial solution F $[Qc^0]$.* Initial solution is given by the option in which the network is uncompensated (without capacitor banks), namely the vector of variables is set to $[Qc^0]=[0, 0,...,0]$. The calculation is done by calling PowerFlow program to determine the currents flowing and nodes voltage (relations (7)÷(10)), power losses (relation (5) and (6)) and the objective function (relation (4)).

Step 4. *ACO Initialization.* The amount of pheromone deposited on each edge *(i,j)* in the transition process from position *i* to position *j* is initialized. Initially, the amount of pheromone has the same value τ_0 for each of the edges ($\tau_{i,j} \leftarrow \tau_0$, *i=0.. Qc_i^{max}, j=1..n*).

Step 5. *Solutions Construction.* We consider an ant colony consisting of *m* ants. In one iteration *p* of the optimization process there are sent *m* ants, and each of them generates a solution for the problem $[Qc]^{p,h}$. The solutions are derived from the probabilities $p^{(h)}(i,j)$, computed for each edge *(i,j)*, using relation (14).

Step 6. *Calculation of the currents flowing in the branches and the nodes voltage.* For the *m* solutions obtained at each iteration *p*, PowerFlow program is calling to compute the power losses in the network and the voltages, then there are verify the restrictions imposed by relations (7)÷(13). If at least one of the restrictions is not verified, then the solution is abandoned and we return to step 5.

Step 7. O*bjective function evaluation.* For feasible solutions the values of the objective function are evaluated, using relation (4). The best solution obtained from all the iterations performed $[Qc]^{opt}$ is retained and also the corresponding objective function value $F^{opt}=\min(F[Qc]^{p,h}])=\min(F[Qc]^{opt}])$. When reaching the maximum number of iterations required N^{max}, $[Qc]^{opt}$ represents the solution considered optimal, otherwise the process continues with pheromone updating. The amount of pheromone for edges *(i,j)* is updated, according to the next step.

Step 8. *Pheromone updating.* It is made for each iteration *p*, in accordance with relations (15) and (16), after all *m* ants have generated a complete solution. At each iteration *p* each ant itinerary is retained, and then the amount of pheromone is updated.

Step 9. *Process stop.* The condition for ending the optimization process is determined either by the maximum number of iterations set N^{max} or by the situation in which all the ants follow the same itinerary, hence generating the same solution. If the number of iterations is exceeded or all the ants follow the same route, the calculation process ends, otherwise it returns to step 5.

The nine steps algorithm has been implemented in a Mathcad module, named *ACOPlacement*.

4. Case Study

The proposed method for optimal placement of capacitor banks using ACO algorithm was implemented in Mathcad, on a personal computer having 1.58 GHz Processor and 896 MB RAM.

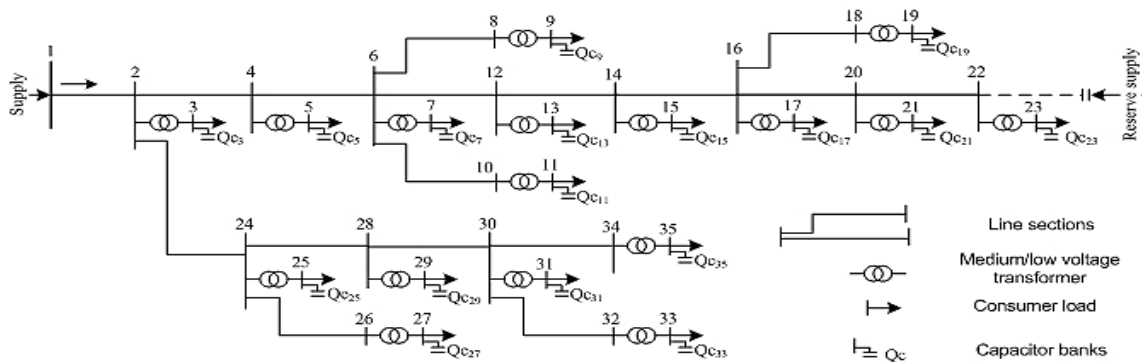For the designed algorithm it was used a

Figure 2. Test distribution network having 35 nodes

20kV / 0.4kV test network (Figure 2), which consists of 35 nodes, reflecting the typical structure of radial distribution networks. We considered there are only low voltages consumers (0.4kV) connected to 17 nodes of the network (nodes which are indicated using odd numbers). Capacitor banks having capacities Qc_i will be set on the low voltage bars, resulting in 17 variables corresponding to the 17 low voltage bars. The network is supplied from node 1, where the voltage remains constant and equal to 20 kV.

Table1. Data of the test-network having 35 nodes

Branch type S or T	From node i	End to node j	R^S or R^T [Ω]	X^S or X^T [Ω]	G^T [S]	B^T [S]	Load in final nodes	
							P_L [kW]	Q_L [kVAr]
S	1	2	1.2456	1.9875	-	-	-	-
T	2	3	0.0442	0.0640	0.0038	0.0213	45	60
S	2	4	2.3478	2.5678	-	-	-	-
T	4	5	0.0129	0.0384	0.0069	0.0453	124	108
S	4	6	1.9860	2.2451	-	-	-	-
T	6	7	0.0129	0.0384	0.0069	0.0453	107	75
S	6	8	2.3457	2.7893	-	-	-	-
S	6	10	3.4568	3.8975	-	-	-	-
S	6	12	2.4982	3.0126	-	-	-	-
T	8	9	0.0442	0.0640	0.0038	0.0213	40	47
T	10	11	0.0129	0.0384	0.0069	0.0453	100	82
T	12	13	0.0069	0.0240	0.0092	0.0663	200	100
S	12	14	4.5691	4.9875	-	-	-	-
T	14	15	0.0129	0.0384	0.0069	0.0453	126	75
S	14	16	3.8919	4.2367	-	-	-	-
T	16	17	0.0129	0.0384	0.0069	0.0453	114	69
S	16	18	1.7502	2.2145	-	-	-	-
S	16	20	4.5679	5.7892	-	-	-	-
T	18	19	0.0129	0.0384	0.0069	0.0453	115	70
T	20	21	0.0442	0.0640	0.0038	0.0213	30	30
S	20	22	3.5607	4.0192	-	-	-	-
T	22	23	0.0069	0.0240	0.0092	0.0663	210	80
S	2	24	2.7894	3.0378	-	-	-	-
T	24	25	0.0129	0.0384	0.0069	0.0453	80	110
S	24	26	3.5671	4.3579	-	-	-	-
S	24	28	2.8961	3.7892	-	-	-	-
T	26	27	0.0129	0.0384	0.0069	0.0453	90	90
T	28	29	0.0129	0.0384	0.0069	0.0453	95	75
S	28	30	5.6781	6.2355	-	-	-	-
T	30	31	0.0129	0.0384	0.0069	0.0453	105	93
S	30	32	4.2579	4.9761	-	-	-	-
S	30	34	3.8921	4.9764	-	-	-	-
T	32	33	0.0129	0.0384	0.0069	0.0453	75	90
T	34	35	0.0129	0.0384	0.0069	0.0453	90	90

The parameters of electric power lines and transformers are presented in Table 1, and also the power consumption of network nodes. Total active and reactive power is P=1746 kW, Q=1344 kVAr respectively. The values of the constants used in calculations are the following: Cp=890 EUR/kW year, Cq=18 EUR/kVAr year, $\Delta U_{adm}^{max}=10\%$ the limits of the voltages for all the nodes of the network are U_i^{min}=0.9 and U_i^{max}=1.1, the limits of the condensers capacity are Q_i^{min}=0 and $Q_i^{max}=Q_{Li}$ in case of the low voltage nodes. For the initial solution, without capacitor banks (uncompensated, Qc^0_{1x17}=[0,0,...0]) the power losses are equal to $\Delta P=\Delta P^S+\Delta P^T$=96.037 kW, and the objective function value is $F[Qc^0]$=85472.93 EUR/year.

ACO algorithm parameters were set to values: α=1, β=2, τ_0=1, ρ=0.05, Q=10, Qc_0=10, m=80 and N^{max}=10. The value of the coefficient ε was determined by running the program one hundred times for values within the interval [0, 10], computing an average value of F. The variation of F average value is presented in Figure 3 and it can be noticed that the coefficient ε optimum is ranging within [2, 3] interval. For our study the coefficient value was set to ε=2.5.
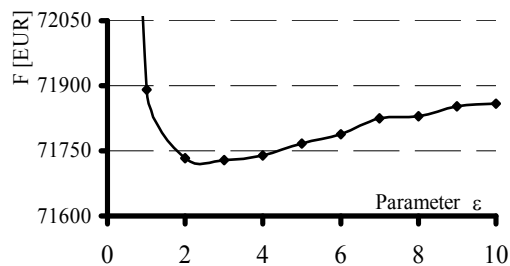

Figure 3. Function F variation according to for coefficient ε values

To check the algorithm performance, it was run one hundred times, retaining the best and the worst value of function F (*Best F* and *Worst F*), the average value (*Average F*), the standard deviation (*SD F*) of function F, and the computation time (*CPU*).

The characteristics of the convergence process for ACO algorithm are presented in Figure 4, representing the function F variation according to the number of iterations.

Next, the effect of the reactive power compensation in the network for the best solution obtained (*Best F*) is presented, and the total power losses (ΔP^{opt}=63.773 kW) are calculated, as well as the value of objective function $F[Qc^{opt}]$=71697.858 EUR/year and the variation of voltage (expressed in proportional units - *pu*) in the network nodes
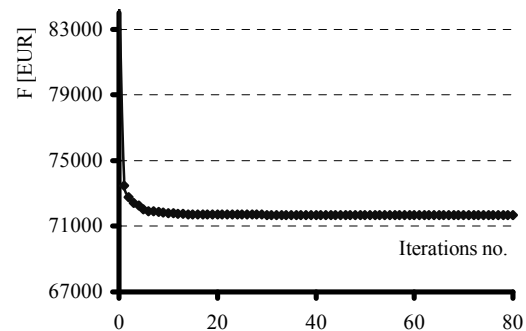

Figure 4. Convergent characteristics of ACO distribution network having 35 nodes

(Figure.5). The vector of the solution obtained for the low voltage nodes has the following values: $[Qc^{opt}]$=[40, 50, 30, 30, 50, 50, 60, 60, 60, 30, 70, 50, 40, 30, 60, 60, 60] in kVAr. Function F values decrease with 13775.07 EUR/year (representing 16.12%) and the power losses decrease with 32.264kW (representing 33.59%) (Table 3).

In Table 4 there are presented the voltage values before and after placing the capacitor banks in the network. Following the data from Table 4 and Figure 5 it can be noticed that initially, in the uncompensated network, the voltage values are lower than those in the compensated (with capacitor banks) variant. Hence, in the final nodes (19, 21, 23 nodes) the voltage value approaches the minimum limit, while in the compensated variant (obtained with ACO algorithm) all voltages have improved values, the highest jumps being recorded in nodes 15, 17, 19, 21 and 23.

**Table 2.** Objective function values obtained using ACO

Best F [EUR/year]	Worst F [EUR/year]	Average F [EUR/year]	SD F [EUR/year]	CPU [sec.]
71697.858	71702.305	71698.499	0.891	10

Table 3. Power losses and function *F* values before and after the capacitor banks placement

Solution	ΔP [kW]	Power Losses reduction [%]	F [EUR/year]	F reduction [%]	Minimum voltage [pu]	CPU [sec]
Initial solution [Qc^0] (without capacitor banks)	96.037	-	85472.93	-	0.90356	-
Solution obtained using ACO algorithm [Qc^{opt}] (with capacitor banks)	63.773	33.59	71697.858	16.12	0.93841	10

Table 4. Relative values of the 35 nodes voltage before and after the capacitor banks placement

Nodes / Solution	1	2	3	4	5	6	7	8	9	10	11	12
Initial solution U[pu]	1	0.9868	0.9484	0.9741	0.9354	0.9645	0.9358	0.9639	0.9318	0.9627	0.9328	0.9554
ACO solution U[pu]	1	0.9913	0.9701	0.9821	0.9568	0.9753	0.9547	0.9749	0.9562	0.9741	0.9574	0.9691
Qc^{opt} [kVAr]	0	0	40	0	50	0	30	0	30	0	50	0

Nodes / Solution	13	14	15	16	17	18	19	20	21	22	23	24
Initial solution U[pu]	0.9296	0.9433	0.9121	0.9353	0.9066	0.9343	0.9052	0.9304	0.9080	0.9274	0.9036	0.9783
ACO solution U[pu]	0.9517	0.9610	0.9459	0.9556	0.9432	0.9550	0.9422	0.9524	0.9435	0.9502	0.9384	0.9854
Qc^{opt} [kVAr]	50	0	60	0	60	0	60	0	30	0	70	0

Nodes / Solution	25	26	27	28	29	30	31	32	33	34	35
Initial solution U[pu]	0.9435	0.9765	0.9458	0.9720	0.9446	0.9633	0.9300	0.9612	0.9315	0.9611	0.9300
ACO solution U[pu]	0.9636	0.9840	0.9638	0.9812	0.9618	0.9756	0.9583	0.9744	0.9605	0.9743	0.9590
Qc^{opt} [kVAr]	50	0	40	0	30	0	60	0	60	0	60
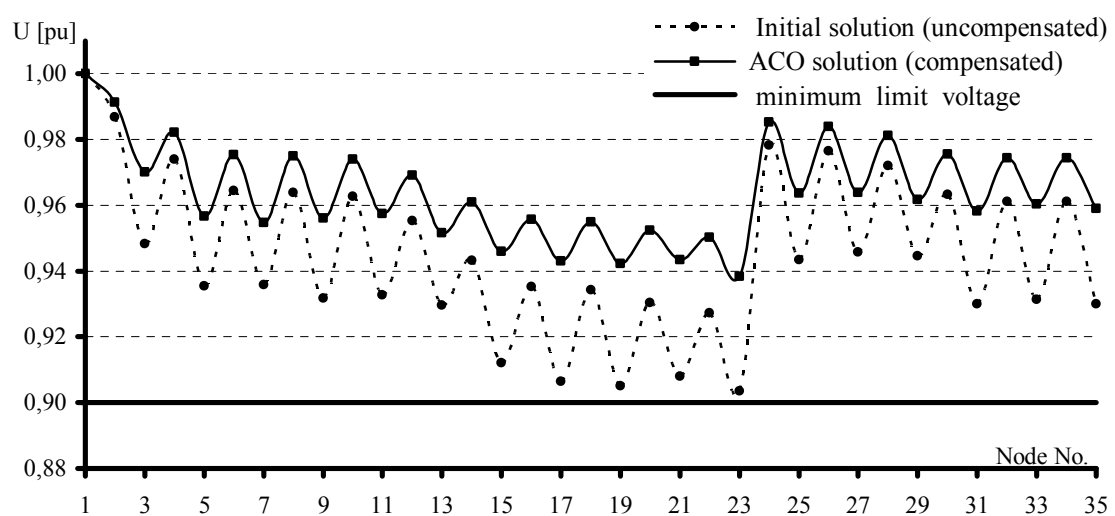
Figure 5. Voltage variation (expressed in proportional units) in the network nodes

5. Conclusions and Future Works

Metaheuristic techniques allow for high quality solutions in case of problems in real, complex and large electrical distribution networks. An algorithm based on ACO metaheuristic was presented and tested for optimal allocation of capacitor banks in a typical PDN having 35 nodes. The algorithm is relatively easy to implement and runs quickly. The results indicate a significant reduction of active power losses in the network and therefore of the costs, and also an improvement of the voltage level. The very close values of the best and the worst solution (*Best F* and *Worst F*) for the objective function obtained for one hundred runs indicate the efficiency of the presented algorithm and the high quality of the solutions.

In the future, the authors will continue their work by developing algorithms based on ACO for multi-objective optimization of capacitor banks allocation in PDN, considering the issues of continuity in consumers supplying.

REFERENCES

1. CHANG, C. F., **Reconfiguration and Capacitor Placement for Loss Reduction of Distribution Systems by Ant Colony Search Algorithm**, IEEE Transactions on Power Systems, vol. 23, no. 4, 2008, pp. 1747-1755.

2. FELEA, I., D. C. SECUI, N. RANCOV, **Automatic Evaluation of Solutions for Reactive Power Compensation Subsystems of Industrial Power Systems**, Annals of the Oradea University, Fascicle of Power Engineering, Oradea, 1995, pp. 60-72, (in Romanian).

3. MALACHI, Y., S. SINGER, **A Genetic Algorithm for the Corrective Control of Voltage and Reactive Power**, IEEE Transactions on Power Systems, vol. 21, no. 1, 2006, pp. 295-300.

4. KILYENI, S., P. ANDEA, M. GROZA, C. BARBULESCU, **Optimal Compensation of Radial Networks**. **Part II. Optimisation Problem's Solution**, Proceedings of the IEEE International Conference EUROCON 2005, Belgrade, Serbia & Montenegro, Vol.2, 2005, pp. 1517-1520.

5. ROTAR, C., **Evolutionary Optimization Techniques**. **Multicriterial Optimization**, Accent Press, Cluj-Napoca, 2008, (in Romanian).

6. LEE, K. Y., F. F. YANG, **Optimal Reactive Power Planning Using Evolutionary Algorithms: A Comparative Study for Evolutionary Strategy, Genetic Algorithm and Linear Programming**, IEEE Transactions on Power Systems, vol. 13, no. 1, 1998 pp. 101-108,.

7. GRAINGER, J. J., S. H. LEE, **Optimum Size and Location of Shunt Capacitors for Reduction of Losses on Distribution Feeders**, IEEE Transaction Power Apparatus Systems, vol. PAS 100, 1981, pp. 1105-1118.

8. CHENG, H. C., C. C. KOU, **Network Reconfiguration in Distribution Systems Using Simulated Annealing**, Electrical Power Systems research, vol.29, No. 3, 1994, pp. 227-238.

9. DAS, D., **Reactive Power Compensation for Radial Distribution Networks Using Genetic Algorithm**, Electrical Power and Energy Systems, vol. 24, 2002, pp.573-581.

10. AHUJA, A., S. DAS, A. PAHWA, **An AIS-ACO Hybrid Approach for Multi-objective Distribution System Reconfiguration**, IEEE Transactions on Power Systems, vol. 22, no. 3, 2007, pp. 1101-1111.

11. KENNEDY, J., R. EBERHART, **Particle Swarm Optimization**, in Proceeding of the IEEE International Conference on Neural Networks, Piscataway, NJ, 4, 1995, pp. 1942-1948.

12. AYAL-D-VIMAL RAJ, P., S. SENTHILKUMAR, S. PRASADH KANNAH, M. SUDHAKARAN, T. G. PALANIVELU, **Swarm Intelligence for Voltage Stability Analysis Considering Voltage-var Compensation**, Elektrika, vol. 10, no. 1, 2008, pp. 43-48.

13. GAVRILAS, M., O. IVANOV, C. V. SFINTES, **Enhanced Particle Swarm**

Optimization Method for Power Loss Reduction in Distribution Systems, 19th International Conference on Electricity Distribution, Paper 0088, Session 5, Austria, 2007.

14. CHIOU, J. P., C. F. CHANG, C. T. SU, **Ant Direction Hybrid Differential Evolution for Solving Large Capacitor Placement Problems**, IEEE Transactions on Power Systems, vol. 19, no. 4, 2004, pp. 1794-1800.

15. SEIFI, A. R., **A New Hybrid Optimization Method for Optimum Distribution Capacitor Planning**, International Journal of Electrical and Electronics Engineering, vol. 3, no. 3, 2008, pp.136-141.

16. DORIGO, M., L. M. GAMBARDELLA, **Ant Colonies for the Traveling Salesman Problem**, BioSystems, 43, 1997, pp. 73-81.

17. ZIDI, S., S. MAOUCHE, S. HAMMADI, **Ant Colony with Dynamic Local Search for the Time Scheduling of Transport Networks**, International Journal of Computers, Communications & Control, Vol. I, No. 4, 2006, pp. 110-125.

18. TANGOUR, F., P. BORNE, **Presentation of some Metaheuristics for the Optimization of Complex Systems**, Studies in Informatics and Control, Vol.17, No. 2, 2008, pp.169-180.

19. FOONG, W. K., A. R. SIMPSON, H. R. MAIER, S. STOLP, **Ant Colony Optimization for Power Plant Maintenance Scheduling Optimization - A Five-station Hydropower System**, Annals of Operations Research, 159, 1, 2007, pp. 433-450.

20. ALLAOUA, B., A. Laoufi, **Optimal Power Flow Solution Using Ant Manners for Electrical Network**, Advances in Electrical and Computer Engineering, vol. 9, no. 1, 2009, pp. 34-39.

21. LIOUANE, N., I. SAAD, S. HAMMADI, P. BORNE, **Ant Systems & Local Search Optimization for Flexible Job Shop Scheduling Production**, International Journal of Computers, Communications & Control, Vol. II, No. 2, 2007, pp. 174-184.

22. NEGULESCU, S. C., C. V. KIFOR, C. OPREAN, **Ant Colony Solving Multiple Constraints Problem: Vehicle Route Allocation**, International Journal of Computers, Communications & Control, Vol. III, No. 4, 2008, pp. 366-373.

23. CRIŞAN, G. C., E. NECHITA, **Solving Fuzzy TSP with Ant Algorithms**, International Journal of Computers, Communications & Control, Vol. III, Supl. issue:S, 2008, pp. 228-231.

24. DZITAC, I., I. MOISIL, **Advanced AI Techniques for Web Mining**, Proceedings of the 10th WSEAS international conference on Mathematical methods, computational techniques and intelligent systems, 2008, pp.343-346.

25. DORIGO, M., **Optimization, Learning and Natural Algorithms**, PhD thesis, Politecnico di Milano, Italy, 1992.