Stability Study of Mamdani's Fuzzy Controllers Applied to Linear Plants

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Abstract: A stability study approach of fuzzy control systems, in the continuous case and the discrete one, is presented in this paper. This approach is based on vector norms corresponding to a Lyapunov's vector function and allowing to establish sufficient conditions for global asymptotic stability of controlled linear systems by Mamdani's fuzzy controllers. The considered fuzzy regulators are of type PI and have particular fuzzy partition for input variables corresponding to Lur'e type systems.

Keywords: Mamdani's fuzzy controllers, global asymptotic stability, vector norms, arrow's form matrix.

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1. Introduction

Although stability analyzes of a fuzzy model is particularly difficult, since it is naturally non linear, it is important when realizing the fuzzy controller to analyze the dynamic behavior of closed-loop system. Stability concept for fuzzy systems has been developed by many authors [1] [3] [5] [7] [10] [12] [17] [18] [19].

Since their appearance in the middle of 1970, many researches have been developed to analyze closed loop system's stability with fuzzy regulators. In particular, Kichert and Mamdani [8] have developed an input/output model of type multi-relay for a fuzzy regulator to use first harmonic method to highlight existence conditions to periodic responses.

One of the approaches often used in the specialized literature consisting of a fuzzy controller which is the nonlinear element used with linear model to be controlled. In this approach the fuzzy model is considered as a particular class of nonlinear models.

In the case of single input controllers, the error, Ray and Majumder [14], after showing that the static input-output controller characteristic belongs to a bounded sector, used a particular case of the circle criterion to obtain sufficient conditions of asymptotic stability in the case of continuous linear systems. In the discrete case, Langari and Tomizuka [9] used the Lyapunov method to propose sufficient stability conditions for controllers sampling only the error. After this, Melin and Vidolov [11] extended the approach proposed in [14] to the case of fuzzy controllers realizing PD and PI type strategies to control SISO linear system.

They proposed sufficient stability conditions by using the Kalman-Yacubovitch theorem. In the same way, Rambault [13] presented sufficient stability conditions by using the Popov theorem in the case of PI fuzzy controllers.

Another approach was developed in [1] for the stability study of TSK fuzzy systems by using vector norms and exploiting the Borne and Gentina criterion.

In this work, we will present the studies proposed in [1] with another view and we will extrapolate them for the discrete case.

Thus, we are interested in PI fuzzy controllers whose inputs are the error e and its variation de and the output is the control variation du. For a particular partition of inputs, we will show that this controller is equivalent to a linear PI with a nonlinear gain. These regulators correspond to systems of type Lur'e.

This paper is setup as follows: in the next section a description of the fuzzy system is presented, the third section is a setting of the system under the Lur'e problem form, the fourth section propose the stability conditions both in the continuous case and in the discrete one, an application example is presented in section five, and finally in the sixth section a conclusion is given.

2. Particular Class of Fuzzy Control

Among the approaches proposed in the specialized literature consisting of controlling a linear model by a fuzzy controller which is considered as a nonlinear element.

The fuzzy PI control system considered in this study is given by Fig.1 which is valid either in the continuous case or in the discrete one.

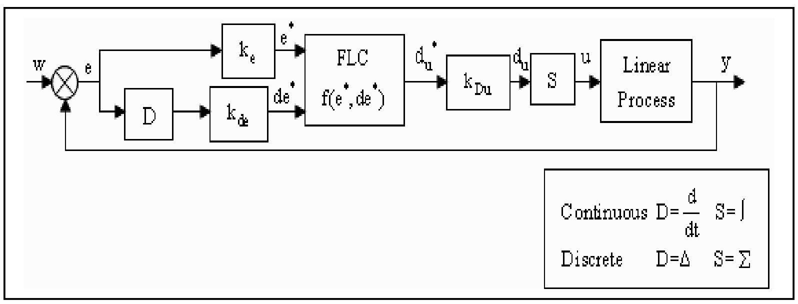

Figure 1. Fuzzy control system

Where k_e, k_{de}, k_{du} are scale factors and w, y are respectively the setpoint and the output.

We suppose that universes of discourse are symmetrical with respect to zero and delimited in the normalized interval $[-1,1]$. we suppose also that the partition of the normalized variables e^*, de^* and du^* is triangular strong whose fuzzy subsets number is odd [4]. For example $r=7$ according to Fig. 2.

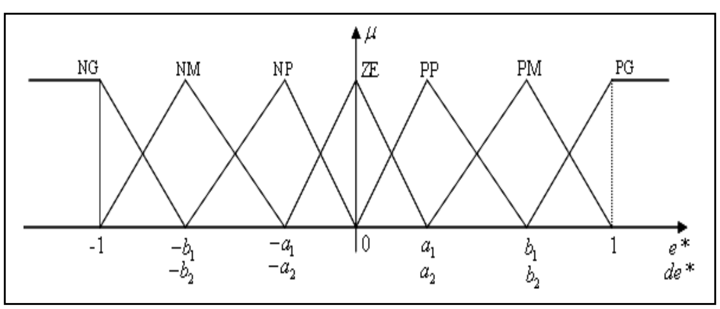

Figure 2. Input variables partition

The rule base considered is an $r \times r$ traditional rule table that is of antidiagonal type such as the

Mac Vicar-Whelan one presented in Table 1 [20].

Table 1. The MacVicar-Whelan table

e \ Δe	NG	NM	NP	ZE	PP	PM	PG
NG	NG	NG	NG	NG	NM	NP	ZE
NM	NG	NG	NM	NM	NP	ZE	PP
NP	NG	NM	NP	NP	ZE	PP	PM
ZE	NG	NM	NP	ZE	PP	PM	PG
PP	NM	NP	ZE	PP	PP	PM	PG
PM	NP	ZE	PP	PM	PM	PG	PG
PG	ZE	PP	PM	PG	PG	PG	PG

Let $\sigma(e^*, de^*)$ the surface in the space (e^*, de^*, du^*), verifying the two properties [15]:

$$\begin{cases} \text{i) If } \sigma=0 \text{ then the input-output characteristic surface } du^*(e^*, de^*)=0 \\ \text{ii) It exists } k>0 \text{ such as } du^*(k\sigma - du^*) \geq 0 \text{ for all } e^* \text{ and } de^*. \end{cases} \tag{1}$$

The first property means that the intersection of the overvaluing surface σ with the plan (e^*, de^*) is a part of the intersection of the characteristic surface $du^*(e^*, de^*)$ with the same plan. The curve σ=0 which is piecewise linear illustrated by Fig.3 verifies the previous property independently of the inference mechanism used.

According to Fig.3, in the plan (e^*, de^*) this curve is symmetrical with respect to zero. Therefore we will explicit the expression of $\sigma = 0$ only in the case of $e^* > 0$, so for r=7, we have:

$$\begin{cases} \dfrac{e^*}{a_1} + \dfrac{de^*}{a_2} = 0 \quad \text{if } 0 < e^* \leq a_1 \text{ and } -a_2 < de^* \leq 0 \\ \dfrac{e^*}{(b_1 - a_1)} + \dfrac{de^*}{(b_2 - a_2)} + \dfrac{a_2 b_1 - a_1 b_2}{(b_1 - a_1)(b_2 - a_2)} = 0 \quad \text{if } a_1 < e^* \leq b_1 \text{ and } -b_2 < de^* \leq a_2 \\ \dfrac{e^*}{(1 - b_1)} + \dfrac{de^*}{(1 - b_2)} + \dfrac{(b_2 - b_1)}{(1 - b_1)(1 - b_2)} = 0 \quad \text{if } b_1 < e^* \leq 1 \text{ and } -1 < de^* \leq -b_2 \\ e^* + de^* = 0 \quad \text{if } e^* > 1 \text{ and } de^* < -1 \end{cases} \tag{2}$$

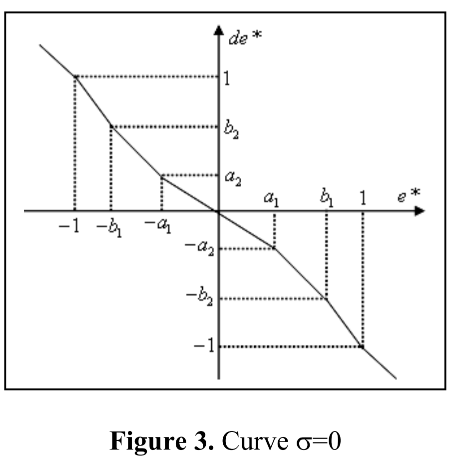

Figure 3. Curve σ=0

In particular, if the partitions are identical for the two inputs e^* and de^*, i.e. $a_1 = a_2$ and $b_1 = b_2$

as shown in Fig.2, then the curve $\sigma = 0$ is the straight line corresponding to the second bisector in the plan (e^*, de^*) whose equation is: $e^* + de^* = 0$. in this case the precedent second property shows that it exists $k>0$ such that the characteristic surface $du^*(e^*, de^*)$ is situated between the plan (e^*, de^*) and the plan given by the equation: $du^* = k\,(e^* + de^*)$. So, the last plan overvalues the control characteristic surface in all its points and it characterizes a linear PD controller that becomes a PI controller when we add the integration . Consequently, we introduce a nonnegative nonlinear gain $f(e^*, de^*)$ such as:

$$du^* = f(e^*, de^*).\sigma = f(.).(e^* + de^*) \qquad (3)$$

3. Setting the System under the Lur'e Problem Form

An important class of nonlinear systems corresponds to the case of systems with separable nonlinearity as presented in Fig.4.

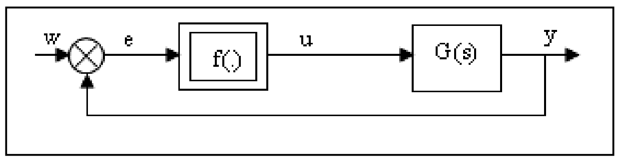

Figure 4. Lur'e problem

In the basic diagram of Lur'e problem, the nonlinear element has a single input of which we can overvalue the curve $f(.).e$ by a straight line. However, in this case we have two inputs the error and its variation. Similarly with the conventional case, we search a plan overvaluing the characteristic $du^*(e^*, de^*)$.

Consequently, the fuzzy control system of Fig.1 can be transformed to the following form (Fig.5).

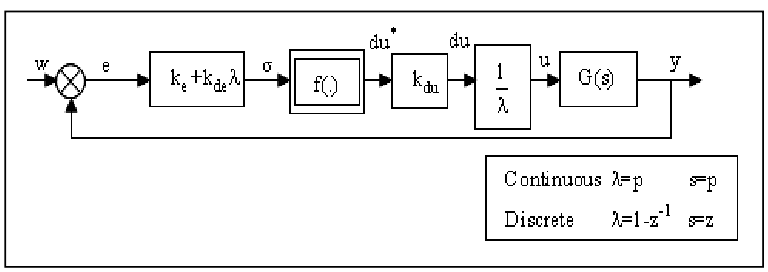

Figure 5. Equivalent diagram of the fuzzy control system

The research of global stabilization conditions of the linear controlled system can be simplified by modifying the previous diagram into the following (Fig.6) and we are interested in the autonomous regime (w=0).

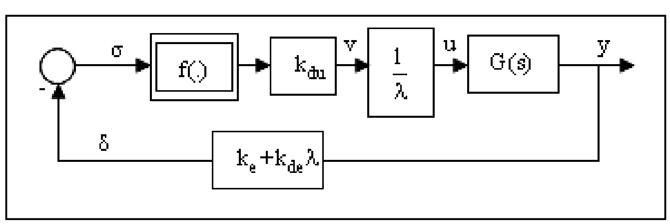

Figure 6. Fuzzy control system in the autonomous regime

4. Proposed Stability Conditions

4.1 Continuous case

Let $G(p)$ the linear continuous system to be controlled. We suppose that $G(p)$ is stable, rational with a relative degree at least equal to 1 and without common factor between numerator and denominator such that:

$$G(p)=\frac{Y(p)}{U(p)}=\frac{b_1p^{n-1}+...+b_n}{p^n+a_1p^{n-1}+...+a_n} \tag{4.a}$$

We set $G(p)$ in the controllable form given by :

$$\begin{cases}\dot{x}=Ax+Bu\\ y=Cx\end{cases} \tag{4.b}$$

$$A=\begin{bmatrix}0 & 1 & \dots & 0\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \dots & 0 & 1\\ -a_n & \dots & \dots & -a_1\end{bmatrix},\ B=\begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix}\ \text{et}\ C^T=\begin{bmatrix}b_n\\ \vdots\\ \vdots\\ b_1\end{bmatrix} \tag{4.c}$$

The linear part of the system is $L(p)$ such that:

$$L(p)=\frac{G(p)}{p}(k_e+k_{de}p) \tag{5.a}$$

The controllable form of $L(p)$ is given by :

$$\begin{cases}\dot{x}^*=A^*x^*+B^*v\\ \delta=C^*x^*\end{cases} \tag{5.b}$$

x^* is of order $(n+1)$, then:

$$A^*=\begin{bmatrix}0 & 0 & 1 & \dots & 0\\ \vdots & \vdots & \ddots & \ddots & \vdots\\ 0 & 0 & \dots & 0 & 1\\ 0 & -a_n & \dots & \dots & -a_1\end{bmatrix},\ B^*=\begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix}\ \text{and}\ C^{*T}=\begin{bmatrix}b^*_{n+1}\\ \vdots\\ \vdots\\ b^*_1\end{bmatrix} \tag{5.c}$$

where $\begin{cases}b^*_1=k_{de}b_1\\ b^*_i=k_eb_{i-1}+k_{de}b_i \quad \text{for}\ 2\le i\le n\\ b^*_{n+1}=k_eb_n\end{cases}$.

By closing the loop, we have:

$v=f(.)k_{du}\sigma=-f(.)k_{du}\delta=-f(.)k_{du}C^*x^*$,

So we can write:

$$\dot{x}^*=A^*x^*-B^*f(.)k_{du}C^*x^*=[A^*-B^*f(.)k_{du}C^*]x^*=A_Cx^* \tag{6.a}$$

Let A_C (which is of order $(n+1)$) such that:

$A_C = A^* - B^* f(.) k_{du} C^*$,

$$A_C = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 \\ -a_{C_{n+1}}(.) & \dots & \dots & -a_{C_1}(.) \end{bmatrix} \tag{6.b}$$

where $\begin{cases} a_{C_{n+1}}(.) = f(.) k_{du} b^*_{n+1} \\ a_{C_i}(.) = a_i + f(.) k_{du} b^*_i \quad \text{for } 1 \le i \le n \end{cases}$

The following basic change :

$$P = \begin{bmatrix} 1 & 1 & \dots & 1 & 0 \\ \alpha_1 & \alpha_2 & \dots & \alpha_n & 0 \\ \alpha_1^2 & \alpha_2^2 & \dots & \alpha_n^2 & \vdots \\ \vdots & \vdots & \dots & \vdots & 0 \\ \alpha_1^n & \alpha_2^n & \dots & \alpha_n^n & 1 \end{bmatrix} \tag{7}$$

where $\alpha_i \neq \alpha_j \quad \forall i \neq j$, allows to put the previous state matrix in the following arrow's form [2]

$$F = P^{-1} A_C P = \begin{bmatrix} \alpha_1 & 0 & \dots & 0 & \beta_1 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \dots & 0 & \alpha_n & \beta_n \\ \gamma_1(.) & \dots & \dots & \gamma_n(.) & \gamma_{n+1}(.) \end{bmatrix} \tag{8.a}$$

such that:

$$\beta_i = \prod_{\substack{j=1 \\ j \neq i}}^{n} (\alpha_i - \alpha_j)^{-1} \quad \forall i = 1, \dots, n \tag{8.b}$$

$$\begin{cases} \gamma_i(.) = -P_{A_C}(., \alpha_i) \quad \forall i = 1, 2, \dots, n \\ P_{A_C}(., \lambda) = \lambda^{n+1} + \sum_{j=0}^{n} a_{C_{n+1-j}}(.) \lambda^j \end{cases} \tag{8.c}$$

$$\gamma_{n+1}(.) = -a_{C_1}(.) - \sum_{i=1}^{n} \alpha_i \tag{8.d}$$

Let $z = P^{-1} x^*$ and $\dot{z} = Fz$

By applying the Borne and Gentina criterion to the previous system, we can state the following theorem:

Theorem 1 [16]***:***

If it exist $\alpha_i < 0$ for $i = 1, \dots, n$, $\alpha_i \neq \alpha j \;\; \forall i \neq j$ and $\varepsilon > 0$ such that $\forall z \in \Re^{n+1}$: :

$$-\gamma_{n+1}(.) + \sum_{i=1}^{n} \left| \gamma_i(.) \beta_i \right| \alpha_i^{-1} \geq \varepsilon > 0 \tag{9}$$

then the equilibrium point $z = 0$ for the continuous system is globally asymptotically stable.

Proof: Let the following comparison system:

$$\dot{z}=Mz \tag{10}$$

where the matrix M is such that:

$$\forall i \;\; m_{ii}=f_{ii} \;\; \text{and} \;\; \forall i \neq j \;\; m_{ij}=\left|f_{ij}(\cdot)\right| \tag{11}$$

and $\forall z \in \Re^{n+1}$:, the matrix M has its out diagonal elements positives and the ones non constants are insulated in the last line.

Thus, by referring to results obtained in [2], the conditions of the previous theorem can be deduced of the Kotelyanski's conditions [6].

If moreover, it exist $\alpha_i, i=1,...,n$ such that:

$$\gamma_i(.)\beta_i>0 \;\; \text{for all} \;\; i=1,...,n \tag{12}$$

the previous theorem can be simplified according to the following corollary.

Corollary 1:

If the following conditions i) and ii) are checked:

i) if it exists $\alpha_i<0$ for $i=1,...,n,\ \alpha_i \neq \alpha j \ \ \forall i \neq j$, such that : $\gamma_i(.)\beta_i>0$

ii) and $\varepsilon>0$ such that :

$$P_{A_C}(.,0) \geq \varepsilon>0 \quad \forall z \in \Re^{n+1} : \tag{13}$$

then the equilibrum point $z=0$ is globally asymptotically stable.

4.2 Discrete case

Let $G(z)$ the discrete system obtained by introducing a zero-order holder to the continuous system to be controlled. We suppose that $G(z)$ is stable, rational with a relative degree at least equal to 1 and without common factor between numerator and denominator such that:

$$G(z)=\frac{Y(z)}{U(z)}=\frac{b_1z^{-1}+...+b_nz^{n}}{1+a_1z^{-1}+...+a_nz^{-n}} \tag{14.a}$$

We set $G(z)$ in the controllable form given by:

$$\begin{cases} x(k+1)=Ax(k)+Bu(k) \\ y(k)=Cx(k) \end{cases} \tag{14.b}$$

$$A=\begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 \\ -a_n & \dots & \dots & -a_1 \end{bmatrix} B=\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} C^T=\begin{bmatrix} b_n \\ \vdots \\ \vdots \\ b_1 \end{bmatrix} \tag{14.c}$$

Let $L(z)$ the system with v as input and δ as output such that:

$$L(z)=\frac{\Delta(z)}{V(z)}=\frac{(k_e+k_{de})-k_{de}z^{-1}}{1-z^{-1}}G(z) \tag{15.a}$$

Its controllable form is given by:

$$\begin{cases}\dot{x}^*(k+1)=A^*x^*(k)+B^*v(k)\\ \delta(k)=C^*x^*(k)\end{cases} \tag{15.b}$$

x^* is of order $(n+1)$ and then:

$$A^*=\begin{bmatrix}0 & 1 & \dots & 0\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \dots & 0 & 0\\ -a^*_{n+1} & \dots & \dots & -a^*_1\end{bmatrix} B^*=\begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix} C^{*T}=\begin{bmatrix}b^*_{n+1}\\ \vdots\\ \vdots\\ b^*_1\end{bmatrix} \tag{15.c}$$

where: $\begin{cases}a^*_1=a_1-1\\ a^*_i=a_i-a_{i-1} \quad \text{for } 2\le i\le n\\ a^*_{n+1}=-a_n\end{cases}$

and

$$\begin{cases}b^*_1=(k_e+k_{de})b_1\\ b^*_i=(k_e+k_{de})b_i-k_{de}b_{i-1} \quad \text{for } 2\le i\le n\\ b^*_{n+1}=-k_{de}b_n\end{cases}$$

In addition $v=f(.)k_{du}\sigma=-f(.)k_{du}\delta=-f(.)k_{du}C^*x^*(k)$

therefore:

$$\dot{x}^*(k+1)=A^*x^*(k)-B^*f(.)k_{du}C^*x^*(k)=[A^*-B^*f(.)k_{du}C^*]x^*(k)=A_Dx^*(k) \tag{16.a}$$

where $A_D=A^*-B^*f(.)k_{du}C^*$

$$A_D=\begin{bmatrix}0 & 1 & \dots & 0\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \dots & 0 & 0\\ -a_{D_{n+1}}(.) & \dots & \dots & -a_{D_1}(.)\end{bmatrix} \tag{16.b}$$

$a_{D_i}(.)=a^*_i+f(.)k_{du}b^*_i \quad i=1,...,n$

We use the same basic change used in the continuous case to transform the matrix A_D into the arrow's form such that:

$z(k+1)=Fz(k)$, $F=P^{-1}A_DP$ and $z=P^{-1}x^*$

We apply the Borne and Gentina criterion to the matrix F in the discrete case, and then we can state the following theorem:

Theorem 2:

If it exist $\alpha_i\in\Re$ and $\varepsilon>0$ such that $\forall z\in\Re^{n+1}$:

i) $1-|\alpha_i|>0 \quad \forall i=1,...,n$

ii) $1-|\gamma_{n+1}(.)|-\sum_{i=1}^{n}|\gamma_i(.)\beta_i|(1-|\alpha_i|)^{-1} \geq \varepsilon>0$ (17)

Then the equilibrum point $z=0$ for the discrete system is globally asymptotically stable.

***Proof*:**

We consider the following comparison system:

$$z(k+1)=Mz(k) \tag{18}$$

where the matrix M is such that:

$$\forall i,j \;\; m_{ij}=|f_{ij}(.)| \tag{19}$$

$\forall z \in \Re^{n+1}$:, the matrix M has its out diagonal elements positives and the ones non constants are insulated in the last line.

In the same way and by referring to results obtained in [2], the application of the Borne and Gentina criterion allows to conclude the stability if the matrix $(I-M)$ verifies the Kotelyanski's conditions.

and if it exist $\alpha_i > 0$ such that:

$$\begin{cases} 0<\alpha_i<1 \quad i=1,...,n \\ \gamma_{n+1}(.)>0 \\ \gamma_i(.)\beta_i>0 \quad i=1,...,n \end{cases} \tag{20}$$

then the overvaluing matrix M becomes identical to itself, what allows to simplify the previous theorem.

Corollary 2:

The discrete system is globally asymptotically stable if the last conditions are satisfied and:

$$P_{A_D}(.,1) \geq \varepsilon>0 \quad \forall z \in \Re^{n+1} \tag{21}$$

5. Application

For the validation of the results found previously, we consider the fuzzy control of a first order process whose transfer function is given by:

$$G(p)=\frac{1}{p+0.4} \tag{22}$$

The fuzzy PI controller uses the MacVicar-Whelan table. The fuzzy partition of the variables e^*, de^* and du^* is identical. The conjunction operator used between inputs is the Min, the aggregation operator is the Max, the implication is the Mamdani's one (Min) and the defuzzification method is the gravity center one.

Scale factors of inputs are fixed to $k_e = 1$ and $k_{de} = 1$. In order to adapt the control law to the piloted system, we adjust only the factor scale k_{du} of the output variable.

By programming, we search the values k_{min} and k_{max} that allow respectively overvaluing and undervaluing the characteristic surface of the fuzzy controller by PI plans. The table 2 presents the results obtained by distorting the discourse universe to 100 points, and the fig.7 shows the action surface for the fuzzy controller corresponding to the Mamdani's implication.

Table 2. Values of slopes k_{min} and k_{max} of plans respectively overvaluing and undervaluing the action surface of the fuzzy controller for each implication

Implication	k_{min}	k_{max}
Mamdani (*min*)	0.47	1.86
Larsen (*prod*)	0.38	1.14

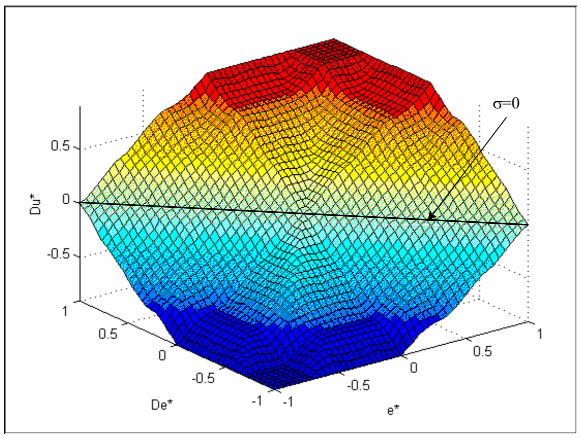

Figure 7. Action surface of the fuzzy controller for Mamdani's implication

By referring to table 2, we have $k=k_{max}=1.86$ for Mamdani's implication and $k=k_{max}=1.14$ for Larsen's implication.

The linear part of the system $L(p)$ is given by:

$$L(p)=\frac{(1+p)}{p(p+0.4)} \tag{23}$$

The application of the theorem allows to deduce the following stability condition:

$\exists\alpha>0$, $-\gamma_2(.)+|\gamma_1(.)\beta|\frac{1}{\alpha}>0$ where $\begin{cases}\beta=1\\ \gamma_1(.)=-\left[\alpha^2+(0.4+f(.)k_{du})\alpha+f(.)k_{du}\right]\\ \gamma_2(.)=-(\alpha+0.4+f(.)k_{du})\end{cases}$

After simplification this condition becomes:

$0<f(.)k_{du}<-\frac{2\alpha(\alpha+0.4)}{2\alpha+1}$ for $-0.4<\alpha<0$.

For $\alpha=-0.27$, overvaluing the upper limit for this condition, we have:

$0<f(.)k_{du}<0.15$.

The maximum gain allowed k_{du} is $\frac{0.15}{1.86}=0.07$ for the Mamdani's implication and $\frac{0.15}{1.14}=0.13$ for the Larsen's one.

By applying the simplified form of the theorem, we obtain the following condition:

$$0<f(.)k_{Du}<-\frac{\alpha(\alpha+0.4)}{\alpha+1} \quad \text{for} \quad -0.4<\alpha<0,$$

which becomes for $\alpha=-0.23$:

$$0<f(.)k_{Du}<0.05.$$

Then we deduce the maximum gain allowed k_{du} ; 0.02 for the Mamdani's implication and 0.04 for the Larsen's one.

We consider now the discrete control of the previous system with the sampling period $T_e=0.6s$. Then, the transfer function of the controlled system becomes:

$$G(z)=Z\left[B_0(p).G(p)\right]=\frac{0.5334}{z-0.7866}$$

where $B_0(p)$ is a zero order holder.

The stability condition proposed in the theorem.2 is ; the system previously described is globally asymptotically stable if it exists α such that $1-|\alpha|>0$ and:

$$1-|\gamma_2(.)|-|\gamma_1(.)\beta|\frac{1}{1-|\alpha|}>0$$

where:
$$\begin{cases} \beta=1 \\ \gamma_1(.)=-\left[\alpha^2+(-1.79+1.06f(.)k_{du})\alpha+(0.79-0.53f(.)k_{du})\right] \\ \gamma_2(.)=1.79-\alpha-1.06f(.)k_{du} \end{cases}$$

With the application of the simplified version of the theorem, given by corollary.2, by choosing $\alpha=-0.88$ we obtain the condition $0<f(.)k_{du}<0.02$ and the gain allowed k_{du} is 0.01.

6. Conclusion

In this paper we have studied the global asymptotic stability of a fuzzy system, where the fuzzy controller is a Mamdani's one and for a particular partition of the input subsets. After presenting the particular class of fuzzy PI controller, we do release some properties allowing the verification of passivity relation of these controllers between the tuning grandeur v and the observation σ , linear combination of the error and its variation.

The global asymptotic stability conditions obtained in both the continuous case and the discrete one are deduced from the application of the Borne and Gentina criterion and the use of vector norms as Lyapunov function.

Finally it suits to remark that this study can be extended to the stability study of non linear systems controlled by the same way, the Mamdani's fuzzy controllers, with a few modifications on the fuzzy controller, and it remains with the report of the system to be controlled and the nature of its non linearity.

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