

A Recognition Algorithm for a Class of Partitionable Graphs that Satisfies the Normal Graph Conjecture

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Abstract: A graph is normal if admits a clique cover and a stable set cover so that every clique may intersect every stable set. The Normal Graph Conjecture says that every $\{C_5, C_7, \overline{C_7}\}$ -free graph is normal. In this paper we prove this conjecture for the class of O-graphs and we give a recognition algorithm for O-graphs.

Keywords: Normal graphs, O-graphs, Partitionable graphs, Partite graphs, Triangulated graphs.

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1. Introduction

Normal graphs form a superclass of perfect graphs and can be considered as closure of perfect graphs by means of co-normal products [9] and graph entropy [8]. Perfect graphs have been characterized as those without odd holes and antiholes as induced subgraphs (Strong Perfect Graph Theorem, [5]). Korner and de Simone [6] observed that $C_5, C_7, \overline{C_7}$ are minimal, not normal graphs. As a generalization of the Strong Perfect Graph Theorem, Korner and de Simone conjectured that every $\{C_5, C_7, \overline{C_7}\}$ -free graph is normal (Normal Graph Conjecture, [11]). Wagler [15] proved the conjecture for the class of circulant graphs.

The entropy [10] of a graph is a functional depending both one the graph itself and on a probability distribution on its vertex set.

In [4] we find a recent survey of results on combinatorial optimization problems in which the objective function is the entropy of a discrete distribution.

In [8], the authors prove that a graphs is perfect if and only if it “splits graph entropy”. Using this derive the following strengthening of the normality of perfect graphs:

Let G be a perfect graph. Then G contains a family \mathcal{A} of independent sets and a family \mathcal{B} of cliques with the following properties:

- i) $|\mathcal{A}| + |\mathcal{B}| = k + 1$;*
- ii) the sets in \mathcal{A} (\mathcal{B}) cover all vertices;*
- iii) the incidence vectors of sets in \mathcal{A} (\mathcal{B}) are linearly independent;*
- iv) every $A \in \mathcal{A}$ intersects every $B \in \mathcal{B}$.*

The class of perfect graphs is important because many problems of interest in practice but intractable in general can be solved efficiently when restricted to the class of perfect graphs [6].

Partitionable graphs contain all the potential counterexamples to Berge’s famous Strong Perfect Graph Conjecture ([3]) which was proved by Chudnovski, Robertson, Seymour, and Thomas in [5]. Partitionable graphs ([7])

are one of the central objects in the theory of perfect graphs due to the following theorem of Lovasz: *A graph is perfect if and only if $\alpha(H)\omega(H)\geq n(H)$ for every induced subgraph H of G .*

In this paper we find a class of partitionable graphs that are not perfect, but are normal. We prove Normal Graph Conjecture for the class of O-graphs and give a recognition algorithm for O-graphs.

Throughout this paper, $G=(V,E)$ is a simple (i.e. finite, undirected, without loops and multiple edges) graph [2]. Let \bar{G} denote the complement graph of G . For $U\subseteq V$ let $G(U)$ denote the subgraph of G induced by U . By $G-X$ we mean the graph $G(V-X)$, whenever $X\subseteq V$, but we often denote it simply by $G-v$ ($\forall v\in V$), when there is no ambiguity. If $v\in V$ is a vertex in G , the neighborhood $N_G(v)$ denotes the vertices of $G-v$ that are adjacent to v . We write $N(v)$ when the graph G appears clearly from the context. The neighborhood of the vertex v in the complement of the graph G is denoted by $\bar{N}(v)$. For any subset S of vertices in G , the neighborhood of S is $N(S)=\cup_{v\in S}N(v)-S$ and $\bar{N}[S]=S\cup\bar{N}(S)$. A clique is a subset of V with the property that all the vertices are pairwise adjacent. The *clique number (density)* of G , denoted by $\omega(G)$ is the cardinal of the maximum clique. A clique cover is a partition of the vertices set such that each part is a clique. $\theta(G)$ is the cardinal of a smallest possible clique cover of G ; it is called the *clique cover number* of G . The *stability number* of G is $\alpha(G)=w(\bar{G})$; the *chromatic number* of G is $\chi(G)=w(G)$.

By P_n, C_n, K_n we mean a chordless path on $n\geq 3$ vertices, the chordless cycle on $n\geq 3$ vertices, and the complete graph on $n\geq 1$ vertices. If $e=xy\in E$, we also write $x\sim y$; we also write $x\not\sim y$ whenever x, y are not adjacent in G . A set A is totally adjacent (non adjacent) with a set B of vertices ($A\cap B=\emptyset$) if ab is (is not) edge, for any a vertex in A and any b vertex in B ; we note with $A\sim B$ ($A\not\sim B$). A graph G is F -free if none of its induced subgraphs is in F .

A graph G is called α -partitionable if $\alpha(G)=\theta(G)$ holds.

A graph G is *perfect* if $\alpha(H)=\theta(H)$ (or, equivalent, $\chi(H)=\omega(H)$) holds for every induced subgraph H of G , i.e. every induced subgraph is α -partitionable.

We call *matching* a set $F\subseteq E$ such that the edges in F are not adjacent.

A $[p,q,r]$ -partite graph is a graph whose set of vertices is partitioned in p stable sets S_1, S_2, \dots, S_p , each of them consisting of exactly q vertices and every subgraph induced by $S_i\cup S_j$ consists of exactly r independent edges, for $1\leq i<j\leq p$.

A *circulant* $C^{k,n}$ is a graph with nodes $1, \dots, n$ where ij is an edge if i and j differ by at most $k(mod n)$ and $i\neq j$.

The subset $A\subseteq V$ is called a *cutset* if $G-A$ is not connected. If, in addition, some $v\in A$ is adjacent to every vertex in $A-\{v\}$, then A is called a *star cutset* and v is called the *center* of A .

The paper is organized as follows. In Section 2 we give sufficient conditions for a graph to be normal. In Section 3 we give a recognition algorithm for O-graphs.

2. The Normal Graph Conjecture is True for O-graphs

Definition 1. *A graph G is called normal if G admits a clique cover C and a stable set cover S such that every clique in C intersects every stable set in S .*

In this section we address the problem of finding another class of α -partitionable graphs that are not perfect, but are normal.

Definition 2. *A graph G is partitionable if $\theta(G)=\alpha(G)$ and $\chi(G)=\omega(G)$.*

Definition 3. [13] *A graph G is O-graph if there exists a coloring of G and a coloring of \bar{G} , the complement of G , such that any class of colors of G intersects any class of colors of \bar{G} .*

Sufficient conditions for a graph to be normal are set by the following result. The equivalent conditions, (i) with (ii) and (i) with (iii), are stated and [12].

Theorem 1. *Let G be a graph with n vertices, m edges, stability number α and density ω . Then the following conditions are equivalent:*

(i) G is O-graph;

(ii) G is partitionable and $n = \alpha\omega$;

(iii) V can be partitioned in $\omega\alpha$ -stable set and $\alpha\omega$ -cliques;

(iv) G is $[\omega, \alpha, \alpha]$ -partite.

Proof. Let G an O-graph. We show that G fulfills condition (ii). Let $\chi(G) = p$, $\theta(G) = \chi(\bar{G}) = q$, $S = (S_1, \dots, S_p)$ a p -coloring of G and $Q = (Q_1, \dots, Q_q)$ a q -coloring of \bar{G} such that $S_i \cap Q_j \neq \emptyset$, for $i=1, \dots, p$ and $j=1, \dots, q$ hold. Then

$$\begin{aligned} \alpha(G) &\geq |S_i| = \left| \bigcup_{j=1}^q (S_i \cap Q_j) \right| \\ &= \sum_{j=1}^q |S_i \cap Q_j| \\ &= q = \theta(G) \geq \alpha(G) \end{aligned}$$

for all $i=1, \dots, p$. Therefore:

$$\alpha(G) = \theta(G) (\chi(G) = \theta(\bar{G}) = \alpha(\bar{G}) = \omega(G)) ;$$

$$|S_i| = \alpha(G), i=1, \dots, p; q = \alpha(G);$$

$$|Q_j| = \omega(G), j=1, \dots, q; p = \omega(G);$$

$$n = \sum_{i=1}^p |S_i| = \alpha(G)\omega(G).$$

Suppose that G satisfies condition (ii) and we show (i). Because $\chi(G) = \omega(G)$ ($=\omega$) and $n = \alpha(G)\omega(G)$, there exists an optimal colouring of G with ω stable sets S_1, \dots, S_ω with $|S_i| = \alpha$ ($\alpha(G)$), $i=1, \dots, \omega$. Similarly, there exists an optimal colouring Q_1, \dots, Q_α of \bar{G} with $|Q_j| = \omega$, $j=1, \dots, \alpha$. Obviously $S_i \cap Q_j \neq \emptyset$ for all $i=1, \dots, \omega$ and $j=1, \dots, \alpha$, which means that G is O-graph.

It is clear that (ii) is equivalent to (iii).

Let G be O-graph. We show that (iv) holds. Let $\{S_1, \dots, S_\omega\}$ a partition of G in $\omega\alpha$ -stable sets, and $\{Q_1, \dots, Q_\alpha\}$ a partition in $\alpha\omega$ -cliques with $S_i \cap Q_k \neq \emptyset$ for all $i=1, \dots, \omega$ and $k=1, \dots, \alpha$. The subgraph induced by $S_i \cup S_j$ ($i, j=1, \dots, \omega$, $i \neq j$) admits a matching with α elements, which obviously is maximal. Indeed, let $\{x_k^i\} = S_i \cap Q_k$. For $k \neq 1$ we have $x_k^i \neq x_k^i$ because $Q_k \cap Q_l = \emptyset$. So $S_i = \{x_1^i, \dots, x_\alpha^i\}$, $i=1, \dots, \omega$. Because $\{x_k^i, x_k^j\} \subseteq Q_k$ it follows that $x_k^i x_k^j \in E(G)$, for $k=1, \dots, \alpha$, $i, j=1, \dots, \omega$ with $i \neq j$. Consequently, the set of edges $\{x_k^i, x_k^j | k=1, \dots, \alpha\}$ is a matching in $G(S_i \cup S_j)$ for $i, j=1, \dots, \omega$ with $i \neq j$. Because \bar{G} is an O-

graph with $\alpha(\bar{G}) = \omega(G)$ and $\omega(\bar{G}) = \alpha(G)$

it follows that \bar{G} is $[\alpha, \omega, \omega]$ -partite.

Suppose that G satisfies (iv). We prove that (i) holds.

As G is $[\omega, \alpha, \alpha]$ -partite, it follows that there exists a partition of V in $S = \{S_1, \dots, S_\omega\}$ α -stable sets and, as \bar{G} is $[\alpha, \omega, \omega]$ -partite it follows that there exists a partition of V in $C = \{Q_1, \dots, Q_\alpha\}$ with Q_i cliques and $|Q_i| = \omega$ ($1 \leq i \leq \alpha$), which means that G is an O-graph and this completes the proof.

Corollary 1. *If G is a graph that satisfies one of the conditions (i), (ii), (iii) or (iv) in the above theorem, then G is a normal graph.*

Remark 1 ([12]). *For an O-graph G with n vertices, stability number α and density ω , the number of edges, m , verifies the following: $\alpha\omega(\omega-1)/2 \leq m \leq \alpha^2\omega(\omega-1)/2$.*

Proof. Because the disjoint reunion of $\alpha\omega$ -cliques is O-graph of minimal length it follows that $\alpha\omega(\omega-1)/2 \leq m$. Because for an ω -colouring $\{S_1, \dots, S_\omega\}$ with α -stable sets of the O-graph G and because for two non-adjacent vertices $x \in S_i$ and $y \in S_j$ ($i \neq j$) the graph $G' = G + xy$ is an O-graph with the same n , α , ω it follows that the ω -partite, complete graph $K_{\alpha, \dots, \alpha}$ is an O-graph of maximal length, that is $m \leq \alpha^2\omega(\omega-1)/2$.

3. An Algorithm for O-graph Recognition

At first, we recall the notion of weakly decomposition.

Definition 4. ([13], [14]) *A set $A \subset V(G)$ is called a weakly set of the graph G if $N_G(A) \neq V(G) - A$ and $G(A)$ is connected. If A is a weakly set, maximal with respect to set inclusion, then $G(A)$ is called a weakly component. For simplicity, the weakly component $G(A)$ will be denoted with A .*

Definition 5. ([13], [14]) *Let $G = (V, E)$ be a connected and non-complete graph. If A is a weakly set, then the partition $\{A, N(A), V - A \cup N(A)\}$ is called a weakly decomposition of G with respect to A .*

The name of "weakly component" is justified by the following result.

Theorem 2. ([13], [14]) *Every connected and non-complete graph $G=(V,E)$ admits a weakly component A such that $G(V-A)=G(N(A))+G(\overline{N}(A))$.*

Theorem 3. ([13], [14]) *Let $G=(V,E)$ be a connected and non-complete graph and $A\subset V$. Then A is a weakly component of G if and only if $G(A)$ is connected and $N(A)\sim\overline{N}(A)$.*

The next result, based on Theorem 2, ensures the existence of a weakly decomposition in a connected and non-complete graph.

Corollary 2. *If $G=(V,E)$ is a connected and non-complete graph, then V admits a weakly decomposition (A,B,C) , such that $G(A)$ is a weakly component and $G(V-A)=G(B)+G(C)$.*

Theorem 3 provides an $O(n+m)$ algorithm for building a weakly decomposition for a non-complete and connected graph.

Algorithm for the weakly decomposition of a graph ([13])

Input: A connected graph with at least two nonadjacent vertices, $G=(V,E)$.

Output: A partition $V=(A,N,R)$ such that $G(A)$ is connected, $N=N(A)$, $A\rightsquigarrow R = \overline{N}(A)$.

begin

$A:=$ any set of vertices such that $A\cup N(A)\neq V$

$N:=N(A)$

$R:=V-A\cup N(A)$

while ($\exists n\in N, \exists r\in R$ such that $nr\notin E$) *do*

begin

$A:=A\cup\{n\}$

$N:=(N-\{n\})\cup(N(n)\cap R)$

$R:=R-(N(n)\cap R)$

end

end

In [13] some applications of weakly decomposition have been depicted. In the following proposition we present two of those applications.

Proposition 1. *Let $G=(V,E)$ be connected, non-complete graph and (A,N,R) a weakly decomposition, with A the weakly component. The following hold:*

a) G is P_4 -free iff $A\sim N\sim R$ and $G(A)$, $G(N)$ and $G(R)$ are P_4 -free;

b) G is $K_{1,3}$ -free iff R and $N(n)\cap A$ are cliques, $\forall n\in N$ and $G-A$ and $G-R$ are $K_{1,3}$ -free.

Each of the above results lead to recognition algorithms for the corresponding graphs.

Proposition 2. *Let $G=(V,E)$ be connected, non-complete graph and (A,N,R) a weakly decomposition, with $G(A)$ the weakly component.*

G is triangulated iff:

1) N is a clique and

2) R and $G-R$ are triangulated.

Proof. We note the fact that N is a minimal cutset, because N is the set of neighbours of A and $N\sim R$. Then there exists a $P_3:anr$, for every $n\in N$, with $a\in A$ and $r\in R$. Because G is triangulated, N is a clique. Graphs $G(R)$ and $G-R$ are triangulated. Conversely, let G a graph that satisfies conditions 1) and 2). Let C_k ($k\geq 4$) a cycle induced in G . As N is a clique, it follows that $|N\cap V(C_k)|\leq 2$. If $N\cap V(C_k)=\emptyset$ then $C_k\subseteq G-N$, contradicting 2) or $A\not\rightsquigarrow R$ does not hold. If $|N\cap V(C_k)|=1$ then $C_k\not\subseteq(N\cup R)$ and $C_k\not\subseteq(N\cup R)$ and $C_k\not\subseteq G(A\cup N)$. So $V(C_k)\cap A\neq\emptyset$. Furthermore, $|V(C_k)\cap R|=1$. Because $A\rightsquigarrow R$, we obtain a contradiction. If $|V(C_k)\cap R|=2$, as $N\sim R$, it follows that $C_k\subseteq G-R$, contradicting 2).

Remark 2. *Let $G=(V,E)$ be connected, non-complete graph and (A,N,R) a weakly decomposition, with $G(A)$ the weakly component. Then we have:*

$$\alpha(G)=\max\{\alpha(G(A))+\alpha(G(R)), \alpha(G(N[A]))\}.$$

Proof. Any stable set of maximum cardinal either intersects R and so it has the cardinal

$$\alpha(G(A))+\alpha(G(R))$$

or it does not intersects R and so it has cardinal $\alpha(G(N[A]))$.

Proposition 3. *Let $G=(V,E)$ be connected, non-complete graph and (A,N,R) a weakly decomposition, with $G(A)$ the weakly component. If G is triangulated then we have:*

$$\alpha(G)=\alpha(G(A))+\alpha(G(R)).$$

Proof. We will use the formula given in Remark 2. Let $T\subset A\cup N=N[A]$ such that T is stable and $|T|=\alpha(G(N[A]))$. As $N=N(A)$ is a clique it follows that $|T\cap N(A)|\leq 1$. If $T\cap N(A)=\emptyset$ then $T\cup\{r\}$ is a stable set in $G(A\cup R)$. If $T\cap N(A)=\{n_0\}$ then $(T-\{n_0\})\cup\{r\}$

is stable in $G(A \cup R)$ ($r \in R$). It follows that, in the expression of $\alpha(G)$, the maximum is always obtained by the first component.

Every non-empty graph has a triangulated induced subgraph since every graph on three or fewer vertices is triangulated.

Balas and Yu ([1]) developed a polynomial-time algorithm to find a vertex-maximal triangulated induced subgraph of a given graph, and devised a branching strategy that has been used in many subsequent research efforts (see also [16]).

We give a recognition algorithm for O-graphs, based on the branching strategy by Balas and Yu, but using Proposition 2 and Proposition 3 to determine a stable set of maximum cardinal for a triangulated graph.

Procedure Stable-O-graph(G)

Input: A connected graph with at least two nonadjacent vertices, $G=(V,E)$.

Output: An answer to the question: is G an O-graph ?

begin

0. While $G \neq \emptyset$ do

1. find a maximal induced subgraph $H=G(T)$ such that H is triangulated
2. build in H a stable set S of maximum cardinal
3. build in H the disjoint cliques $K_1, \dots, K_{|S|}$
4. let $U = \bigcup_{i=1}^{|S|} K_i$
5. let $V-U = \{x_1, \dots, x_k\}$
6. for every i from 1 to k : $V_i = \overline{N}_G(x_i) - \{x_j \mid j < i\}$ determine a stable set S_i in $G(V_i)$ using Stable-O-graph ($G(V_i)$)
7. let S_0 a maximal stable set (one of S or $S_1 \cup \{x_1\}$ or $\dots S_k \cup \{x_k\}$)
8. $G \leftarrow G - S_0$
9. If all sets S_0 have the same cardinal then G is an O-graph.

$\alpha(G) = |S_0|$; $\alpha(G) = n / \alpha(G)$;

$\theta(G) = \alpha(G)$; $\lambda(G) = \alpha(G)$

end

In what follows, we give some remarks on the algorithm.

Step 1 is $O(m+n)$, according to [1].

Step 2 is the following:

begin

$S \leftarrow \emptyset$

$L \leftarrow \{H\}$ // L is a list of graphs

while ($L \neq \emptyset$) *do*

begin

extract an element F from L

if (F is complete) *then*

Return: $S \leftarrow S \cup \{v\}, \forall v \in V(F)$

else

begin

determine a weakly decomposition (A, N, R) for F

put in L the subgraph induced by A and the connected component of the subgraph induced by R

end

end

end

where:

$S = \{s_1, \dots, s_{|S|}\}$

The test " F is complete" is done as follows: if there exists a vertex v in F whose neighbor list (or the degree of v) is not $V(F) - \{v\}$ (corresponding to $|V(F)| - 1$) then F is not complete.

Step 2 is $O(nm)$.

Step 3 is the following:

begin

for $i \leftarrow 1$ to $|S|$ *do*

begin

$K_i \leftarrow \{s_i\}$

for every v in $T - S$ *do*

if $\{v\} \sim K_i$ *then*

$K_i \leftarrow K_i \cup \{v\}$

$T \leftarrow T - K_i$

end

end

Step 3 is $O(n^3)$.

In Step 4 we have:

$K_1, \dots, K_{|S|}$ is a covering with cliques of $G(U)$

and $\alpha(G(U)) = \theta(G(U)) = |S|$

In Step 5 we have an arbitrary order of the vertices in $V-U$.

In Step 6 we use Stable-O-graph for $G(V_i)$.

In Step 7, either S or $S_1 \cup \{x_1\}, \dots, S_k \cup \{x_k\}$ is a maximum stable set for G ([16]).

It follows that the complexity of the recognition algorithm for O-graphs is $O(n^3)$.

4. Conclusions and Future Work

In this paper we have proved that the normal graph conjecture is true for O-graphs and we have presented a recognition algorithm for these graphs.

Our future work will verify the conjecture on classes of graphs characterized by means of forbidden subgraphs.

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A Usability Assistant for the Heuristic Evaluation of Interactive Systems

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Abstract: The increasing demands for usable interactive systems in the context of limited project budgets bring in front the need for faster and cheaper evaluation methods. Heuristic evaluation is a kind of inspection method that proved to be cost effective. Typically, the method involves a small number of evaluators that are testing the interactive system against a set of usability principles called heuristics. A way to increase the efficiency of usability evaluation methods is to provide evaluators with software tools able to assist in documenting and recording of usability problems. This paper presents a software assistant for usability evaluation which provides with various facilities to conduct heuristic evaluation: definition of the tasks set, specification of heuristics used, and documenting of usability problems. In order to support the specific requirements of a target application domain a set of usability guidelines could be specified that are detailing the heuristic set. These guidelines could be consulted during usability problem identification and specification. This way, a broader range of evaluator preferences and requirements could be accommodated.

Keywords: usability, heuristic evaluation, usability evaluation assistant, software tools, tools for working with guidelines

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1. Introduction

The increasing demand for usable interactive systems in the context of a limited project budget and strict deadlines creates an extra pressure for evaluators and designers. This reveals the need for faster and cheaper evaluation methods.

Depending on the purpose and the moment when it is done, usability evaluation could be formative or summative (Scriven, 1991). Formative usability evaluation is performed in an iterative development cycle and aims at finding and fixing usability problems as early as possible (Teofanos and Quesenbery, 2005). The sooner these problems are identified, the less costly the effort to fix them is.

Formative usability evaluation can be carried on by conducting an expert-based usability inspection and / or by conducting user testing with a small number of users. In this last case, the evaluation is said to be user-centered, as opposite to expert-based formative evaluation.

Heuristic evaluation is a kind of inspection method which typically involves a small number of evaluators that are testing the

interactive system against a set of usability principles called heuristics. This method proved to be cost effective and is widely used by the usability practitioners' community (76% according to UPA Survey, 2005).

Heuristic evaluation provides with two kinds of measure: quantitative (number of usability problems per severity level) and qualitative (detailed descriptions of individual usability problems).

The quality of usability problem description is critical for the usefulness of a usability report. On the other hand, there is a lot of work to be done in order to properly describe each usability problem. A way to increase the efficiency of any evaluation method is to provide evaluators with suitable tools able to assist them during the evaluation process. As shown by Hvannberg et al. (2007), not only these problem registration tools are improving the immediate management of usability problems but they are also supporting a structured usability problem reporting.

This paper presents a software tool for usability evaluation which provides several facilities to conduct a heuristic evaluation: