Parallelizing Neuro-fuzzy Economic Models in a GRID Environment

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Abstract: Modeling and simulating large economic systems, with hundreds of players, is a basic requirement for understanding economic processes even at the scale of a large town where tens of shops are competing. Based on fuzzy logic models, we parallelize computations with the purpose to make them affordable even when larger systems are simulated. After briefly introducing the basic models involving several strategies of the commercial players, we extend the models to group-based hierarchically organized players. The overall models are nonlinear, which favors a rich dynamic behavior. Then, we present the parallelizing procedure and simulation results. Special attention is paid to the dynamic behavior of the market, including transitory regimes, asymptotic stability, and periodicities.

Keywords: GRID computing, fuzzy systems, market model, economy, nonlinear dynamics

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1. Introduction

In this paper, we deal with models of the market involving essentially small companies that buy products from large manufacturers and distribute the products. The models dealt with are appropriate for small vendors (commercial agents) who sell common goods. For such agents, the buying price of the products they commercialize may be assumed constant in time and the same for all agents, as they all buy from essentially the same manufacturers or great distributors. Moreover, the products have similar quality and may be assumed identical in all respects. The models are therefore appropriate for agents like boutiques and small shops. Examples of such vendors are small shops selling general merchandise, shops in the street selling eatables, fashionable shops, boutique wineries, and over the counter drugstores. Subsequently, whenever confusion cannot arise, we name the small commercial agents companies, vendors, agents, or firms.

Markets composed of such vendors are often fluctuating widely and may show a large range of prices, not necessarily motivated by economic factors, but frequently motivated by the strategy subjectively adopted by the managers of the shops. To explain at least partly the behavior of these markets and to analyze the outcomes of the strategy adopted, in a certain context – represented by the other vendors – we have developed several models, involving various market strategies [16-19]. The strategies differ in the way the vendors change the price of the products depending on the prices their competition practices and on the profits that the vendors assume to obtain for a specified selling price. The computation of the profits involves reasoning based on fuzzy logic [6-8], [13-15].

The vendors' strategies may vary from the reasonable seeking of profit maximization to the less reasonable seeking of a profit that is larger than that of the competition. Unreasonable behaviors of the economic agents are well documented in the literature [2-5], [9-11]. It may range from unreasonably hopeful to revenge- and hate-driven behaviors [16-19]. An unknown the vendors have to deal with regards the prices used by the competing agents; the vendors have to learn these prices. However, they do so after some delay. The values of the delays greatly influence the market dynamic behavior [9]. The dynamics is determined based on some arbitrary unit of time; that unit is equal to the time the agents are able to change their prices. We assume all vendors have the same time unit.

The simulation of the market dynamics requires the implementation of models with various strategies, possibly with associations of shops, coordinated by a "central" manager, as well as large number of agents. The use of fuzzy logic in modeling the decision making processes makes the model more appropriate to the qualitative manner the vendors reason, but increases the computation load in the simulations. As the individual decisions are based on the knowledge of the prices used by a large number of vendors, the complexity of the computations is quadratic. Even for medium markets, comprising a few tens of agents, the computation time on a PC becomes prohibitive (hours to tens of hours). For this reason, we developed GRID versions of the models [1], [21-22]. In this paper, we report on the GRID version of our models and on the results obtained by the implementation of the models with a few tens of agents.

In Section 2, we briefly overview the fuzzy models developed in previous papers [16-19]. In Section 3, we present hierarchical (group) models, involving a "main (central) manager and shop" and "dependent shops" that rely on the decisions made by the central manager. Section 2 is devoted to the analysis of the components of the economic fuzzy systems. A basic version of the parallelization of the algorithm is presented in Section 4, while in Section 5 we exemplify results related to the computation time. The final section is devoted to a discussion and to conclusions.

2. Basic Fuzzy Models [16-19]

We have addressed in previous papers [16-19] the modeling of several possible strategies the commercial agents might use. A detailed description of these models can be found in [19]. The time horizon of the adaptation of the vendors to the market conditions is a *one step ahead horizon*, meaning that the vendors modify the prices of their products with the aim of attaining a specified behavior of their profit, possibly correlated with the estimated profits of the competing vendors.

We have adopted a simple model of reasoning and decision making of the commercial agents. Every agent takes compares its own current price for the product with the prices of the competitive vendors.

IF (price used by agent #i is Ai)   AND   (competition price is Bj)   THEN profit of Ai is Ck

The linguistic model is described in Table 1, after [16]. Notice the very simple model of reasoning, based only on two factors in the premises and only on three linguistic degrees used in the premises and five linguistic attributes in the conclusions.

Table 1. Fuzzy Inference Rules For Profit Computing (from [16])

Price P1	Competitor price P2		
	Low	Medium	High
Low	M	More or less M	VH
Medium	L	H	Somewhat H
High	VL	L	L

We summarized in Table 2 the four basic strategies of the individual competitors implemented discussed in this paper.

Table 2. Basic Strategies (from [16])

Criterion	Type of increment	Acronym of the strategy
Maximize profit by modifying price with an increment that is fixed or variable	fixed	max-fix
	variable, computed using fuzzy logic rules	max-fuzzy
Obtain at least the same profit as the competition (comparative, "me-too" imitative strategy)	fixed	comp-fix
	variable, computed using fuzzy logic rules	comp-fuzzy

Notice that two fuzzy decision processes are involved in the computations, namely, the estimation of the profit, and the price adjustment. The first process determines the profit $\pi_k(n+1)$ of the k-th vendor at the next time step, $n+1$, as a function of the price it uses, $p_k(n+1)$, of the (average) price used by the competing vendors as known with the related delays, $p_{av}(n)$, and of the estimated profit of the competitors, $\pi_{av}(n)$. The second process takes place only in the variable-increment model and refers to the computation of the change of the price at the next time step, change that is applied to price at the current time step. For details on this process, refer to [16-19].

To estimate the future profit, every vendor needs to compare the price of the product in its own shop with the prices of the product practiced by the competitor. We denote by τ_{ij} the delay experienced by the vendor with current number i when learning the price used by the competitor j . We assume these delays are fixed during the timeframe of the evolution of the market, as used in the simulations. A number of $N \times N$ delays characterizes a market with N vendors; formally, we assume $\tau_{ii} = 0 \quad \forall i$. Every agent performs at every time step the following operations:

i) Learn the prices used by the competitors, $p_{i \neq k}(n)$, for all competitors ($i \neq k$). This step requires $N \times N$ memory readings, where N is the number of competitors.

ii) Based on these prices, estimate the own profit at next time step, $\pi_{k\_i} = \pi(p_k(n), p_i(n - \tau_{ki}))$; the own profit is computed, in the present form of the algorithm, by a) estimating the own profit assuming a single competitor, ; b) averaging the estimated "one-to-one" own profits, $\pi_{k\_i}(n)$, over all competitors, for obtaining the estimated own profit at time moment n, $\pi_k(n) = \sum_i \pi_{k\_i}(n)$. This step requires $N \times N$ memory $O(N^2)$ operations.

iii) Estimate the profit of the competitors at the next time step; this operation is performed only for the comparative-type strategies; estimate the average profit on the market. The average profit is considered in the process of determining the next price value, in case of the *comparative* strategy. This step requires $O(N^2)$ operations.

iv) Determine the best price for the next step, either using a fixed increment, or a variable increment computed according to the fuzzy strategy of price change. This step requires $O(N)$ operations.

Notice that the overall complexity is $O(N^2 S)$, where S is the number of time steps in the simulation.

Assuming a fixed price increment (or decrement) is a "strong" hypothesis; it is justified by the slightly reduced computation time. Beyond being too schematic, the assumption of fixed increments has the drawback of producing long transitory regimes of the models. Using continuously variable adjustments of the price is more realistic, but it does not take into account the discrete nature of the monetary units. Variable adjustments on a discrete scale are, however, the most time-consuming; we have not implemented them until now.

Several factors determine the dynamics of the overall system standing for the market; the most important influence have the values of the delays and the relationship between the profit, on one side, and the price of the agent and the price of the competition, on the other side. This relationship is itself determined by the rules and the corresponding membership functions determining the profit. The equivalent surface profit vs. own price and competition price may have local maxima and minima inside the definition domain, as shown in Figure 1. The surface shape and properties depend on the rules and on the membership functions of the antecedents and consequence in the rules. The market models consist of vendor models with behavior described by fuzzy rules. The overall market models are nonlinear, which favors a rich dynamic behavior.

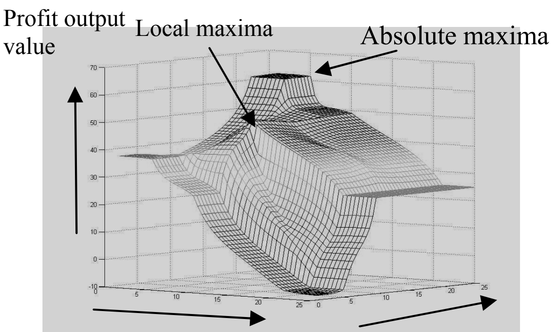

Figure 1. 3D visualization of the input/output function of the profit for a firm with the current price p_1 and for the competitor price p_2 [16]

We use two types of market models, namely the single-strategy market and the mixed-strategies market. The first model assumes that all agents in the market have the same strategy. This is a highly simplifying hypothesis for real-life markets; it is suitable only for modeling small markets, with a low number of agents. The mixed models allow for two strategies, with whatever percentage of agents using one of the strategies.

3. Economic Models with Group Strategies [20]

After building a set of models with independently acting companies, the next step is to develop models of markets with small companies aggregating themselves into groups. The companies that aggregate may do so because they are the property of the same group of investors, or because they have some kind of agreement. (Such agreements may be unlawful, yet it is difficult to prevent friends of people related by family connections or other kinship to act in some agreement).

Groups are characterized by some degree of coherence and by total connectivity. Coherence refers to the decision-making process, which is at least partly homogeneous, in the sense that some decisions apply to all the members of the group. Total connectivity means that all the members of the group have tight connections, with information circulating virtually instantaneously, at least from the satellite shops to the leadership of the group. We will assume that the group leadership runs a shop, a "main", "central-to-business" shop in the group. The group, however, does not need to consist of uniform members; some shops may practice higher prices than others do, because of their downtown position, or due to other commercial factors. However, in a totally-coherent group, how to change the prices is decided by the "chief" member of the group, even if prices practiced by the members of the group differ. The decision may be a qualitative one, like "increase the price", "keep the price constant", or "decrease the price", with no precise value for the price increment or decrement. Coherence requires that the decision made by the "chief" firm regarding the price modification is applied at the next inference step without any supplementary delay to all the shops in the group. However, the total coherence group strategy (TCGS) as described above is only one possible group strategy. At the other end is the loose group (or mutual-information-only) strategy (LGS), which requires only a mutual information through the central manager of the prices practiced by the competitor, but decision is left to each individual agent.

A consequence of the total connectivity is that the leadership of the group learns about the prices of the concurrence through the fastest way available to the group; this is a consequence of the zero-delay for information propagation in the group. When a member of the group learns about a change of the prices used by the competition, that member reports on the change; such information can come from several members of the groups, characterized by different delays in scrutinizing the market. However, only the information from the group member that has the minimal delay counts. Because we assume that the communication time between the firms of the same group is negligible, the "central" agent learns about the price used by the competition with the minimal delay:

$$\tau_{\mathrm{C}j} = \min_{k=1}^{J}\left(\tau_{\mathrm{S}k\mathrm{C}j}\right), \tag{1}$$

where J represents the number of members of the group which have a connection with the competing vendor Cj.

In all models based on groups, we compute the profit of firm #i at the time moment t according to the formula:

$$\pi_i(t) = \frac{\sum_{j=1}^{\hat{J}} \pi_{ij}(t)}{\hat{J}} = \frac{\sum_{j=1}^{\hat{J}} f\left(p_i(t), p_j(t-\hat{\tau}_{ij})\right)}{\hat{J}} \tag{2}$$

where π_{ij} is the profit of the firm #i; it is related to the competing firm #j; $\hat{J}$ represents the total number of competitors (companies with connection with current firm #i which does not belong to the same group). Here, J denotes the total number of firms connected with #i (if the delays τ_{ij} are not null). Notice that we assume that the profit depends only on the prices; we compute it accordingly.

The total coherence group strategy (TCGS) is more rigid and therefore it may have a homogenizing effect on the prices in the market, especially when the groups are large. We have not yet compared the profits obtained on similar markets (same number of groups, same number of companies, identical initial prices) when the TCGS strategy is replaced by the LGS, but we expect profits may be higher for the second, because of the higher flexibility and because TCGS is a limit case for LGS.

4. Parallelizing Neuro-fuzzy Economic Model

For the interfacing and performing the communications between the computing nodes, MPI – Message Passing Interface offers several connectives [1]. We have used the version of MPI implemented for the C programming environment. Using basic MPI routines/functions, the running parameters (the prices) are transmitted during the process; the values of the profits are received as returned results.

For the parallel algorithm, we use a modular description of the economic model, starting with the basic components – fuzzy systems SF1(for profit computing) and SF2 (for computation of crisp value of increment when the increment type is fuzzy), and ending with the neuro-fuzzy networks BCk . The parallel computing algorithm is:

1) **Initialization phase**

- This phase is made only in the central process, which distributes, respectively receives information from other processes. They are called the MPI initialization functions.
- For every firm from the parallelized economic model the following issues are set:

- the lists (queues) with the initial prices,
- the descriptive fuzzy membership function for the „*price*" and „*profit*" concepts,
- a line #*k* from the delayed matrix will be used by the computing nodes which compute the profit of firm #k,
- the type of strategy used to establish the new selling prices,
- the type of increment (require a SF2 component) and
- the fuzzy rules set (if these rules are not the same for all the firms in the model).

- The following items are also defined: the stop condition for the system (maximal number of steps *P* – iterative cycles, the reaching of stabilization requirement, the exceeding of the maximum or the minimum value accepted which correspond to an abnormal state, the repeating of a scenario by a pre-established number of times - the entering in a loop)

2) **while** step *(* $p \geq 1$*)* or (the stop condition aren't activated) *do*

3) **Parallelization phase**

for k = 1 *to N*, each of the *N* firms is associated to a computing node in the GRID. According to the input parameters (strategy, increment, price vector etc.) is computed the average profit of firm #k at the moment of time *t*, and eventually the average profit of concurrence using 'delayed' prices:

$\pi_{med\ k}[t]$ and $\pi_{med\ delayed\ \bar{k}}[t]$

The transmitting of the running parameters to every node #k is performed with the function MPI_SEND (the price vectors are completed according to the line #k of delay matrix).

This type of parallelization of the economic model requires N+1 computing nodes - one for the central process and N for the process associated to every firm #k (for k=1...N). The computing time varies with each node and is dependent on the type of strategy adopted by the firm and the type of increment. The most expensive from the point of view of the computing time is the strategy „max-profit" with a "fuzzy increment"; their complexity is: 4N ·O(SF1) + O(SF2).

The computing node includes the blocks {BCk(pk,[pτi]), BCk(p+k,[pτi]), BCk(p-k,[pτi])} and for the first block is computed also the average profit of the concurrence $b_{med\ delayed\ \bar{k}}$

4) Each block BCk includes N-1 fuzzy systems SF1
for i = 1 to N-1

- compute the profit $\pi_{k,i}[t]$, $\pi_{i,k\ delayed}[t]$ according to the prices $p_k[t]$, $p_i\,[t-\tau_{k,i}]$, using the fuzzy inference rules (reunion operation with the outputs of the rules),
- compute the system output after defuzzification .

endfor

The complexity of SF1 is expressed using the notation: NP - number of premises, R - number of rules, k - number of firms:

$$O(SF1) = (R \cdot NP + R + 2) \cdot k \approx R \cdot NP \cdot k$$

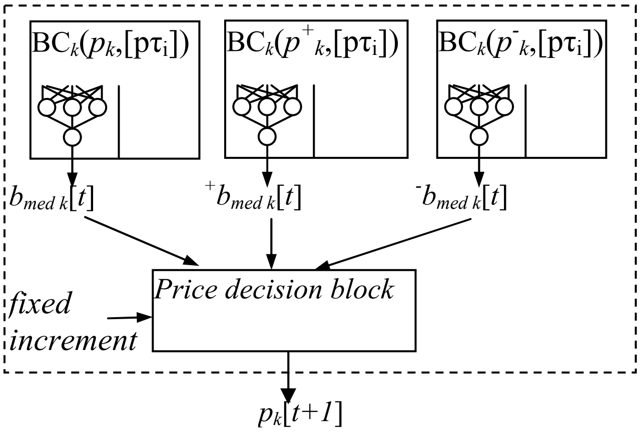

Figure 2. Sketch of the benefit computing procedure

5) Modify the selling price using the “decision block”. For this, we use three rules (described in [16-19]). If the increment is fuzzy, we need a calling of the fuzzy system SF2 in order to compute the crisp value of the increment. The complexity of SF2 is: $O(SF2) \approx R \cdot k = NA \cdot k$

6) One transmits the result $p_k[t+1]$ at the root process using the MPI communicators

7) **Data centralization in the root process** (at every iteration step - discrete moment of time)
In the root process the vector of prices are reconfigured (like in a FIFO queue) with the returned values from every computing node of the GRID network. These data are collected with the MPI functions with bidirectional transfer MPI_ISEND, MPI_IRECV.

8) Decrementing the step $p \leftarrow p-1$, and return to phase 2

For the economic models with group strategies TCGS, in the previous algorithm the phase 5 (the decision regarding the selling price) is performed only by the “parent” firm of each group. In order to do this modification we need intermediary communication.

5. Results

We compare the performances obtained for the implementations of the models under two programming languages, C and FuzzyCLIPS [12], under two operating systems, Windows and Linux, in two versions of the program – serial and distributed (GRID), as well as related to the capabilities of the machines, for the serial versions of C code and of FuzzyCLIPS. The results reported here refer to the computation time only; the memory resources are not considered.

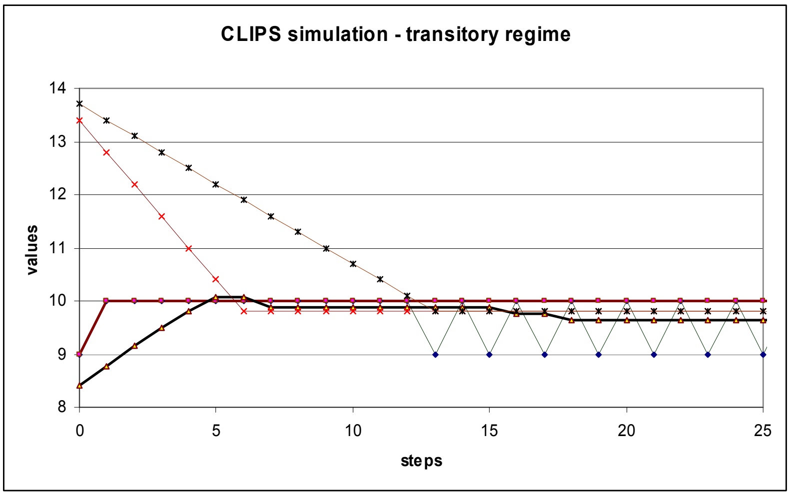

Figure 3. Transitory regime of about 20 steps (mixed strategies marked)

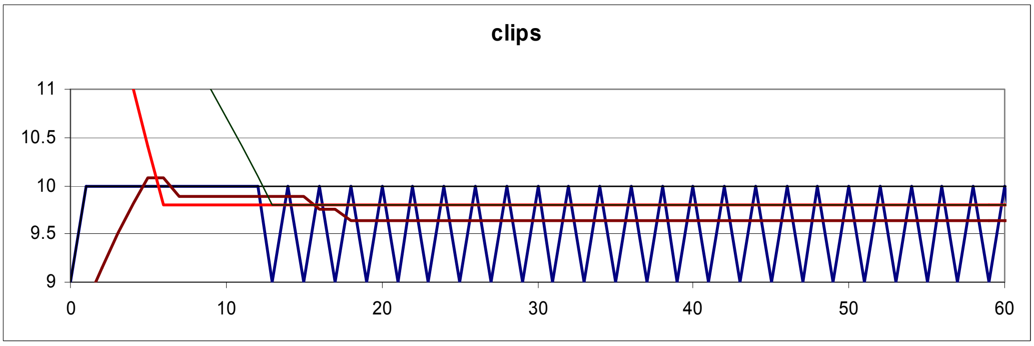

Figure 4. Constancy and periodical variations in the stable regime

In this section, we compare the results obtained on running the simulation of networks of networks of economic agents under FuzzyCLIPS, serial C, and GLOBUS GRID program versions; the programs run on three serial computers and on the *GRAI* GRID network.

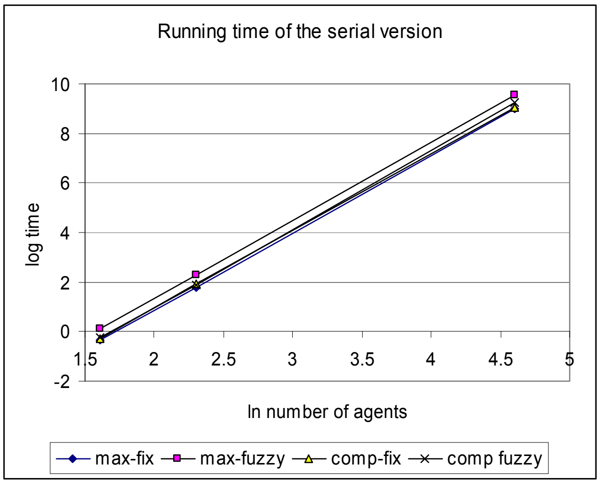

Figure 5. Comparison of the computation time for four strategies of the agents in the market.
The horizontal axis uses the natural logarithm, while the vertical axis uses the basis 2 logarithm

The power law of the computation complexity is evidenced by the log-log plot of the computation times (see Fig. 5). The exponent of the computation time increase, which is equal to the slope of the lines in the log-log plot, differs slightly between the three strategies. The exponent is slightly larger for the *comp-fuzzy* strategy and for the *max-fuzzy* strategy, as expected. The *max-fuzzy* strategy has a larger multiplying factor than the *comp-fuzzy* strategy – see the vertical shift of the former in Fig. 5.

The complexity graphs must be regarded with some reserve, because the configuration of the computer – including the operating system – significantly influence the computation times. As a demonstration, we show in Fig. 6 the graphs obtained for the *comp-fix* strategy on three computers: the results seem to indicate different complexities, while the run program is the same (an .exe program). Notice the slight difference in the slopes obtained on the three computers, as well as the difference in the "overhead" computations.

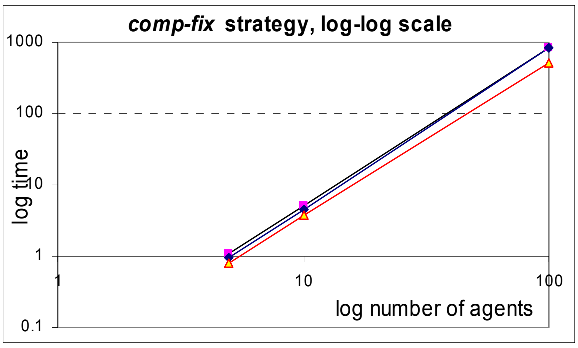

Figure 6. Variation of the computation time depending on the machines

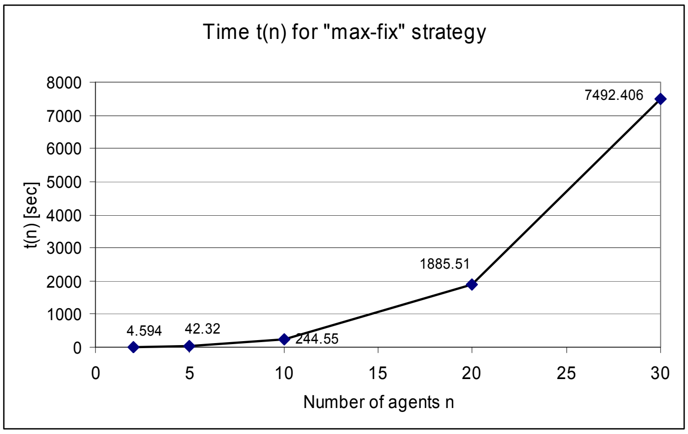

Figure 7. Computation time for the "max-fix" strategy, under FuzzyCLIPS

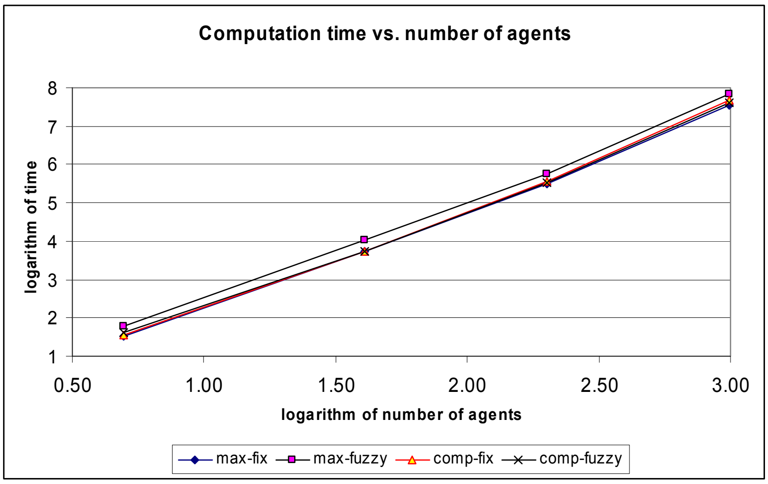

Figure 8. Logarithmic dependence of the computation time with respect to the number of agents. FuzzyCLIPS

The results for the running time on three different machines, for the serial computation (ANSI C program, WINDOWS operating system) are shown in Figure 9. For every algorithm (strategy of the economic agents), and for every number of agents in the network, the program has been run for ten times. The running time on any of the machines is quite stable, for all the cases above mentioned.

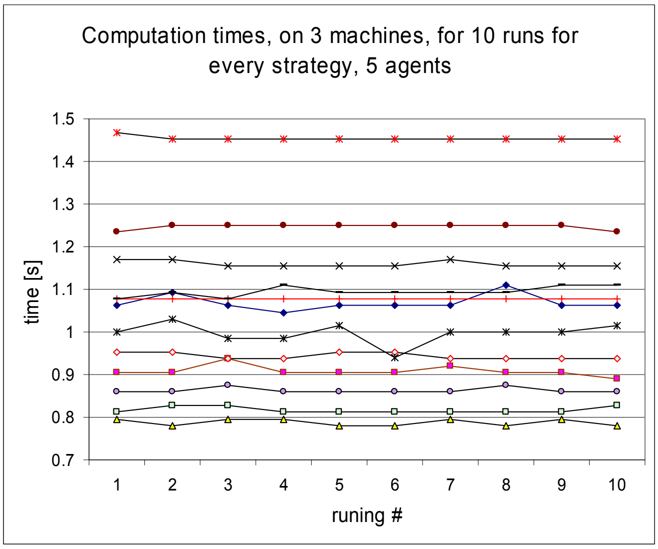

Figure 9. Computation time fluctuations for various machines and strategies of the agents

Based on the graphical results shown in Figures 5-8 , we hypothesize two complexity models: the power model,

$$t(N) = AN^B \tag{3}$$

and a third-order polynomial model, $t(N) = a + bN + cN^2 + dN^3$. The second hypothesis is based on the fact that in the power model approximation, we have obtained values for b lower than 3 (see Table 3). From (3), for two values of N, we obtain the exponent as

$$\ln(t(N_1)) - \ln(t(N_2)) = B(N_2 - N_1). \tag{4}$$

Table 3. Power Law Approximation for the Computation Time of the Economic Models, FuzzyCLIPS

max-fix				**max-fuzzy**			**comp-fix**		
n	**t(n)**	**ln(t)**	power obtained for the larger values of n	**t(n)**	**ln(t)**	power obtained for the larger values of n	**t(n)**	**ln(t)**	power obtained for the larger values of n
5	42.32	3.75	2.9468	55.84	4.02	2.992	42.02	3.74	3.07
10	244.55	5.50		318.56	5.76		257.30	5.55	
20	1885.51	7.54		2534.31	7.84		2154.24	7.68	

The use of polynomial laws needs the use of at least four points $(N, t(N))$, to derive the four coefficients for the cubic law. The use of large values for N provides a more accurate understanding of the asymptotic behavior, but a poor understanding of the computation overhead, which appears more clearly at lower values of N. Moreover, using only large values of N may produce a curve that, at lower values of N, has large errors. To avoid errors at higher and at lower N values, we used two approximations for these regions of the N variable.

Experimental findings for the computation times on GRID are shown in Table 4 and Figure 10.

Table 4. Running Time Results for Larger values of *N*

Strategy	Number of agents	Average time	Power coefficient (B in equ. (3))
max-fix	30	10.75	2.20
	40	20.33	2.21
	44	21.92	0.79
max-fuzzy	30	13.85	2.00
	40	25.13	2.06
	44	30.66	2.08
comp-fix	30	10.97	2.20
	40	20.19	2.12
	44	26.05	2.67
comp-fuzzy	30	11.34	2.29
	40	21.91	2.28
	44	26.36	1.94

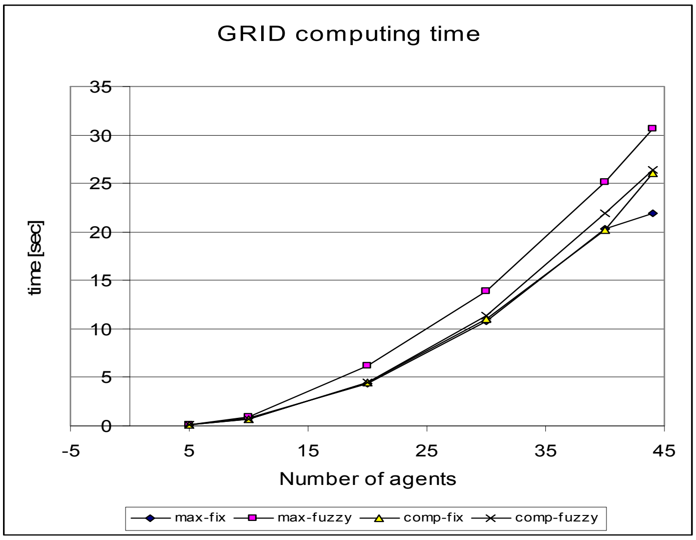

Figure 10. Computation times for the GRID application (4 strategies)

The polynomial model for complexity has confirmed the presence of the cubic term, but the results have been found too much dependent on the values of N used in computations; therefore, we do not report on the polynomial models here.

6. Discussion and Conclusion

We reported in this paper on the parallelization and computation in a distributed, GRID environment of several market models. Regarding the dynamic behavior of the larger systems simulated under GRID, we can conclude, based on the results, that the increase of the number of vendors in the model does not produce significant changes of the dynamics. As expected, in markets where all vendors use the same strategy, the delays in the network largely decide the behavior, including the duration of the transitory regimes and the possibility of occurrence of periodic oscillations. Also expected, larger values of the increment – at least for reasonable values of the increment – tend to stabilize faster the behavior in the strategies using fixed increment. In case of markets with vendors that use different strategies (mixed strategies markets), one of the strategies might dominate the behavior, especially the length of the transitory regime.

Regarding the computation time, the use of a distributed environment reduces the computing time approximately by a factor of N, that is, decreases the perceived complexity from $O(N^{\approx 3})$ to about $O(N^{\approx 2})$. The complexity as obtained for relatively small numbers of agents (less than 100) appears to be $O(N^{\approx 3})$, while our theoretical estimation provides a complexity of order $O(N^2)$. We do not have yet an explanation for this problem except that the overhead, for $N < 100$, is still very important and simulations do not discern between overhead and the influence of N.

Further work is needed to implement variable price increments on a discrete scale, markets with more than two strategies. In addition, improved modes of distributing the computations must be developed, to reduce the communication time. Indeed, in the present version, the communication complexity is of the order of $O(N^2)$. One obvious way to reduce data transfers between nodes is to assign a whole group of vendors to a computing node; this distribution procedure is valid only for group (hierarchic) markets.

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Notice. The contribution of the authors is as follows. Sections I to III, V, and VI have been written by the first author, with the consulting of the other authors. The second author, with consulting from the first author, has written section IV. The paper is based on the models proposed by the first author and presented in the papers [16-19], authored by the first two authors. The opinion of the first author is that a better parallelization could be obtained by assigning more nodes to a processor and fears that the parallelization proposed here is not very effective, due to large communication times. The second author, with some help from the first, wrote the FuzzyCLIPS code, moreover he wrote the C code for the applications. The third author has helped with obtaining the computation times and in the parallelization process.

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