Mixed Variables Fuzzy Programming Algorithm

Bogdana Pop

Department of Computer Science, Transilvania University of Braşov,

50 Iuliu Maniu, Braşov 500091, Romania

Ioan Dziţac

Department of Business Informatics, Agora University of Oradea,

8 Piaţa Tineretului, Oradea 410526, Romania

e-mail: bpop@unitbv.ro, idzitac@univagora.ro

Abstract: In the classical problems of mathematical programming the coefficients of the problems are assumed to be exactly known. This assumption is seldom satisfied by great majority of real-life problems. The compromise to accept the uncertainty into the mathematical models must be done by working with complex systems. To consider fuzzy constraints is one of possible steps to be done in modelling real systems. One of the most important situations when mixed variables are used is the case of variables having quantified values in connection with regular continuous variables. Starting from the idea of Wang and Liao (2001) for solving fuzzy non-linear integer programming problem and taking into account the multiple criteria optimization in fuzzy environment, a solving method for fuzzy multiple objective integer optimization problem is developed first. Considering multiple criteria optimization in the environment of fuzzy constraints and mixed variables another solving algorithm is introduced.

Key words: optimization, fuzzy constraints, mixed variables, multiple criteria.

Bogdana Pop graduated in Mathematics and Computer Science from "Transilvania University of Braşov, in 1995 and obtained her Ph. D in Mathematics in 2003 from the Romanian Academy. Currently she is an assistant professor at Computer Science Department of Transilvania University of Braşov. Her research interests include different aspects of Fuzzy Optimization and Mathematical Fundamentals of Computers.

Ioan Dziţac received M. Sc in Mathematics (1977) and Ph. D in Information Sc. (2002) from "Babes-Bolyai" University of Cluj-Napoca. Now, he is associate professor and head of Business Informatics Department at "Agora" University of Oradea, Romania. His current research interests include different aspects of Parallel and Distributed Computing and Economic Mathematics and Informatics.

1. Introduction

In real life the certainty in and the solidity of data accuracy are illusory. To obtain an optimal solution is also under the influences of some data missing. Uncertainty has to be taken into consideration when real systems are analyzed. The compromise to accept the uncertainty into the mathematical models must be done working with complex systems.

To interpret the information in fuzzy manner is natural in order to build and to solve real life problems. The philosophy of fuzzy sets theory is related to the model of human thinking and making decision ([6,7]). Concepts of fuzzy sets theory "crowded" into a lot of research fields since 1980, when fuzzy logic has had a great success in the control systems theory. Real advantages of a fuzzy approach to solving optimization problems could be highlighted when a comparison is made with the stochastic methods to deal with imprecision ([5,11]).

Starting from Ergott's idea that "life is about decision" ([3]) the importance of multiple objective models becomes obvious. Ergott's introductive examples make distinction between continuous (maximizing storage capacity of a water dam and, at the same time, minimizing water loss due to evaporation and construction cost) and discreet (choosing a car considering price efficiency and power) optimization. Mixed variables (continuous and integer or even boolean) could appear in practical problems as taking decisions in a transportation problem where warehouses are boolean selected and quantities of products are continuous described. Adding fuzzy constraints more proper models are built.

In 1983 ([4]), Grossman developed an algorithm which decomposes a linear programming problem with flexible constraints into a finite number of linear programming problems without enlarging its dimensions.

In [9], Perkgoz et al. focused on multiple objective integer programming problems with random variable coefficients in objective functions and/or constraints. In [13], Sakawa and Kato presented an interactive fuzzy satisfying method for nonlinear integer programming problems through a genetic algorithm. In [12] α-Pareto optimal solutions were determined for the multi objective integer nonlinear programming problems having fuzzy parameters in the constraints together with the corresponding stability set of the first kind.

Paper [8] presented a method useful in solving a special class of large-scale multi objective integer problems based upon a combination of the decomposition algorithm coupled with the weighting method together with the branch-and-bound method.

Taking into account the multiple criteria optimization in the environment of fuzzy constraints and mixed variables, a solving method is developed here.

"The best compromise" (or Pareto-optimality, efficiency, non-domination) is the central concept in multiple criteria optimization only because an optimal solution for one objective function is not necessary optimal solution for others. In the classic theory weak-efficiency, strong-efficiency, proper-efficiency are gradual definition for the same essence. Starting for classical definition the following concepts appear in the fuzzy multiple objective optimization: M-Pareto optimality (related to the membership functions of "equal" goals), α -Pareto optimality (related to the multiple objective programming problems associated to each α -level set), M-α-Pareto optimality (related to the multiple objective programming problems associated to each α-level set after that "fuzzy equal" goals were defined) ([1, 2]).

In what follows a multiple objective fuzzy constraints mixed variables optimization (MOFCMVO) problem will also be considered. Section 2 draft presents the MOFCMVO model. An algorithm to solving multiple objective fuzzy constraints mixed variables optimization problem is introduced in Section 3. Computational results are inserted in Section 4 and short concluding remarks are made in Section 5.

2. The MOFCMVO Model

Multiple objective programming problems with fuzzy constraints and mixed variables

$$\text{"min"}\left\{z(x) = \left(z_1(x), z_2(x), \ldots, z_p(x) \mid x \in \overline{X}\right)\right\}, \tag{1}$$

where

- $\overline{X} = \left\{x \in Z^k \times R^r \mid Ax \,\bar{\leq}\, b, x \geq 0\right\}$ is the feasible set of the problem,
- A is an $m \times n$ constraint matrix, *x* is an *n*-dimensional vector (k-integer, r-real) of decision variable and $b \in R^m$,
- $p \geq 2$,
- $\left(z_i(x)\right)_{i=\overline{1,p}}$ are objective functions which could be linear, linear fractional or convex functions (in order to make the new method workable),
- "$\bar{\leq}$" defines fuzzy constraints (in sense of having a flexible inequality right hand side (RHS) - left hand side (LHS) and being described by a membership function),

defines the MOFCMV model.

The term *"min"* used in Problem (1) is for finding all efficient solutions in a minimization sense in terms of the Pareto optimality.

A possible way to handle constraints imprecision is to consider the following parametric problem

$$\text{"min"}\left\{z(x) = \left(z_1(x), z_2(x), \ldots, z_p(x) \mid x \in X(\theta)\right)\right\}. \tag{2}$$

where $X(\theta) = \left\{x \in Z^k \times R^r \mid Ax \leq b + \theta b', x \geq 0\right\}$ is the feasible set of the problem, $\theta \in [0,1]$ and *b'* a given perturbation vector ([7]).

The problem of multidimensional mathematical programming (1) consists in finding out a vector $x^* = \left(x_1^*, x_2^*, \ldots, x_n^*\right)$ which should be "the best" (in the sense that it realizes the best compromise) from the point of view of the ensemble of the objective-functions.

Definition 2.1 It is called marginal solution of a multidimensional programming problem a vector $x^* = \left(x_1^*, x_2^*, \ldots, x_n^*\right)$ which is an optimal solution from a certain point of view (meaning a vector x^* for which there an index $h = \overline{1,p}$ exists such that $z_h\left(x^*\right) = \min_{x \in X} z_h(x)$. $z_h\left(x^*\right)$ is called marginal value of function z_h.

Definition 2.2 It is called efficient point of a multidimensional programming problem a vector $x^* = \left(x_1^*, x_2^*, ..., x_n^*\right)$ for which if $z_h(x) \leq z_h\left(x^*\right), \forall h \in \overline{1, p}, \forall x \in X$ it follows that $x = x^*$ (in other words, a vector for which there is no other vector which improves the values of all the criteria).

Remark 2.1 For a solution of the problem to be accepted, firstly the point in which the function is realized has to be an efficient point (in the sense of the given definition).

3. The Solving Method

In [10] an algorithm to solving multiple objective linear fractional programming (MOLFP) is presented. We insert below a briefly description of this algorithm.

- **Step 1.** Establish the problem constant and put $i = 0$.
- **Step 2.** Compute marginal values using ERA.
- **Step 3.** Choose x^0 as a feasible solution of multiple objective problems (MOP).
- **Step 4.** If $\nabla z\left(x^i\right) = 0$ then x^i is the optimal solution. Otherwise go to Step 5.
- **Step 5.** (Acceptability test) If x^i is an acceptable solution of MOP then favorable STOP. If x^i is not an acceptable solution of MOP but a possible improving exists go to Step 6. Otherwise unfavorable STOP. Convenient solution does not exist for MOP.
- **Step 6.** Search a direction of minimization *s* for as many objective functions as possible. If there is no such a direction *s* then let *s* be the direction of minimization for the criterion which realized the largest slack in Step 5. Compute $\lambda_{\max}$ and according with ERA go to Step 7 with $x^{i+1} = x^i + \lambda_{\max} s$.
- **Step 7.** If $S\left(x^{i+1}\right) \leq S\left(x^i\right)$ then go to Step 4 with $i = i + 1$. Otherwise, compute $x^{i+1} = x^i + \lambda s$ for $\lambda < \lambda_{\max}$ and return to the test of Step 7.

where $S(x) = \sum_{h=1,2,...,p} \left(z_h - z_h(x)\right)^2$ and z_h is the marginal value of $z_h(x)$ over the feasible set *X*.

Under the name $MultiObj(\theta)$) the algorithm is first used to solve a fuzzy multiple objective integer programming problem and $MultiObjAlg(\theta)$ algorithm is obtained.

Procedure *back(p,r,y)* described below will be incorporated in $MultiObjAlg(\theta)$ to modify non integer values *x* in integer ones *[x]* or *[x]+1* such that the deviation of the perturbation vector to decrease.

Procedure $back(p, r, y)$:

- If $p = n$ then if $Ax \leq b + \theta_r b'$ then $y = \left(x^r, \theta_r, z\left(x^r\right)\right)$; return.
- If *u[1]=0* then *x[p]=[x[p]]*; *u[1]=1*; $back(p + 1, r, y)$; *u[1]=0*.
- If *u[2]=0* then *x[p]=[x[p]]+1*; *u[2]=1*; $back(p + 1, r, y)$; *u[2]=0*.

Wang and Horng [14] proposed an approach to perform complete parametric analysis in integer programming by considering all of the possible candidates of θ. They defined the principal candidates of θ as been that θ which makes $b_i + \theta b'_i$ an integer for $i = \overline{1, m}$. We also work with these θ's.

In [15], Wang and Liao proposed a heuristic algorithm to analyze the same fuzzy problem. We will use their method to solving multiple objective programming with integer variables and fuzzy constraints.

3.1. Integer Variables

Practical problems having integer variables sometimes are studied in connection with integer coefficients. The case of integer coefficients and possible simplified versions of here developed algorithms is not under discussion in this paper.

Considering *k = 0* and *r = n* in Problem (1) MOFCIVO (multiple objective, fuzzy constraints, integer variables optimization) model is obtained.

$MultiObjAlg(\theta)$ is inserted below.

- **Step 1.** Define the thresholds $0 = \theta_1 < \theta_2 < ... < \theta_q = 1$ using the principal candidates of parameters θ. Put *q=1*.
- **Step 2.** For *k=q* down to 1 do
  - Compute values $x^k = (x_1, x_2, ..., x_n)$ using $MultiObj(\theta_k)$.
  - If $x^k \in Z^n$ then favorable STOP with x^k the θ_k −fuzzy degree acceptable solution of the problem.
  - Otherwise call $back(1, k, y^k)$ and obtain $x^k = (x'_1, x'_2, ..., x'_n)$. Identify $k_{\min}$ such that $Ax \le b + \theta_{k_{\min}} b'$. Then $x^{k_{\min}}$ is the $\theta_{k_{\min}}$ −fuzzy degree acceptable solution of the problem.
- **Step 3.** Describe the problem solution as $(y^k)_{k=\overline{1,q}}$.

This algorithm is used to solve Problem (2) meaning the deterministic problem with the feasible set $X(\theta)$. To improve the interactivity of the method different values for θ will be considered in experiments.

3.2. Mixed Variables

For a fixed value θ we start defining Problem (3) as Problem (2) without the integrity restriction of variables.

$$"\min"\{z(x) = (z_1(x), z_2(x), ..., z_p(x)) \mid Ax \le b + \theta b', x \ge 0\} \tag{3}$$

We obtain an efficient solution for Problem (3) using $MultiObj(1)$. It is a solution for Problem (2) if and only if its components are integer numbers. In this case it is also solution for Problem (1) but with minimal degree in fuzzy environment. Consequently, our next goals are to transform the solution into a mixed (k-integer, r-real) one and also to improve its fuzzy degree.

The solving algorithm could be described as follows.

- **Step 1.** Define the thresholds $\theta = (\theta_1, \theta_2, ..., \theta_q)$.
- **Step 2.** For *j =q* down to 1 do
  - Compute values $x^j = (x_1, x_2, ..., x_n)$ using $MultiObj(\theta_j)$.
  - If $x^j \in Z^k \times R^r$ then favorable STOP with x^j the θ_j −fuzzy degree acceptable solution of the problem.
  - Otherwise call $back(1, j, y^j)$ and obtain $x^j = (x'_1, x'_2, ..., x'_n)$. Identify $j_{\min}$ such that $Ax \le b + \theta_{j_{\min}} b'$. Then $x^{j_{\min}}$ is the $\theta_{j_{\min}}$ −fuzzy degree acceptable solution of the problem.
- **Step 3.** Describe the problem solution as $(y^j = (x^r, \theta_r, z(x^r))^j)_{k=\overline{1,q}}$.

One of the most important situations when mixed variables are used is the case of variables having quantified values in connection with regular continuous variables. Moreover, binary decision variables could be connected with regular variables. To transform integer variables in binary ones is matter of combinatorial optimization. The complexity of solving algorithms will increase respecting the number of type's changed variables.

4. Computational Results

In order to illustrate our solving method let us consider the following deterministic linear fractional program

$$"\min"\left(z(x) = \left(\frac{2x_1 - 7x_2 + 10}{-x_1 - 2x_2 + 50}, \frac{-x_1 - x_2 + 17}{x_1 - x_2 + 13} \right) \right) \quad (4)$$

Subject to

$$\begin{cases} -x_1 + 3x_2 \bar{\leq} 6, \\ 3x_1 + x_2 \bar{\leq} 12, \\ x_1, x_2 \geq 0, \\ x_1, x_2 \in Z. \end{cases} \quad (5)$$

The fuzzy feasible set of Problem (4) meaning (5) is treated as a parameter feasible set (6) considering $\theta \in [0,1]$ and

$$\begin{cases} -x_1 + 3x_2 \leq 6 + 3\theta, \\ 3x_1 + x_2 \leq 12 + 6\theta, \\ x_1, x_2 \geq 0, \\ x_1, x_2 \in Z. \end{cases} \quad (6)$$

Step 1 gave $\theta = (0, 0.166, 0.334, 0.5, 0.666, 0.834, 1)$. The solving algorithm gave fuzzy integer solutions contained in Table 4.1. Consequently, a solution of Problem (4) is

$$Y = \left[y^1 = ((3,3), 0, (-0.21, 0.366)), y^2 = ((4,4), 0.834, (-0.26, 0.3)) \right]$$

k	θ	x_1	x_2	$f(x_1)$	$f(x_2)$	$x_1' \in Z$	$x_2' \in Z$
1	0	3	3	-0.121	0.366	3	3
2	0.166	3.249	3.249	-0.155	0.35	3	3
3	0.334	3.501	3.501	-0.19	0.33	3	3
4	0.5	3.75	3.75	-0.225	0.316	3	3
5	0.666	3.9	3.9	-0.24	0.3	3	3
6	0.834	4.251	4.251	-0.3	0.28	4	4
7	1	4.5	4.5	-0.34	0.26	4	4

Table 4.1

To deal with linear fractional optimization we used classic solving methods which are described in detail in [14]. Also, to make basic optimization calculus we used classic tools.

5. Concluding Remarks

In this paper we have proposed a method of solving fuzzy (constraints) multiple objective integer (decision variables) optimization problems. We worked with the concept of principal candidates for the fuzzy environment parameters (and to improve the interactivity of the method different values for parameters θ were considered) and obtained a fuzzy solution meaning possible solution values versus fuzzy degree of this.

We have applied an algorithm which solves deterministic classic problems and then we transformed the solution into an integer one (developing procedure *back(p,r,y)*) also improving its fuzzy degree. Computational results inserted in Section 5 highlighted the aim of the proposed solving method. To perform global computational analyze we implemented our algorithms (ERA [10], *MultiObj(*θ*), back(p,r,y)*).

Considering that mixed variables are used in the case of variables having quantified values in connection with regular continuous variables or when binary decision variables are connected with regular variables, further work could deal with transforming integer variables in binary ones and modify here developed algorithms in order to keep complexity under control. Current method doesn't take into consideration optimization algorithm's complexity. Only the part of transforming non integer variables to integer ones deals with such aspects.

REFERENCES

1. BADRA, N.M., **Stability of Multiciteria Nonlinear Programming Problems Using Fuzziness**, Fuzzy Sets and Systems, 125, pp. 343-354, 2002.
2. CHAKRABORTY, M. and GUPTA, S., **Fuzzy Mathematical Programming for Multi Objective Linear Fractional Programming Problem**, Fuzzy Sets and Systems, 125, pp. 335-342, 2002.
3. EHRGOTT, E., **Multicriteria Optimization**, SPRINGER-VERLAG, Heidelberg, 2000.
4. GROSSMAN, CH., **An Effective Method for Solving Linear Programming Problems with Flexible Constraints**, ZOR, 27, pp. 107-122, 1983.
5. INGUICHI, M and RAMIK, J., **Possibilistic Linear Programming: a Brief Review of Fuzzy Mathematical Programming and a Comparison with Stochastic Programming in Portfolio Selection Problem**, Fuzzy Sets and Systems, 111, pp. 3-28, 2000.
6. KLIR, G.J. and YUAN, B, **Fuzzy Sets and Fuzzy Logic - Theory and Applications**, PRENTICE HALL, 1995.
7. KOSKO, B., **Fuzzy Thinking**, FLAMINGO, 1993.
8. OSMAN, M.S., SAAD, O.M. and HASAN, A.G., **Solving a Special Class of Large-Scale Fuzzy Multiobjective Integer Linear Programming Problems**, Fuzzy Sets Systems, 107(3), pp. 289-297, 1999.
9. PERKGOZ, C., KATO, K., KATAGIRI, H. and SAKAWA, M., **An Interactive Fuzzy Satisfying Method for Multiobjective Stochastic Integer Programming Problems through Variance Minimization Model**, Sci. Math. Jpn., 60(2), pp. 327-336, 2004.
10. POP, B., **Algorithm to Solving Multiple Criteria Linear Fractional Optimization**, An. Univ. Bucuresti, Inform., 1, pp. 105-114, 2004.
11. ROMMELKFANGER, H., **The Advantages of Fuzzy Optimization Models in Practical Use**, Fuzzy Optimization and Decision Making, 3, pp. 295-309, 2004.
12. SAAD, O.M., **On the Solution of Fuzzy Multiobjective Integer Nonlinear Programming Problems with a Parametric Study**, J. Fuzzy Math., 7(4), pp. 835-843, 1999.
13. SAKAWA, M. and. KATO, K., **An Interactive Fuzzy Satisfying Method for Multiobjective Nonlinear Integer Programming Problems through Genetic Algorithms**, Lect. Notes Comput. Sci., 2715, pp. 710-717, 2003.
14. STANCU-MINASIAN, I. M., **Fractional Programming: Theory, Methods and Applications**, KLUWER ACADEMIC PUBLISHERS, 1997.
15. WANG, H.-F. and HORNG, J.S., **Structural Approach to Parametric Analysis of an IP on the Case of the Right-hand-side**, Europ. J. Oper. Res., 92, pp. 148-156, 1996.
16. WANG, H.-F. and LIAO, Y.-C., **Fuzzy Non-linear Integer Program by Parametric Programming Approach**, Fuzzy Sets Systems, 122(3), pp. 245-251, 2001.