Solving Dynamic Vehicle Routing: An Alternative Metaheuristic Approach

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Abstract: An adapted Evolution strategy is proposed for solving the Dynamic (General) Vehicle Routing Problem (DVRP). Several Mutation and crossover operators are designed to deal with real-time demand information which is available only at the day of operation. A simulation was carried out in which intelligent planning of new online orders are dealt with. Several problems were generated to test the proposed algorithm. The problems were solved twice. First, they are solved off-line in which all orders are known prior to the day of operation. Second, they were solved in which orders are dynamic. The competitive ratio gave an average of 0.65.

Keywords: Dynamic Vehicle Routing; General Vehicle Routing; Adapted Evolution Strategy; Intelligent Real-time Planning.

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1. Introduction

Dynamic (General) Vehicle Routing Problem (DVRP) can be considered as a good example of a distribution context, because of the fact that intelligent manipulation of real-time information can distinguish between one company and another by superior on-time service. Problems of both generic vehicle routing (V. R. P.) and dynamic vehicle routing (DVRP) are identical. But in VRP all routing and demand information are certainly known prior the day of operation; whereas in DVRP part or all of the necessary information is available only at the day of operation. Significance of DVRP is crystallized by the variety of environments it can model. Additional assets are the transportation of elderly or physically handicapped and emergency services (e.g. police, fire and ambulance dispatching).

The paper is organized as follows: Section 2 gives the previous researches using meta-heuristics. The mathematical formulation of the problem is in section 3. The proposed mutation and crossover operators are explained in section 4 and 5. The proposed algorithm is forwarded in section 6 and simulation and results in section 7.

2. Previous Studies Using Metaheuristics

Braysy and Gendreau [1, 2] have prepared a comprehensive survey on the utilization of Meta-heuristics for vehicle routing problem with time windows (VRPTW). Examples are simulated Annealing, Genetic Algorithms, Ant Systems, and Tabu Search. Simulated Annealing helps to allow moves resulting in solutions of worse quality to keep away from locally optimal solutions [5].

Genetic Algorithms, Ant Systems and Tabu Search are memory based methods considered as Adaptive Memory Programming (AMP) Methods [13]. Examples of (AMP) Methods are the Genetic Algorithms for the dynamic pickup and drop problem (PDP) presented by Pankratz [8] and an Ant System for the dynamic VRP by Montemanni [7]. Tabu Search Algorithms for the dynamic PDP by Mitrovic - Minic [6]. Polacek [9] presents an algorithm for the multi-depot (VRPTW) as an example of VNS algorithms (Variable Neighborhood Search Algorithm) for vehicle routing problem. Size of the design of a neighborhood search approach is tackled by Schrimpf [11] and Ropke and Pisinger [10] as they presented similar LNS (Large Neighborhood Search) Algorithms utilizing fast insertion heuristics for the re-insertion of transportation requests. The LNS approach is very well suited for rich vehicle routing problems in which data may change dynamically (Goel and Gruhn [3]).

3. Mathematical Formulation of the Problem

The Contribution of each vehicle $v \in V$ to the objective function is

$$\sum_{o \in O} y^{v}_{n(0,1)} \, p_o - \sum_{(n,m) \in A} x^{v}_{nm} \, c^{v}_{nm} \tag{0}$$

The first term represents the accumulated revenue of served orders, the second term represents the accumulated costs for vehicle movements. [3]

The General Vehicle Routing Problem (GVRP) is maximize

$$\sum_{v \in V} \left(\sum_{o \in O} y^{v}_{n(0,1)} \, p_o - \sum_{(n,m) \in A} x^{v}_{nm} \, c^{v}_{nm} \right) \tag{1}$$

Subject to

$$\sum_{(n,m) \in A} x^{v}_{nm} = \sum_{(m,n) \in A} x^{v}_{mn} \quad \text{for all } v \in V, n \in N \tag{2}$$

$$y^{v}_{n} = \sum_{(n,m) \in A} x^{v}_{nm} \quad \text{for all } v \in V, n \in N \tag{3}$$

$$\sum_{v \in V} y^{v}_{n} \leq 1 \quad \text{for all } n \in N \tag{4}$$

$$\rho_{n(v,1)} = r_{n(v,1)} \quad \text{for all } v \in V \tag{5}$$

$$\text{for all } v \in V, (n,m) \in A \text{ with } m \neq n_{(v,1)} : \text{if } x^{v}_{nm} = 1 \text{ then } \rho_m = \rho_n + r_m \tag{6}$$

$$\text{for all } v \in V, n \in N : \text{if } y^{v}_{n} = 1 \text{ then } 0 \leq \rho_n \leq r^{v} \tag{7}$$

$$\text{for all } v \in V, (n,m) \in A \text{ with } m \neq n_{(v,1)} : \text{if } x^{v}_{nm} = 1 \text{ then } t_n + d^{v}_{nm} \leq t_m \tag{8}$$

$$t^{\min}_{n} \leq t_n \leq t^{\max}_{n} \quad \text{for all } n \in N \tag{9}$$

$$t_{n(v,\mu)} \leq t_{n(v,\mu+1)} \quad \text{for all } v \in V, 1 \leq \mu \leq \lambda_v \tag{10}$$

$$t_{n(o,\mu)} \leq t_{n(o,\mu+1)} \quad \text{for all } o \in O, 1 \leq \mu \leq \lambda_o \tag{11}$$

$$y^{v}_{n(v,\mu)} = 1 \quad \text{for all } v \in V, 1 \leq \mu \leq \lambda_v \tag{12}$$

$$\sum_{\mu=1}^{\mu \leq \lambda_o} y^{v}_{n(o-\mu)} = \lambda_o y^{v}_{n(o,1)} \quad \text{for all } o \in O, v \in V \tag{13}$$

$$y^{v}_{n(o,1)} \leq \delta_{ov} \quad \text{for all } o \in O, v \in V \tag{14}$$

$$x^{v}_{nm} \in \{0,1\} \text{ for all } v \in V, (n,m) \in A, \quad y^{v}_{n} \in \{0,1\} \text{ for all } v \in V, n \in N \tag{15}$$

The objective function (1) represents the accumulated revenue of all served orders reduced by the costs for all arcs used in the solution. Equation (2) represents the flow conservation constraints which impose that each vehicle reaching a node $n \in N$ also departs from the node. Constraints (3) and (4) impose that each node is visited at most once. Constraints (5) to (7), and (8) and (9) represent capacity and time window constraints (10) and (11) are the precedence constraints imposed on the sequence in which nodes associated to vehicles and orders are visited. Equation (12) imposes that each vehicle visits all nodes associated to it. Equation (13) represents the grouping constraints which imposes that all locations belonging to an order are visited by the same vehicle. Inequality (14) represents the compatibility constraints which impose the orders are only assigned to vehicle capable of serving the order. Eventually, Constraints (15) impose that the values of x_{nm}^{v} and y_{n}^{v} are binary.

The GVRP is a generalization of the classical models described in the previous sections. In contrast to the classical models, not all transportation requests must be served in the GVRP. The requirements that all transportation requests are served, however, can be fulfilled by assigning sufficiently large revenue p_o to each transportation request $o \in O$. This guarantees that any solution in which all transportation requests are served has a higher objective function value than every solution in which at least one transportation request is not served.

4. Mutation Operators

4.1 Mutation Type 1

The first mutation operator is the basis of all insertion methods and inserts all locations belonging to an unscheduled order into the tour of a vehicle, subject to compatibility and precedence constraints imposed on the GVRP.

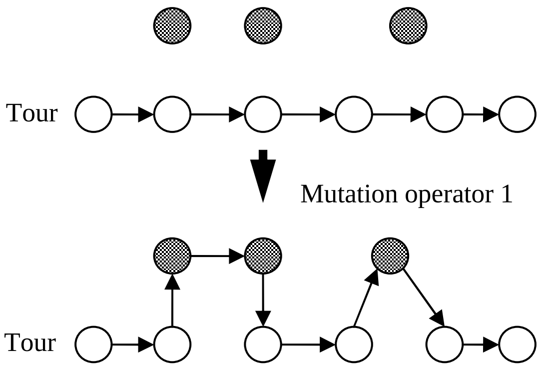

Figure 1. First Mutation Operator

Figure 1 illustrates the insertion of an unscheduled order to the tour of a vehicle.

4.2 Mutation Type 2

The second mutation operator is the inverse of the first operator and removes all locations belonging to a scheduled order from a tour.

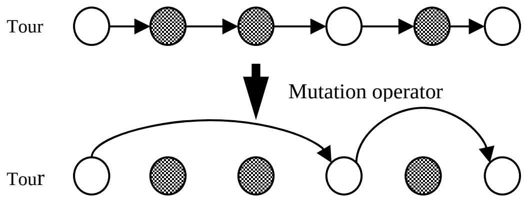

Figure 2. Second Mutation Operator

Figure 2 illustrates the removal of a transportation request from the tour of a vehicle.

4.3 Mutation Type 3

The third mutation operator rearranges locations belonging to one order to other positions within the same tour, subject to precedence constraints imposed on the GVRP.

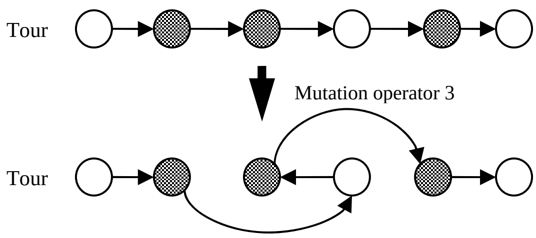

Figure 3. Third Mutation Operator

The third operator is illustrated in figure 3 and can be interpreted as combination of the first and second mutation operators.

5. Crossover Operators

5.1 Crossover Type 1

The first crossover operator moves all locations belonging to an order from one tour to another, subject to compatibility and precedence constraints imposed on the GVRP.

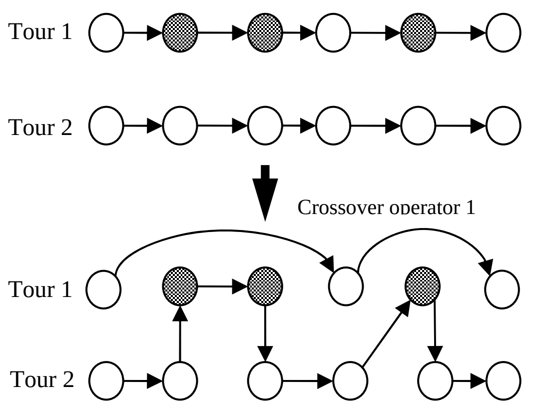

Figure 4. First Crossover Operator

This operator is illustrated in figure 4.

5.2 Crossover Type 2

The second crossover operator is a combined first operator which shifts the locations belonging to two orders in different tours to the respective other tour. Compatibility and precedence constraints imposed on the GVRP are taken into account by this operator.

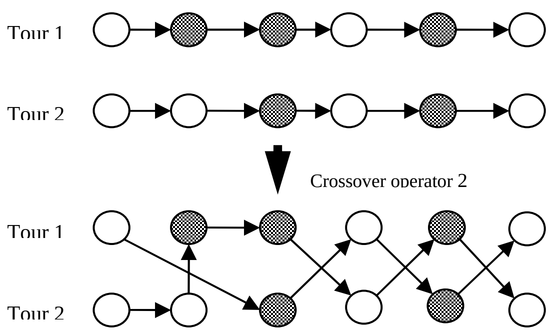

Figure 5. Second Crossover Operator

This operator is illustrated in figure 5.

6. Adapted Evolution Strategy

The $(1,\lambda)$ evolution strategy [4] is adopted where there is one initial parent and $\lambda=4$ offspring are produced. The initial parent is not included in the selection process rather the parent of the following generation should be selected only from λ offspring. The algorithm of this strategy is formulated as follows:

Step 0: (Initialization)

An initial population of one parent, is characterized by its genotype of *n* genes (*n* tours) which unambiguously determine the vitality or fitness for survival.

Step 1: (Variation)

The parent produces $\lambda=4$ offspring so that a total of λ new individuals are available by the application of:

Step 1-a:

Apply mutation operators with equal probabilities (on the initial parent)

Step 1-b:

Apply crossover operators with equal probabilities (on produced offspring)

The genotypes of the descendents differ from the initial parent. The number of genes, however, remains to be *n* in the following, i.e., neither gene duplicate nor gene deletion occurs.

Step 2: (Filtering)

Only one best of the λ offspring becomes the parent of the following generation.

7. Simulation and Results

First, the number of vehicles $|V|$ (the no. of tours), the number of orders known at the beginning of the simulation $|O_0|$, the number of orders that arrive dynamically at timestep *t* $|O_t|$ and the length of the time windows (in hours) τ are given. The distance between sources and destinations are also given. Travel distances are based in the direct distances. In order to consider the average deviation occurring in transport, they are multiplied by 1.4 travel costs are proportional to travel distances. The revenue is gained by the completion of an order.

Computational experiments were performed on a personal computer with core (2) Duo 2.33 GHz processor and 4 GB of RAM. In the simulation of 12 hours of dynamic planning, the algorithm was allowed only 1 minute of computing time per timestep (representing one hour in the simulation scenario). At each timestep all previously unscheduled transportation requests and new transportation requests are inserted to the tours by the operators discussed. The capacity K for each vehicle is constant.

To measure the performance of this on-line algorithm, "competitive analysis" introduced by sleator and Trajan [12] is used. The competitive ratio *cr* is defined as follows:

$$cr = \sup_I \frac{Z(I)}{Z^*(I)}$$

Where $Z(I)$ is the cost of solution by the proposed algorithm for instance I and $Z^*(I)$ is the optimal cost found by an ideal offline algorithm which had to access the entire instance I including dynamic requests beforehand. The competitive analysis framework offers a measure for evaluating the performance of a certain on-line routing policy based on the worst- case ratio between this policy and optimal offline policy. In other words, this ratio quantifies the loss of cost-efficiency stemming from the lack of full information. The following table gives the results for some instances generated to test this proposed method.

Table 1. Simulation Results

problem					Z (I)	Z^* (I)	cr
No.	$\|V\|$	$\|O\|$	$\|O_t\|$	t			
1	100	300	20	2	164523	113521	0.69
2	100	300	20	12	266190	178348	0.67
3	250	750	50	2	526011	336647	0.64
4	250	750	50	12	701461	441921	0.63
5	500	1500	100	2	1112185	667312	0.6
6	500	1500	100	12	1402106	813222	0.58

8. Conclusion

An adapted Evolution Strategy is forwarded with operators specially designed to solve the real-time dynamic (general) vehicle routing problem. Not all of the orders were known prior to the day of distribution. This incurred the intelligent dynamic planning and re-planning of routes (tours). Several test problems were generated and solved both by an offline suitable algorithm and the proposed real-time algorithm giving an average of 65% competitive ratio.

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