Design of Optimized State Feedback Controller Using ACO Control Law for Nonlinear Systems Described by TSK Models

Hend Liouane, Ali Douik, Hassani Messaoud

Ecole Nationale d'Ingénieurs de Monastir (ATSI), rue Ibn El Jazzar, 5019 Monastir, Tunisie

Hend.liouane@issatkr.rnu.tn, Ali.douik@enim.rnu.tn, hassani.messaoud@enim.rnu.tn

Abstract: In the present paper we propose a new optimal design method to determine the parameters of the optimized state feedback controller for non-linear systems described by continuous-time Takagi-Sugeno-Kang (TSK) models. The proposed method uses Ant Colony Optimization (ACO) technique, which consists of a new population-based approach proposed to solve several optimization problems. The simulation results show that the proposed method is indeed adaptive, robust and the optimized control is provided in a reasonable computing time.

Keyword: non-linear systems, TSK fuzzy models, ACO control law, Lyapunov method, LMI formulation.

Liouane Hend received her BSc degree in Physics science from the FSM-Monastir of Monastir in 2001 and the MSEE degree in Energetic science from the ENIM-Monastir in 2003. She is currently a PhD student in the Electrical Engineering Department at ENIM of Monastir. Her research interests include robust control, evolutionary optimization control, multi-agent control system, ant system and fuzzy system.

Douik Ali is an Associate Professor in the Electrical Engineering Department at the ENIM-Monastir. He received a MSEE degree in 1990 and a PhD degree in 1996, both from the ENSET at Tunis University. Currently he is Chair of the Electrical engineering department at the ENIM-Monastir. His research interests include robust identification and control systems, evolutionary computing algorithms, multi-agent control system, soft computing and fuzzy system.

Hassani Messaoud has prepared his PhD at the University of Sophia Antipolis in France and the habilitation diploma at the University of Birmingham in UK. Currently he is an Associate Professor at Monastir Engineer School in Tunisia. He is majoring in robust identification and control algorithm systems for linear and non linear physical processes where different techniques such as Ant System and fuzzy System have been investigated.

1. Introduction

Using conventional techniques and designing controllers for non-linear dynamic plants can be quite challenging. By using soft computing techniques, intelligent control techniques can greatly simplify the synthesis of a controller for such plants. However, the analysis of intelligent techniques such as fuzzy systems, neural networks, evolutionary programming and Ant Colony Optimization has in many cases been widely studied and has shown promising results [1, 8]. These techniques are used occasionally in practical cases mainly because of the lack of experimental implementation and of analysis of these techniques.

Furthermore, based on the multi-model concept [12], cooperative solutions to control systems have recently been used as counterparts to single-entity problems solvers. There has been a paradigm shift in technology, whereby efficient solutions to control problems were developed through the use of multi-agent strategies rather than a single-agent. Indeed, the multi-agent strategies have a very sophisticated metaphor. In control process, the multi-model concept and the multi-agent approach present very good analysis tools. There significance in application performance can be noted as has been shown by the transformation of the multi-agent system into an active research area.

Fuzzy system and multi-model representation have the public attention due to their better approximation and generalization abilities to handle complex and ill-defined systems. Hence, the TSK fuzzy plants [15, 16, 18] are non-linear systems approximated by a set of local linear models, which is described by a set of fuzzy "*IF-THEN*" rules. It is widely established that such models can describe or approximate a wide class of non-linear systems. Therefore it is important to exploit the combination of the TSK approach and the ACO heuristic [8, 9] for the study of the process optimized control.

Moreover, the activities of social insects have inspired successful applications in many areas [3, 14], especially ant system, and yielded many promising cooperative solutions and algorithms for complex tasks.

For example, ants, using their simple individual interactions mediated by pheromones (one ant follows the chemical scent of another), can collectively determine the shortest path from their nest to the food source without using visual cues.

In our case, the design of the optimized state feedback controller for TSK plants is based on the ACO metaheuristic [6, 7], which imitates the behavior of real ants [8, 3, 14] to find the shortest path between a food source and their nest. Ants communicate with each other by means of pheromone trails and exchange information about which way should be followed. The bigger the number of ants, tracing a given path, the more attractive this path becomes and has been undertaken by other ants by depositing their own pheromones. This autocatalytic and collective behavior results in the establishment of the shortest path.

The method improved by modeling real ant behavior uses exactly the following specifications:

- The communication between ants is established through pheromone trail;
- Paths with more pheromone deposits are preferred;
- Pheromone trail increases rapidly on short paths.

Most ACO algorithms use the algorithmic procedure given below:

Initialize Ant Colony parameters

While (until the termination criteria are met) **do**

Generate Promising Solutions

Evaluate Solutions

Update Pheromone Trail

End

In this paper, we present a new approach for searching the controller parameters of non-linear plants described by the TSK fuzzy model. To achieve this objective we use the quadratic Lyapunov function to analyze the global stability by verifying the existence of a common positive definite matrix which guarantees the stability of all local subsystems. Then, we exploit the ACO control law.

This paper is organized as follows; in section 2 continuous TSK model is presented. Section 3 deals with the stability of the TSK fuzzy models by using Lyapunov stability theory. The design of the optimized state feedback controller using ACO approach is discussed in detail in section 4. In section 5 the performances and effectiveness of the optimized command law are shown through the control of an inverted pendulum.

2. Continuous TSK Fuzzy Model

In the TSK method [15, 16, 18], the non-linear plant takes the following form:

$$\begin{cases} \dot{X}(t) = f\left(X(t)\right) + b\left(X(t)\right) u(t) \\ y(t) = g\left(X(t)\right) + d\left(X(t)\right) u(t) \end{cases} \tag{1}$$

with: $X(.) \in IR^{p}$, $u(.) \in IR^{m}$, $y(.) \in IR^{l}$, $f\left(X(.)\right) \in IR^{p}$, $g\left(X(.)\right) \in IR^{l}$, $b\left(X(.)\right) \in IR^{p.m}$ *and* $d\left(X(.)\right) \in IR^{l.m}$, System 1 is approximated by a set of linear local models, which is described by a set of fuzzy *'IF-THEN'* rules, where fuzzy sets are the antecedents and local linear time invariant systems are the consequents.

The *i-th* rule of the TSK fuzzy model has the following form:

$$IF\ z_1(t)\ is\ F_1^i, \ldots, z_j(t)\ is\ F_j^i, \ldots, z_q(t)\ is\ F_q^i\ \ THEN \begin{cases} \dot{X} = A_i\ X(t) + B_i\ u(t) \\ y(t) = C_i\ X(t) \end{cases} \tag{2}$$

where $i = 1, \ldots, n$ with n the number of rules; the z_j $(j = 1, \ldots, q)$ are the premise variables; the F_j^i is the fuzzy set; X is the state vector; u is the input vector; y is the output vector; A_i , B_i and C_i are matrices of adequate dimensions. Therefore, the global state of TSK fuzzy model is based on the interpolation between several LTI local models as follows:

$$\dot{X} = \sum_{i=1}^{n} \mu_i \, (z(t)) \, \left(A_i \, X(t) + B_i \, u(t) \right) \tag{3}$$

The normalized activation function $\mu_i \, (z(t))$ in relation with the *i-th* submodel is as follows:

$$\begin{cases} \sum_{i=1}^{n} \mu_i \, (z(t)) = 1 \\ \mu_i \, (z(t)) \geq 0 \qquad \forall \, i \in \{1, ..., n\} \end{cases}$$

3. Stability Analysis of Closed-loop TSK Fuzzy Model

The study of the stability of the TSK fuzzy model is based on the Lyapunov theorem [11, 2]; that transforms the problem of finding the common matrix P into a linear matrix inequality (LMI) problem and uses convex programming techniques for the solution [17]. Hence, the Lyapunov-based method is used to determine the state feedback controller which guarantees the global stability for the concerned a non-linear plant modeled by TSK fuzzy model.

To ensure this stabilization a proportional state feedback controller is proposed.

$$u(t) = -K \, X(t) \tag{4}$$

The latter determines the global stability condition and the nominal system performances.

The vector $K = [K_1, K_2, ..., K_p]$ represents the global feedback gain which stabilize the TSK fuzzy model.

Substituting (4) in (3), the closed-loop continuous TSK fuzzy model is:

$$\dot{X} = \sum_{i=1}^{n} \mu_i \, (z(t)) \, \left(A_i - B_i \, K \right) \, X(t) \tag{5}$$

This model is stable according to Lyapunov if there is a common quadratic Lyapunov function $V(X) = X^T \, P \, X$ [5, 10] such as:

1) $V(X)$ is positive definite and continuously differentiable;

2) $\dot{V}(X) \leq 0$.

To determinate the vector K, we use the following theorem:

Theorem* 1*: the closed-loop continuous TSK fuzzy model described by (5) is asymptotically stable if there exists a common symmetric positive definite matrix P such that:

$$G_i^T \, P + P \, G_i \, \langle \, 0 \qquad \forall \, i = 1, 2, ..., n \tag{6}$$

with $G_i = A_i - B_i \, K$.

Relation 6 can lead to an LMI formulation [4] to find simultaneously the matrices $P > 0$ and the gain vector K.

4. Design of the ACO Optimized Controller

This controller consists in using the ACO approach to find the optimized global state feedback gain $K_{opt} = [K_{1opt}, K_{2opt}, ..., K_{popt}]$. The exploitation of the ACO approach is described below:

After computing the global gain K which stabilizes the TSK fuzzy model by using the LMI formulation, the main idea is to randomly generate an *Expected Promising Gain Matrix* (*EPGM*) which contains the expected gains for all the components of the state vector that may be used to determine the optimized ACO controller gain.

4.1 The *EPGM* Representation:

The *EPGM* entries used in the optimized ACO controller are represented by $N_{i,j}$ values. These latter represents the gain which correspond to the *j-th* gain vector and corresponds to the *i-th* state variable that the ACO would use in its searching process. For each component of the gain vector *K,* the variation of $N_{i,j}$ is modelled by generating a simulated actual value for each gain by sampling a uniform random distribution in the range $K_{max} = [K_{1max}, K_{2max}, ..., K_{pmax}]$ and $K_{min} = [K_{1min}, K_{2min}, ..., K_{pmin}]$. For example $K_{min} = 0.8\,K$ and $K_{max} = 1.2K$ $\forall$ i $\in$ [1, 2, …, P].

Let an *EPGM* matrix have *p* rows and *m* columns. Random *EPGM* matrix is generated as follows :

1. Let $K = [K_1, K_2, ..., K_p]$ a gain vector which stabilizes the TSK fuzzy model;

2. Let *m* an arbitrary constant quantifying the number of randomly generated gain vector;

3. Let $K_{min} = 0.8\,[K_1, K_2, ..., K_p]$ and $K_{max} = 1.2\,[K_1, K_2, ..., K_p]$ represent the lower bound and the upper bound of the feedback gains that guarantee the stability domain of the TSK plant model;

4. Generate the *EPGM* matrix which is constituted by a set of elements $N_{i,j}$ ($i = 1,..., p$ and $j=1, ..., m$) with the following algorithm:

For *i* from 1 to *p*

For *j* from 1 to *m*

Pick a value for N_m from the uniform random distribution with range $[K_{i.min}, K_{i.max}]$

$N[i, j] = N_m$

endfor

endfor

Table1. The EPGM representation

	K_1	K_2	K_3	K_4	K_5	K_6 …	K_m
x_1	$N_{1,1}$	$N_{1,2}$	$N_{1,3}$	$N_{1,4}$	$N_{1,5}$	$N_{1,6}$ …	$N_{1,m}$
x_2	$N_{2,1}$	$N_{2,2}$	$N_{2,3}$	$N_{2,4}$	$N_{2,5}$	$N_{2,6}$ …	$N_{2,m}$
x_3	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$	$N_{3,4}$	$N_{3,5}$	$N_{3,6}$ …	$N_{3,m}$
⋮	⋮ ⋮	⋮	⋮	⋮ ⋮	⋮		
x_p	$N_{p,1}$	$N_{p,2}$	$N_{p,3}$	$N_{p,4}$	$N_{p,5}$	$N_{p,6}$ …	$N_{p,m}$

In the table 1, x_i ($i = 1, ...,p$) represents the *i-th* state variable, K_j ($j = 1, ...,m$) is the *j-th* gain vector, and $N_{i,j}$ is the correspondents promising gain.

4.2 Construction Graph and Constraints

Graphically, the optimized controller problem can be represented by the bipartite oriented graphs which are composed of two categories of nodes. The state variables are associated with a x_i node and promising gains are associated with a K_j node. The cost of the connection x_i to K_j is directly linked to the gain $N_{i,j}$. To model the process in a more straightforward manner we use the construction graph that is derived from the *EPGM* representation. The expected promising gain $N_{i,j}$ defines the one distance between the gain K_j and the state variable x_i. Figure 1 represents the construction graph.

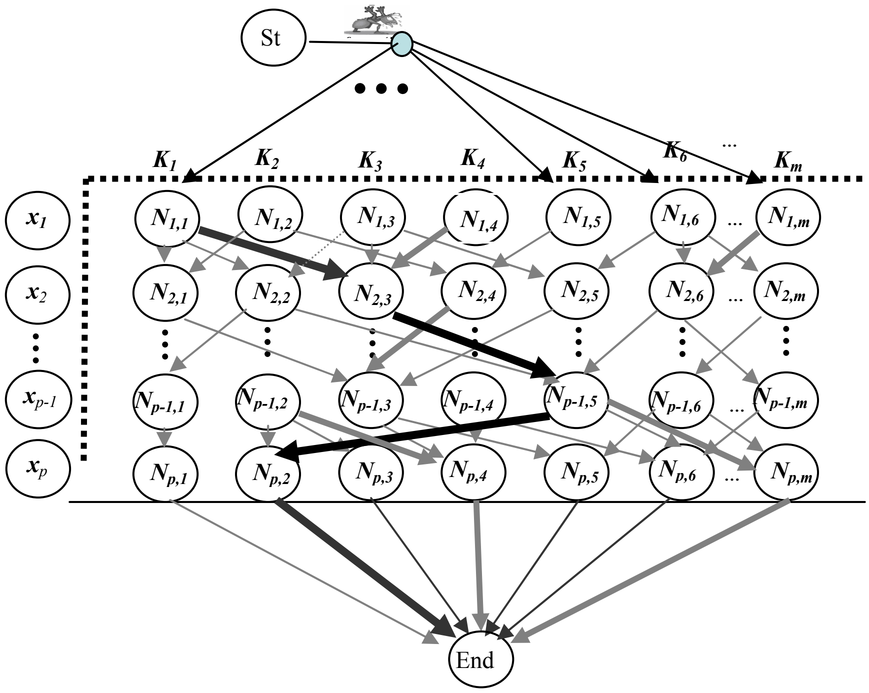


Figure 1. The construction graph associated with the EPGM representation**.**

This construction graph represents a simple solution for the journey of the ants on the path. Specifically, an ant seeks to travel across the construction graph in such a way that all of the following constraints will be satisfied: one and only one node is visited in each row of the graph. We note that there are several paths the ants undertake which differ according to the luminosity. This latter depends on the amount of pheromone which is proportional to the number of ants travelling through these paths. In the rest of this paper, “path”, “tour” , “solution” and “branch” are used interchangeably; the pair *(K_j, x_i)* means: gain K_j is assigned to state variable x_i .

4.3 The ACO Control Law

Typically, ants deposit the chemical pheromone when they move around; they are also able to detect and follow pheromone trails.

In our case, the pheromone trail describes how the ant system builds the optimized solution to the ACO problem. On the construction graph, the probability of choosing a branch at a certain time depends on the total amount of pheromone on the branch which is proportional to the number of ants they visited the branch until that time. $P^a_{i,j}$ represents the probability of the *a-th* ant to assign the gain K_j to the state variable x_i . Each of the ants builds a solution using a combination of the information provided by the pheromone trail $\tau_{i,j}$ and the heuristic function which is defined by $\eta_{i,j} = \frac{1}{|N_{i,j}|}$.

$$P^a_{i,j} = \begin{cases} \dfrac{(\tau_{i,j})^\alpha (\eta_{i,j})^\beta}{\sum_{i,j \in D} (\tau_{i,j})^\alpha (\eta_{i,j})^\beta} & if\ (i,j) \in D \\ 0 & otherwise \end{cases} \tag{7}$$

In this equation, D denotes the set of nodes visited by the a-th ant, where α and β are the parameters that control the relative importance of the pheromone trail versus visibility. Therefore, the transition probability is a trade-off between visibility and pheromone trail intensity at a given time.

4.4 Pheromone Update

To allow the ants to share information about the best solutions, pheromone trail must be updated. After each iteration of the ant system algorithm, equation (8) describes in detail the pheromone update used when each ant has completed its own optimized controller solution S^{ant} characterized by the criterion J^{ant}. This latter corresponds to the length of the path. In order to guide the ants system toward good solutions, a mechanism is required to assess the quality of the best solution. The optimal choice would be to use the iteration-best criterion J^{min} of all solutions given by a set of ants at the current iteration:

$$\Delta \tau_{i,j}^{a} = \begin{cases} \dfrac{J^{\min}}{J^{ant}} & if\ (i,j) \in T^{ant} \\ 0 & otherwise \end{cases} \tag{8}$$

Once all the ants have completed their tours, the trail levels on each branch need to be updated. The evaporation factor ρ which damages the pheromone trail, ensures that the pheromone is not accumulated infinitely and indicates the quality of the pheromone that is approved for the next algorithm iteration. Equation (9) represents the pheromone-level-update:

$$\tau_{i,j} = \rho \,.\, \tau_{i,j} + \sum_{1}^{N^a} \Delta \tau_{i,j}^{a} \tag{9}$$

where N^a defines the number of ants to use in the colony.

In figure 2 the graph is constituted of four state variables and seven vector gains. After some iteration, we note that the pheromone trail increases on the path of the best solution and decreases on the path of the worst solutions.

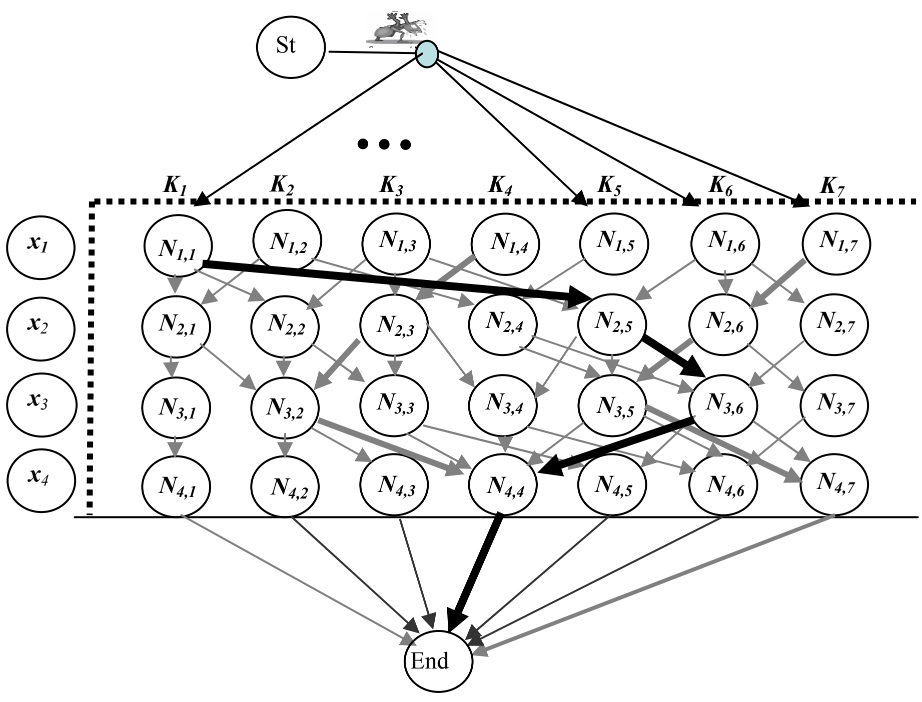


Figure 2. State of the construction graph after some iterations

At the final iteration, only the path of the best solution is maintained The best path labeled as critical path corresponds to the best solution $K_{opt} = [\ K_{1opt} \quad K_{2opt} \quad K_{3opt} \quad K_{4opt}\ ] = [\ N_{1,1} \quad N_{2,5} \quad N_{3,6} \quad N_{4,4}\ ]$.

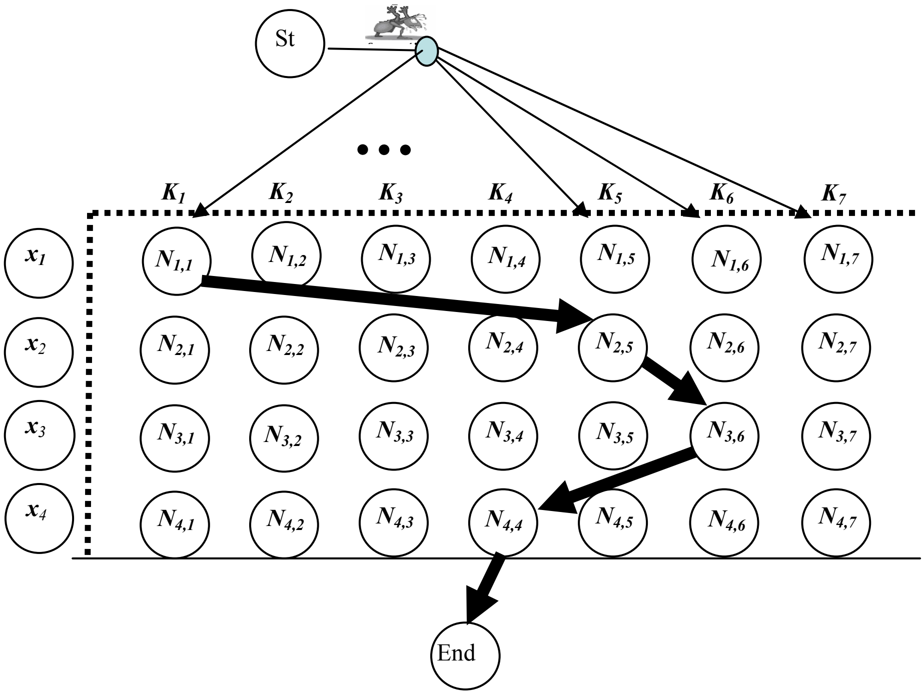


Figure 3. The ants tend to favor the path of the best solution

The corresponding *EPGM* representation is giving by table 2:

Table 2: The EPGM representation that corresponds to the best solution.

	K_1	K_2	K_3	K_4	K_5	K_6	K_7
x_1	$N_{1,1}$	$N_{1,2}$	$N_{1,3}$	$N_{1,4}$	$N_{1,5}$	$N_{1,6}$	$N_{1,7}$
x_2	$N_{2,1}$	$N_{2,2}$	$N_{2,3}$	$N_{2,4}$	$N_{2,5}$	$N_{2,6}$	$N_{2,7}$
x_3	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$	$N_{3,4}$	$N_{3,5}$	$N_{3,6}$	$N_{3,7}$
x_4	$N_{4,1}$	$N_{4,2}$	$N_{4,3}$	$N_{4,4}$	$N_{4,5}$	$N_{4,6}$	$N_{4,7}$

4.5 The Tabu Search

A simple tabu search was also implemented to improve this ACO controller problem. The approach adopted here allows the ants to build their solutions as described in section 4.2, and then the resulting solutions are taken to a local optimum by the tabu search mechanism.

Each of these best ant solutions is used in the pheromone update stage. The tabu search is performed on every best new solution, but needs to be fairly fast. In the case of the ACO controller problem a simple tactic is to randomly pick a gain $N_{i,j}$ on the critical path and randomly change its probability $P_{i,j}$, then verify the value criterion. If the solution is approved we change critical path else we maintain it.

The main steps in the strategy of the ACO controller system by ACO approach and tabu search algorithm are given below:

- Initialize parameters N^a, α, β, τ_0 and ρ;
- Compute the feedback controller K that stabilizes the TSK plant model;
- Apply the *EPGM* creation procedure given in section 4-1;
- Create an initial *solution* and an empty tabu list of a given size.

In order to generate feasible and diverse solutions, initial ants are represented by solutions issued from K,

K_{min} , K_{max} and random method. They are used to approximate an optimized solution as close as possible.

The ACO controller algorithm using tabu search is given as follow:

Repeat (until the termination criteria are met)

- Find *new solution* by ACO controller procedure given in section 4-3
- Compute the value of the criterion described in section 4-4
- Evaluate the quality of the new solution. If a new solution is improved then the current best solution becomes new solution; otherwise we apply the tabu search given in section 4-5
- Add solution to the tabu list; if it is full then delete the oldest entry in the list
- Apply the updating pheromone trail procedure.

END Repeat

4.6 The Setup Parameter Values

The setup parameter values used in the ACO controller algorithm are often very important in getting good results; however the exact values are very often entirely problem dependent and cannot always be derived from features of the problem itself.

- α determines the degree to which pheromone trail is used as the ants build their solution. The lower values imply that the less 'attention' the ants pay to the pheromone trail, but the higher values imply that the ants are performing too little exploration, after testing values in the range 0.25 - 0.75 this algorithm works well with relatively high values (around 0.5 - 0.75).

- β determines the extent to which heuristic information is used by the ants. Again, values between 0.1-0.75 were tested, and a value around 0.5 appeared to offer the best trade-off between following the heuristic and allowing the ants to explore the research space.

- τ_0 is the value to which the pheromone trail values are initialized. Initially the value of the parameter should be moderately high to encourage initial exploration, while the pheromone evaporation procedure will gradually stabilize the pheromone trail.

- ρ is the pheromone evaporation parameter and is always set to be in the range [$0< \rho < 1$]. It defines how quickly the ants 'forget' past solutions. A higher value makes for a more aggressive search; it tests a value of around 0.5-0.75 to find good solutions.

- N^a defines the number of ants to use in the colony, a low value speeds up the algorithm because few searches are done, a high value slows the search down, as more ants run before each pheromone update is performed. A value of 10 appeared to be a good compromise between the execution speed and the quality of the solution achieved.

It is interesting to note that for each value of parameters the ACO controller yields a good solution. Moreover, its convergence speed depends essentially on the number of used ants: the larger N^a is, the more acceptable research solutions are, and therefore the easier and more flexible the search is.

5. Simulation Example

The non-linear system to be controlled is the classical inverted pendulum system described in figure 4.

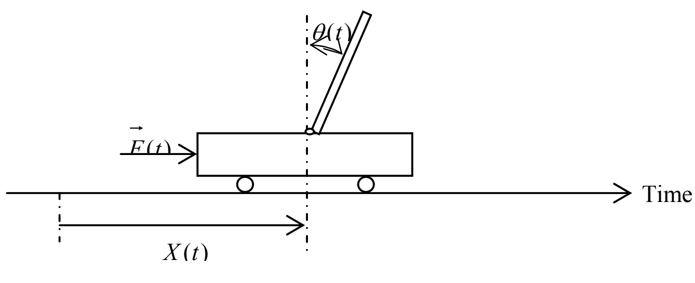


Figure 4. Schematic representation of inverted pendulum and cart

A force $\vec{F}(t)$ is applied to the cart. The goal is to stabilize the pendulum to a position $\theta = 0$ from an initial position θ_0 following a reference trajectory.

The dynamic model of inverted pendulum is described by the following equations:

$$\ddot{x} = \frac{1}{J + \frac{m_i l}{2}(1 - \cos 2\theta)} \left[-m_i a \dot{\theta}^2 \sin\theta + \frac{1}{2} m_i g l \sin 2\theta - l f_p \dot{\theta} \cos\theta + a P_1 u - a(f_c - P_2)\dot{x} \right]$$

$$\ddot{\theta} = \frac{1}{J + \frac{m_i l}{2}(1 - \cos 2\theta)} \left[P_1 l u \cos\theta - \frac{1}{2} m_i l \dot{\theta}^2 \sin 2\theta + m_i g \sin\theta - f_p \dot{\theta} - \dot{x}(f_c - P_2) l \cos\theta \right]$$

with:

$$m = m_c + m_p$$

$$m_i = l\,(m_c + m_p)$$

$$J = J_p - l^2\,(m_c + m_p)$$

$$a = l^2 + \frac{J}{m}$$

Table 2: Parameters of the inverted pendulum.

Name	Description
m_c	Mass of cart
m_p	Mass of pendulum
m	Equivalent mass of cart and pendulum
l	Distance from axis of rotation to center of mass of system
g	Gravity
f_c	Dynamic cart friction coefficient
P_2	Control force to cart velocity ratio
P_1	Control force to PWM signal ratio
f_p	Rotational friction coefficient
J_p	Moment of inertia of pendulum with respect to axis of rotation
J	Moment of inertia related to the mass centre
$x(t)$	longitudinal position of the cart
$\theta(t)$	Angle position of the pendulum

The state vector is given by $X(t) = \begin{bmatrix} x(t) & \theta(t) & \dot{x}(t) & \dot{\theta}(t) \end{bmatrix}^T$ where the positions are measurable.

The complete fuzzy TSK model obtained by polytopic transformation is comprised of two rules [13]:

Rule 1: If *θ(t)* is F_1^1 then $\dot{X}(t) = A_1 X(t) + B_1 u(t)$

Rule 2: If *θ(t)* is F_1^2 then $\dot{X}(t) = A_2 X(t) + B_2 u(t)$

with $z(t) = \theta(t)$ is the premise variable , F_1^1 and F_1^2 represents the fuzzy sets defined in a universe of discourse for the state variable θ . The membership functions $F_1^1(\theta(t))$ and

$F_1^2(\theta(t))$ corresponding to the fuzzy sets are represented as follows :

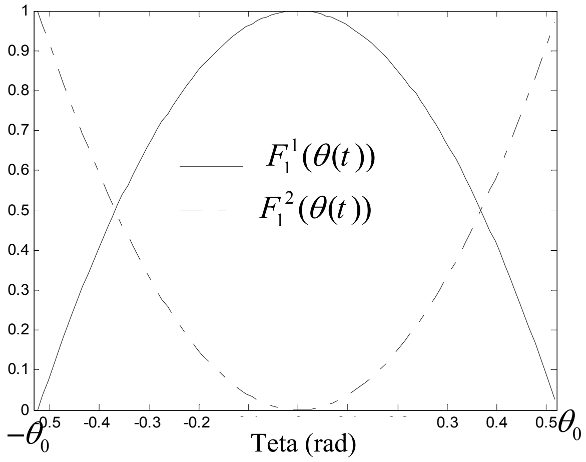


Figure 5. Membership functions**.**

The two sub-models are described respectively by the matrices A_1, B_1 and A_2, B_2 as follow [13]:

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{m_i \ 1 \ g}{J} & -\dfrac{a}{J}\,(f_c - P_2) & -\dfrac{l \ f_p}{J} \\ 0 & \dfrac{m_i \ g}{J} & -\dfrac{(f_c - P_2)\ l}{J} & -\dfrac{f_p}{J} \end{bmatrix}, \ B_1 = \begin{bmatrix} 0 \\ 0 \\ \dfrac{a \ P_1}{J} \\ \dfrac{l \ P_1}{J} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{m_i \ 1 \ g \ \sin 2\theta_0}{2 \ A \ \theta_0} & -\dfrac{a}{A}\,(f_c - P_2) & -\dfrac{l \ f_p \ \cos\theta_0}{A} \\ 0 & \dfrac{m_i \ g \ \sin\theta_0}{A \ \theta_0} & -\dfrac{(f_c - P_2)\ l \ \cos\theta_0}{A} & -\dfrac{f_p}{A} \end{bmatrix}, \ B_2 = \begin{bmatrix} 0 \\ 0 \\ \dfrac{a \ P_1}{A} \\ \dfrac{l \ P_1 \ \cos\theta_0}{A} \end{bmatrix}$$

Therefore, the global state of TSK fuzzy model is based on the interpolation between several LTI local models as follows:

$$\dot{X}(t) = \frac{\sum_{i=1}^{2} w_i(\theta(t)) \ \{ A_i \ X(t) + B_i \ u(t) \}}{\sum_{i=1}^{2} w_i(\theta(t))} = \sum_{i=1}^{2} \mu_i(\theta(t)) \ \{ A_i \ X(t) + B_i \ u(t) \}$$

where: $\mu_i(\theta(t)) = \dfrac{w_i(\theta(t))}{\sum_{i=1}^{2} w_i(\theta(t))}$ is the weighting function.

The ACO technique used to find the optimized feedback gain vector which minimizes the cost function (that corresponds to the length of the path described in section 4-4), given by:

$$J(Q,R)=\int_0^{\infty}(X^T Q X + R U^2)dt$$

where the matrices $Q=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$ and $R=1$ are weighting matrices of the minimized criterion. The ACO approach can be used to solve the problem of computing the optimized gain vector K_{opt}. The optimized control law is implemented by the following equation :

$$u(t) = -K_{opt} X(t)$$

where K_{opt} is the optimized gain vector of the global model obtained by using ACO technique and tabu search.

Then we obtain the following closed-loop continuous TSK model:

$$\dot{x} = \sum_{i=1}^{2}\mu_i(\theta(t))\,(A_i - B_i K_{opt})\,X(t)$$

To determine the *EPGM* representation used in the optimized ACO controller we must compute in the first step the feedback gain *K* by using theorem 1 and LMI formulation. In the second step we randomly generate a set of vector gains belonging to the domain which guarantees the global stability of the TSK fuzzy model. The simulation results yield P and K as:

$$P = \begin{bmatrix}0.0168 & -0.0309 & 0.0099 & -0.0035\\ -0.0309 & 0.2340 & -0.0421 & 0.0259\\ 0.0099 & -0.0421 & 0.0229 & -0.0074\\ -0.0035 & 0.0256 & -0.0074 & 0.0103\end{bmatrix}$$

$$K = [0.0865 \quad 0.0970 \quad -0.0197 \quad -0.0919]$$

The *EPGM* matrix which is constituted by a set of gain $N_{i,j}$ is represented as follows:

($i= 1,..., 4$ and $j=1, ..., 10$)

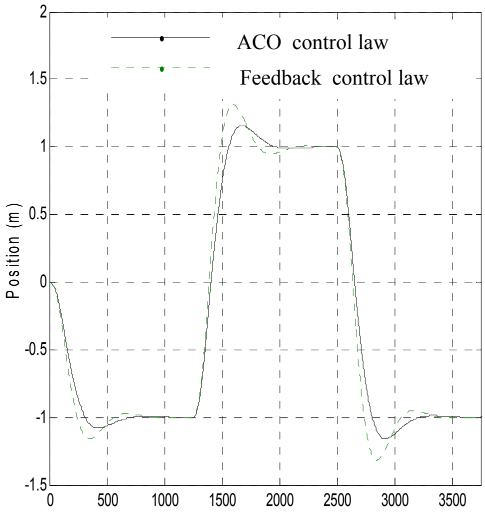
Table3. The EPGM representation

	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	K_{10}
x	0.0699	0.1038	0.0999	0.0865	0.0905	0.0692	0.0730	0.0961	0.07688	0.0922
θ	0.0862	0.1164	0.0948	0.0970	0.0991	0.0819	0.1077	0.0840	0.0776	0.1129
$\dot{x}$	-0.0175	-0.0166	-0.0184	-0.0197	-0.0159	-0.0192	-0.0210	- 0.0157	-0.0207	-0.0236
θ	-0.0879	-0.0745	-0.1096	-0.0919	-0.0735	-0.0828	-0.0980	- 0.0939	-0.0776	-0.1102

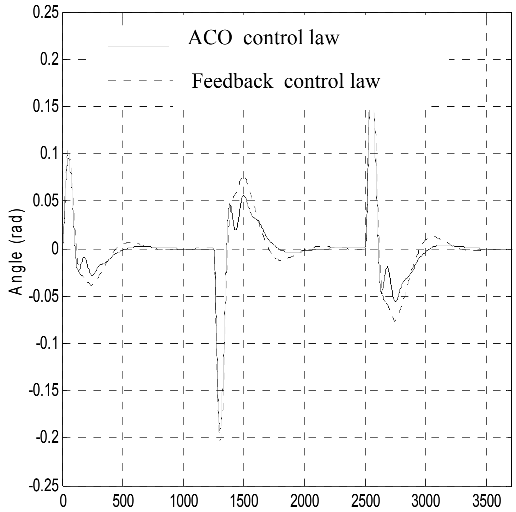
Under simulation conditions $N^a=10$; $\alpha=0.65$; $\beta=0.5$; $\tau_0=0.45$; $\rho=0.25$ and by use of the ACO control law and tabu search we obtain the following optimized vector gain K_{opt} :

$$K_{opt} = [0.0692 \quad 0.1164 \quad -0.0157 \quad -0.0735]$$

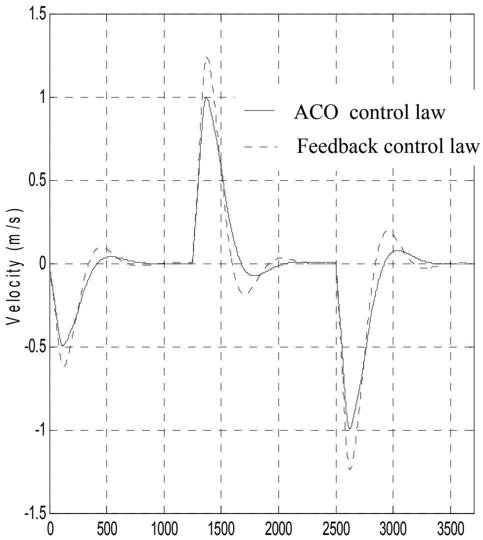
The curves below show the simulation results of the inverted pendulum system responses:



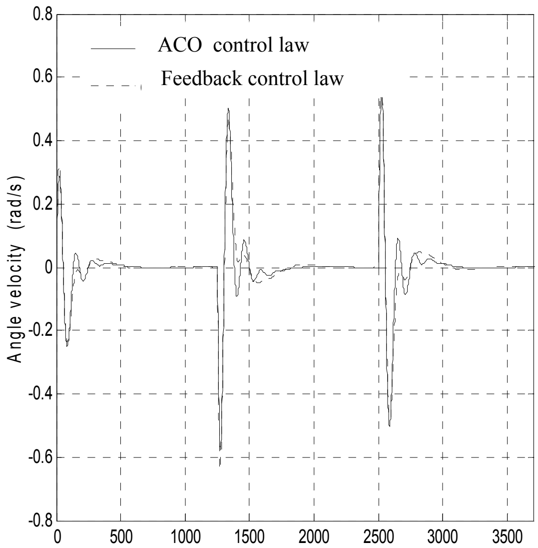
a- *Longitudinal position of the cart: x(m).*



b- *Angle position of the pendulum: θ(rad) .*



c- *Velocity of the cart:* $\dot{x}$ *(m/s).*



d- *Angle velocity of the pendulum* $\dot{\theta}$ *(rad/s).*

Figure 6. System responses of the inverted pendulum with the proposed ACO command law compared with the feedback command law**.**

6. Conclusion

This work proposes a new design technique of robust controllers; firstly using the TSK fuzzy plant model for the analysis of the global stability conditions and secondly the Ant Colony Optimization metaheuristics for the computing of the optimized control law.

The exploitation of the TSK plant model and the LMI tools solve the global system stability problem and gives the nominal system performances while the ACO mechanisms attempt to adjust the system's performance by tuning the controller parameters to find the optimized control law.

In conclusion we emphasize that the TSK fuzzy plant model, when related with ACO controller, can be successfully applied to the optimized control law calculation. Hence, an illustrative inverted pendulum example was provided to summarize the proposed methodology and verify its effectiveness.

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