An Optimal Instrumental Variable Identification Approach for Left Matrix Fraction Description Models

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Abstract: The main contribution of this paper is the extension of the Simplified Refined Instrumental Variable (SRIV) identification algorithm for SISO systems to the identification of MIMO systems described by a Left Matrix Fraction Description (LMFD). The performance of the extended algorithm is compared to the well-known MIMO four-step instrumental variable (IV4) algorithm. Monte Carlo simulations for different signal to noise ratios are conducted to assess the performance of the algorithm.

Keywords: Multivariable System Identification, SRIV, LMFD, IV4, Steiglitz–McBride

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1. Introduction

Advanced engineering applications require suitable mathematical models structures. These model structures are either obtained mathematically using physical laws or experimentally using system identification techniques.

Basically, System identification deals with the problem of obtaining "approximate" models of dynamic systems from measured input-output data.

Many different identification methods have been proposed for both SISO and MIMO systems. Among these we can mention the PEM and n4sid [1] for the identification of state space models, and the ARX, IVX and IV4 methods [1] for systems modeled by a Left Matrix Fraction Description. These methods have been implemented and are available in the Matlab System identification toolbox [2] .

An interesting identification algorithm was proposed by Young [3] [4] and is referred to as the Simplified Refined Instrumental Variable (SRIV). It is an optimal instrumental variable algorithm proposed for the identification of noisy SISO systems.

It is the purpose of this paper to extend the algorithm for the identification of noisy MIMO systems described by a Left Matrix Fraction Description. The performance of the extended algorithm is then compared to that of the MIMO IV4 algorithm used as a benchmark..

In this paper the m-input p-output noisy multivariable system is assumed to be modeled in matrix fraction description form as :

$$y[k] = A^{-1}(q^{-1})B(q^{-1})u[k] + e[k] \tag{1}$$

where

$$A(q^{-1}) = I_p + A_1 q^{-1} + ...A_{na} q^{-na}$$

$$B(q^{-1}) = B_1 q^{-1} + ...B_{nb} q^{-nb}$$

$e[k]$ is a white noise vector and q is the shift operator

2. The MIMO IV4 Algorithm

Given a MIMO system modeled as :

$$y[k] = A^{-1}(q^{-1})B(q^{-1})u[k] + e[k] \quad (2)$$

The objective is to identify the matrix coefficients $A_i \in R^{p \times p}$ and $B_i \in R^{p \times m}$ of the matrix polynomials $A(q^{-1})$ and $B(q^{-1})$. Defining a new vector $v[k] = A(q^{-1})e[k]$ we can write:

$$\begin{aligned} v^T[k] &= y^T[k] + y^T[k-1]A_1^T ... + y^T[k-na]A_{na}^T \\ &\quad - u^T[k]B_0^T ... - u^T[k-nb]B_{nb}^T \end{aligned} \quad (3)$$

or, $v^T[k] = y^T[k] - \varphi^T[k]\theta$ (4)

where

$$\varphi^T[k] = \begin{bmatrix} -y^T[k-1] & . & . & -y^T[k-na] & u^T[k-1] & . & . & u^T[k-nb] \end{bmatrix}$$

and $\theta = \begin{bmatrix} A_1^T & . & . & . & A_{na}^T & B_1^T & . & . & . & B_{nb}^T \end{bmatrix}^T$

The MIMO IV4 algorithm may be summarized as follows :

Algorithm (IV4 Algorithm)

- Determine the Least Squares estimate $\hat{\theta}_{ls}$ *using i/o data as*

$$\hat{\theta}_{ls} = [\Phi^T \Phi]^{-1} \Phi^T Y \quad (5)$$

where

$$Y = \begin{bmatrix} y^T[n+1,:] \\ . \\ . \\ . \\ y^T[N,:] \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \Phi_y \vdots \Phi_u \end{bmatrix}$$

$$\Phi_y = \begin{bmatrix} \begin{bmatrix} -y^T[n,:] \\ . \\ . \\ . \\ -y^T[N-1,:] \end{bmatrix} \begin{matrix} \dots \\ \dots \\ \dots \\ \dots \\ \dots \end{matrix} \begin{bmatrix} -y^T[n-n_a+1,:] \\ . \\ . \\ . \\ -y^T[N-n_a,:] \end{bmatrix} \end{bmatrix}$$

$$\Phi_u = \begin{bmatrix} \begin{bmatrix} u^T[n,:] \\ . \\ . \\ . \\ u^T[N-1,:] \end{bmatrix} \begin{matrix} \dots \\ \dots \\ \dots \\ \dots \\ \dots \end{matrix} \begin{bmatrix} u^T[n-n_b+1,:] \\ . \\ . \\ . \\ u^T[N-n_b,:] \end{bmatrix} \end{bmatrix}$$

and n=na

- Simulate the model output

$$z[k] = A^{-1}(q^{-1})B(q^{-1})u[k] \quad (6)$$

$z[k]$ *to get*

- Estimate the parameters $\hat{\theta}_{iv}$ *as*

$$\hat{\theta}_{iv} = [\Psi^T \Phi]^{-1} \Psi^T Y \quad (7)$$

where

$$\Psi = \left[\Phi_z \vdots \Phi_u \right]$$

$$\Phi_z = \begin{bmatrix} \begin{bmatrix} -z^T[n,:] \\ . \\ . \\ . \\ -z^T[N-1,:] \end{bmatrix} \begin{matrix} \dots \\ \dots \\ \dots \\ \dots \\ \dots \end{matrix} \begin{bmatrix} -z^T[n-n_a+1,:] \\ . \\ . \\ . \\ -z^T[N-n_a,:] \end{bmatrix} \end{bmatrix}$$

- Compute the residual as

$$e_{iv}[k] = A(q^{-1})y[k] - B(q^{-1})u[k] \quad (8)$$

where $A(q^{-1})$ *and* $B(q^{-1})$ *are extracted from* $\hat{\theta}_{iv}$

- Compute a new residual vector $e_s[k]$ *as :*

$$e_s[k] = e_{iv1}[k] ... + e_{ivp}[k] \quad (9)$$

- Estimate a SISO AR model of order p(na+nb) for* $e_s[k]$ *:*

$$e_s[k] = F^{-1}(q^{-1})e[k] \quad (10)$$

where

$$F(q^{-1}) = 1 + f_1 q^{-1} + ... + f_{p*(na+nb)} q^{-p*(na+nb)}$$

- Perform SISO filtering on the components of $u[k]$ *and* $y[k]$*:*

$$u_{if}[k] = F^{-1}(q^{-1})u_i[k] \quad i=1 \text{ to } m \tag{11}$$

$$y_{jf}[k] = F^{-1}(q^{-1})y_j[k] \quad j=1 \text{ to } p \tag{12}$$

- Compute the auxilary filtred model :

$$z_f[k] = A^{-1}(q^{-1})B(q^{-1})u_f[k] \tag{13}$$

- Estimate the final parameters by the IV method using the filtered signals:

$$\hat{\theta} = [\Psi_f^T \Phi_f]^{-1} \Psi_f^T Y_f \tag{14}$$

where

$$\Phi_f = \left[\Phi_{yf} \vdots \Phi_{uf} \right]$$

$$\Psi_f = \left[\Phi_{zf} \vdots \Phi_{uf} \right]$$

$$\Phi_{yf} = \begin{bmatrix} \begin{bmatrix} -y_f^T[n,:] \\ . \\ . \\ . \\ -y_f^T[N-1,:] \end{bmatrix} \cdots \begin{bmatrix} -y_f^T[n-n_a+1,:] \\ . \\ . \\ . \\ -y_f^T[N-n_a,:] \end{bmatrix} \end{bmatrix}$$

$$\Phi_{uf} = \begin{bmatrix} \begin{bmatrix} u_f^T[n,:] \\ . \\ . \\ . \\ u_f^T[N-1,:] \end{bmatrix} \cdots \begin{bmatrix} u_f^T[n-n_b+1,:] \\ . \\ . \\ . \\ u_f^T[N-n_b,:] \end{bmatrix} \end{bmatrix}$$

$$\Phi_{zf} = \begin{bmatrix} \begin{bmatrix} -z_f^T[n,:] \\ . \\ . \\ . \\ -z_f^T[N-1,:] \end{bmatrix} \cdots \begin{bmatrix} -z_f^T[n-n_a+1,:] \\ . \\ . \\ . \\ -z_f^T[N-n_a,:] \end{bmatrix} \end{bmatrix}$$

3. The Extended SRIV Algorithm

The SISO SRIV is concerned with the problem of estimating the model parameters in terms of the following least squares cost function,

$$J = \sum_{k=1}^{N} \hat{e}[k] \tag{15}$$

where $\hat{e}[k]$ is the following error function obtained directly by inspection of the model

$$\hat{e}[k] = y[k] - \frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})} u[k] \tag{16}$$

while N is the total sample size and the "hat" indicates estimated values.

This error function is clearly nonlinear in the parameters of the unknown polynomials. However, it can be written alternatively as,

$$\hat{e}[k] = \frac{1}{\hat{A}(q^{-1})} \left\{ \hat{A}(q^{-1}) y[k] - \hat{B}(q^{-1}) u[k] \right\} \tag{17}$$

or, $\hat{e}[k] = \hat{A}(q^{-1}) y^{*}[k] - \hat{B}(q^{-1}) u^{*}[k]$ (18)

where $y^{*}[k]$ and $u^{*}[k]$ are the "prefiltered" signals defined as follows,

$$\hat{y}[k] = \frac{1}{\hat{A}(q^{-1})} y[k] \tag{19}$$

$$\hat{u}[k] = \frac{1}{\hat{A}(q^{-1})} u[k] \tag{20}$$

Equation (18) is now linear-in-the-parameters of the transfer function model, so that normal IV methods could be used to estimate the parameters if it were possible to perform the prefiltering operations in (19) and (20). In practice, of course, the parameters of $A(q^{-1})$ are unknown a priori and so this prefiltering operation will be made adaptive, with the algorithm "learning" the parameters of the polynomials in an iterative basis.

The extended SRIV algorithm makes use of the Kronicker product and the col{.} operator that transforms a matrix into a column vector by stacking its columns on top of one another.

Expanding equation (2) gives:

$$\begin{aligned} A(q^{-1}) e[k] = {} & y[k] + A_1 y[k-1] \ldots + A_{na} y[k-na] \\ & - B_1 u[k-1] \ldots - B_{nb} u[k-nb] \end{aligned} \tag{21}$$

Equation (21) can be written using the Kronicker operator as

$$\begin{aligned} A(q^{-1}) e[k] = {} & [I_p \otimes y[k]^T] col(I_p) + [I_p \otimes y[k-1]^T] col(A_1^T) \ldots \\ & + [I_p \otimes y[k-n_a]^T] col(A_{na}^T) - \\ & [I_p \otimes u[k-1]^T] col(B_1^T) \ldots \\ & - [I_p \otimes u[k-n_b]^T] col(B_{nb}^T) \end{aligned} \tag{22}$$

Solving for $e[k]$ gives

$$e[k]=A^{-1}(q^{-1})[I_p\otimes y[k]^T]col(I_p)+[A^{-1}(q^{-1})[I_p\otimes y[k-1]^T]...+A^{-1}(q^{-1})[I_p\otimes y[k-n_a]^T]-A^{-1}(q^{-1})[I_p\otimes u[k-1]^T]...$$

$$-A^{-1}(q^{-1})[I_p\otimes u[k-n_b]^T]\begin{bmatrix} col(A_1^T) \\ . \\ . \\ . \\ col(A_{na}^T) \\ col(B_1^T) \\ . \\ . \\ . \\ col(B_{nb}^T) \end{bmatrix} \quad (23)$$

Or simply

$$e[k]=A^{-1}(q^{-1})[I_p\otimes y[k]^T]col(I_p)-\varphi_f^T[k]\theta \quad (24)$$

The MIMO SRIV Algorithm is as follows :

Algorithm (SRIV algorithm)

- *Initialize* $A(q^{-1})=I_p$ *(25)*

- *Perform MIMO least squares to get an initial estimate of* θ

$$\hat{\theta}=[\Phi_f^T\Phi_f]^{-1}\Phi_f^T Y_f \quad (26)$$

- *Compute the auxiliary signal*

$$z[k]=A^{-1}(q^{-1})B(q^{-1})u[k] \quad (27)$$

- *Perform MIMO filtering on the signals* $u[k]$, $y[k]$ *and* $z[k]$

$$y_{ff}[k]=A^{-1}(q^{-1})[I_p\otimes y[k]^T]col(I_p) \quad (28)$$

$$y_f[k]=A^{-1}(q^{-1})[I_p\otimes y[k]^T] \quad (29)$$

$$u_f[k]=A^{-1}(q^{-1})[I_p\otimes u[k]^T] \quad (30)$$

$$z_f[k]=A^{-1}(q^{-1})[I_p\otimes z[k]^T] \quad (31)$$

- *Estimate* θ *using IV method*

$$\hat{\theta}=[\Psi_f^T\Phi_f]^{-1}\Psi_f^T Y_f \quad (32)$$

where

$$Y_f = \begin{bmatrix} y_{ff}[1+p*na] \\ . \\ . \\ . \\ y_{ff}[p*N] \end{bmatrix}$$

$$\Phi_f = \left[\Phi_{yf} \vdots \Phi_{uf}\right]$$

$$\Psi_f = \left[\Phi_{zf} \vdots \Phi_{uf}\right]$$

and Φ_{uf}, Φ_{yf} *and* Φ_{zf} *are constructed as follows :*

$$\Phi_{uf} = \left[\begin{bmatrix} u_f[1+p*(na-k),:] \\ . \\ . \\ . \\ u_f[p*(N-k),:] \end{bmatrix} \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{matrix}\right] \quad k=1 \text{ to } nb$$

$$\Phi_{yf} = \left[\begin{bmatrix} -y_f[1+p*(na-k),:] \\ . \\ . \\ . \\ -y_f[p*(N-k),:] \end{bmatrix} \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{matrix}\right] \quad k=1 \text{ to } na$$

$$\Phi_{zf} = \left[\begin{bmatrix} -z_f[1+p*(na-k),:] \\ . \\ . \\ . \\ -z_f[p*(N-k),:] \end{bmatrix} \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{matrix}\right] \quad k=1 \text{ to } na$$

- *If no convergence, go to step 3.*

Remark 1: The convergence test used in the last step of the algorithm is the relative error of the parameters in percent defined as :

$$100 \left\| \frac{\hat{\theta}(i+1) - \hat{\theta}(i)}{\hat{\theta}(i)} \right\|_2 < \varepsilon \tag{33}$$

where $\hat{\theta}_{(i)}$ denotes the estimated parameter vector at iteration i, and ε is a given tolerance in percent for terminating the iterative search.

Remark 2: A stability check must be performed for both algorithms to force all the roots of the polynomial $\det[A(q^{-1})] = 0$ to lie within the unit circle.

Remark 3: Steiglitz and McBride [5][6] have suggested an iterative approach to identify a SISO linear system subject to white noise measurement noise . Extension to MIMO systems can be done as follows :

Algorithm (Extended Steiglitz-McBride)

- *Initialize* $A(q^{-1}) = I_p$ *(34)*

- *Perform MIMO filtering on the signals u[k] and y[k]*

$$y_{ff}[k] = A^{-1}(q^{-1})[I_p \otimes y[k]^T]col(I_p) \quad (35)$$

$$y_f[k] = A^{-1}(q^{-1})[I_p \otimes y[k]^T] \quad (36)$$

$$u_f[k] = A^{-1}(q^{-1})[I_p \otimes u[k]^T] \quad (37)$$

- *Compute* $\hat{\theta}$ *using LS method*

$$\hat{\theta} = [\Phi_f^T \Phi_f]^{-1} \Phi_f^T Y_f \quad (38)$$

where

$$Y_f = \begin{bmatrix} y_{ff}[1 + p * na] \\ . \\ . \\ . \\ y_{ff}[p * N] \end{bmatrix}$$

$$\Phi_f = \begin{bmatrix} \Phi_{yf} \vdots \Phi_{uf} \end{bmatrix}$$

and Φ_{uf}, Φ_{yf} *and are constructed as follows :*

$$\Phi_{uf} = \begin{bmatrix} \begin{bmatrix} u_f[1 + p*(na-k), :] \\ . \\ . \\ . \\ u_f[p*(N-k), :] \end{bmatrix} \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{matrix} \end{bmatrix} \text{ k=1 to nb}$$

$$\Phi_{yf} = \begin{bmatrix} \begin{bmatrix} -y_f[1 + p*(na-k), :] \\ . \\ . \\ . \\ -y_f[p*(N-k), :] \end{bmatrix} \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{matrix} \end{bmatrix} \text{ k=1 to na}$$

- *If no convergence, go to step 2.*

The Steiglitz-McBride technique is therefore close to the SRIV technique. The main difference lies in the fact that the SRIV method uses the IV method while the Steiglitz-McBride technique utilizes the LS method only.

4. Simulation Example

A simulation example is presented to illustrate the performance of the MIMO SRIV method as compared to that of the MIMO IV4 and MIMO Least squares estimation methods. Let's consider the 2-input 2-output process (ie, p=m=2) described in LMFD as

$$y[k] = A^{-1}(q^{-1})B(q^{-1})u[k] + e[k] \tag{39}$$

where

$$A(q^{-1}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.5 & -0.4 \\ 0.3 & -0.6 \end{bmatrix} q^{-1} + \begin{bmatrix} -0.1 & -0.3 \\ 0.2 & 0.3 \end{bmatrix} q^{-2}$$

$$= \begin{bmatrix} 1+0.5q^{-1}-0.1q^{-2} & -0.4q^{-1}-0.3q^{-2} \\ 0.3q^{-1}+0.2q^{-2} & 1-0.6q^{-1}+0.3q^{-2} \end{bmatrix}$$

$$B(q^{-1}) = \begin{bmatrix} -0.1 & -0.9 \\ 0.2 & 0.3 \end{bmatrix} q^{-1} + \begin{bmatrix} -0.8 & -0.3 \\ 0.1 & 0.7 \end{bmatrix} q^{-2}$$

$$= \begin{bmatrix} -0.1q^{-1}-0.8q^{-2} & -0.9q^{-1}-0.3q^{-2} \\ 0.2q^{-1}+0.1q^{-2} & 0.3q^{-1}+0.7q^{-2} \end{bmatrix}$$

The aim is to estimate the matrix polynomials $A(q^{-1})$ and $B(q^{-1})$ from I/O data contaminated by white noise. A PRBS data sequence of length N=1000 is used to excite the system.

A Monte Carlo simulation of 100 experiments has been performed for signal to noise ratio equal to 10 db for both outputs.

The Monte Carlo Simulation (MCS) results are presented in table1 where the mean and standard deviation of the estimated parameters are displayed.

Table 1. MCS results

	MIMO SRIV	MIMO IV4
$\hat{A}_1$	$\begin{bmatrix} 0.5029 \pm 0.0235 & -0.3974 \pm 0.0279 \\ 0.3040 \pm 0.0265 & -0.6018 \pm 0.0274 \end{bmatrix}$	$\begin{bmatrix} 0.5018 \pm 0.0256 & -0.3982 \pm 0.0302 \\ 0.3057 \pm 0.0267 & -0.5970 \pm 0.0319 \end{bmatrix}$
$\hat{A}_2$	$\begin{bmatrix} -0.0974 \pm 0.0250 & -0.3024 \pm 0.0292 \\ 0.1988 \pm 0.0285 & 0.2995 \pm 0.0241 \end{bmatrix}$	$\begin{bmatrix} -0.0979 \pm 0.0283 & -0.3007 \pm 0.0312 \\ 0.2030 \pm 0.0333 & 0.2955 \pm 0.0281 \end{bmatrix}$
$\hat{B}_1$	$\begin{bmatrix} -0.0979 \pm 0.0147 & -0.9008 \pm 0.0138 \\ 0.2010 \pm 0.0173 & 0.2999 \pm 0.0158 \end{bmatrix}$	$\begin{bmatrix} -0.0974 \pm 0.0169 & -0.9010 \pm 0.0169 \\ 0.2001 \pm 0.0219 & 0.2979 \pm 0.0195 \end{bmatrix}$
$\hat{B}_2$	$\begin{bmatrix} -0.8001 \pm 0.0181 & -0.3012 \pm 0.0222 \\ 0.0959 \pm 0.0215 & 0.6948 \pm 0.0277 \end{bmatrix}$	$\begin{bmatrix} -0.8016 \pm 0.0191 & -0.3010 \pm 0.0275 \\ 0.0969 \pm 0.0232 & 0.6957 \pm 0.0275 \end{bmatrix}$

It can be seen from table 1 that both the MIMO SRIV and the IV4 algorithms deliver unbiased and quite accurate results.

To see the influence of the noise level on parameter estimation, some Monte Carlo simulations of 100 experiments have been performed for different values of SNR ratios varying from 1 to 20 dB. For each run of a Monte Carlo Simulation new noise sequences are generated in order to give independent realizations.

The performance index used for comparison is the Mean Normalized Errors (MNE) which is a measure of bias of the estimates from the true value and is defined as:

$$MNE = 100 \frac{\|\theta - \theta_{mean}\|_2}{\|\theta\|_2} \tag{40}$$

where θ_{mean} is the mean of the estimation parameter value and θ is the true parameter value.

The results are shown in table 2.

Table 2. MNE for different values of SNRs.

SNR (dB)	MNE (SRIV)	MNE (IV4)	MNE (LS)
1	1.7760	2.7446	44.4156
5	0.6392	0.8673	30.6596
10	0.5536	0.6188	16.9639
15	0.2582	0.3429	8.4333
20	0.0750	0.0761	3.5382

Graphical representations for the evolution of the MNE criterion for different values of SNR's and methods are shown in figures 1 and 2.

From table 2, figure 1 and figure 2 we can see that the MIMO SRIV is more immune to noise than MIMO IV4 . Of course the MIMO Least Squares gives bad estimates as expected.

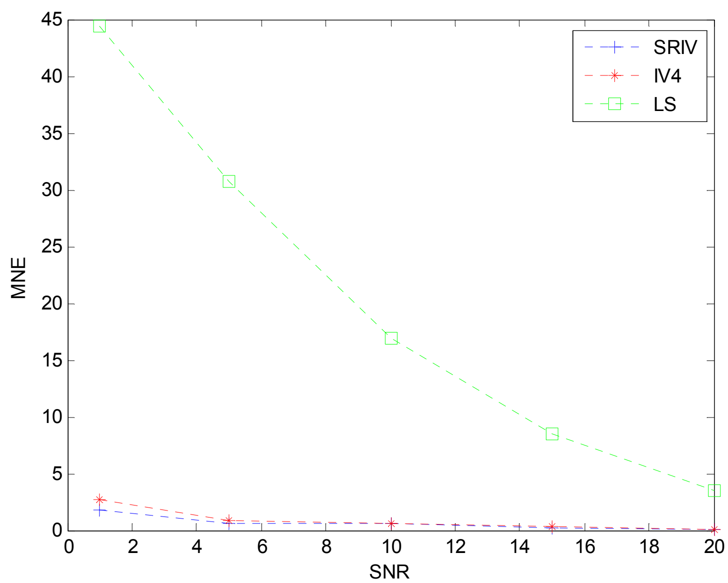

Figure 1. Monte Carlo Simulation results for SRIV, IV4 and LS methods.

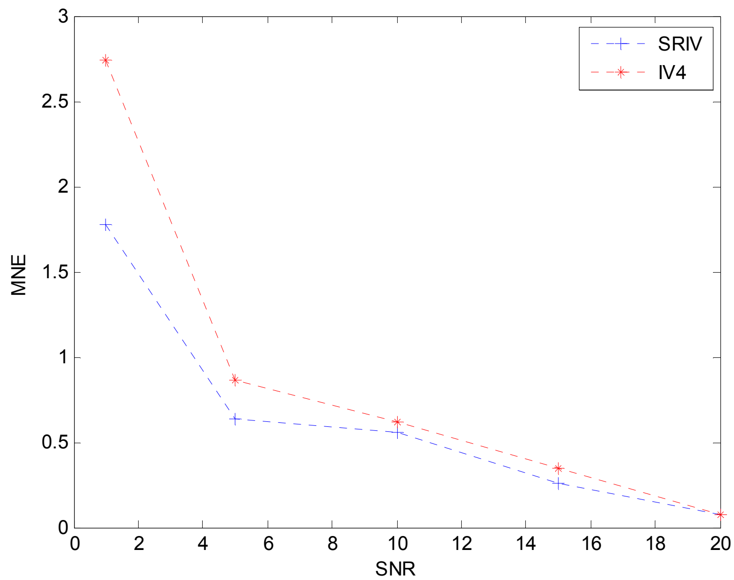

Figure 2. Monte Carlo Simulation results for SRIV and IV4 methods.

5. Conclusions

This paper has presented an extension of the SRIV algorithm to MIMO systems described by a Left Matrix Fraction Description using the Kronicker product. Block filtering of the input/output as well as iterativity are the main features of the algorithm.

A simulations example illustrated the superiority of the MIMO SRIV algorithm over the MIMO IV4 and the MIMO least squares algorithms.

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