Ant System and Fuzzy Controller for Multi-Objective Optimization of the Flexible Job Shop Scheduling Problems

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Abstract: The multi-objective scheduling research derives its importance from the need to address the real scheduling problem, which is transformed in a single objective. A schedule that is good for one objective function may in fact be quite insignificant for another. Decision-makers must carefully evaluate the trade-offs involved in considering several different criteria in practical scheduling applications.

In this paper, a hybrid approach that combines ant system optimisation and fuzzy logic concept is introduced to facilitate the multi-objective optimisation Flexible Job Shop Scheduling Problem (FJSP). The scheduling problem has two main characteristics namely the flexibility of machines that have the possibility to process all the operations with different processing times and the taking into account different and independent criteria that must be optimized simultaneously. The considered objectives are to minimise makespan, the workload of the critical machine and the total workload of machines. Based on the concept of the ant system and fuzzy controller, the ant system parameters are controlled for multi-objective evolution. The efficiency of the proposed approaches is compared with other approaches by defining the lower bounds for each criterion.

Keywords: Flexible production, job-shop scheduling, Ant colony, Fuzzy controller, Tabu search, makespan, multi-objective optimisation, lower bounds.

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1. Introduction

Recently, it has been verified that approaches issues from the soft computing, such as fuzzy logic, genetic algorithms, ant system and tabu search, are simple, practical, adaptable and computationally efficient to solve a flexible job shop scheduling problem including hard single industrial objective. However, the single objective is insufficient in practical case.

Frequently, the decision-maker can not clearly give a particular preference to a criterion. For example, the minimisation of the makespan C_{max} may be of primary importance and, the minimisation of the workload W machine and the total workload W_T of secondary interest.

Various approaches have been developed in the multi-objective optimisation domain. Two classes of multi-objective optimisation procedure exist (Collette *et al*., 2002) (Hsu *et al*, 2002):

- The Non-Pareto approach consists in the transformation of a multi-objective problem into a mono-objective based on an aggregation operator mixing the different objectives into a weighted sum. The weighted sum translates multiple objectives into a single objective value. Recently, (Kacem *et al.,* 2002) consider the homogenisation approach for dealing with this potential scale difference. (Cochran *et al*., 2003) discuss several schemes for weighting the objectives.

- The Pareto approach is based on the Pareto optimisation concept. This approach satisfies two objectives: converge to the Pareto front and also obtain diversified solutions scattered all over the Pareto front, (Zitzler and Thiele, 1999). A few researchers have dealt with multi-objective scheduling in flexible job shop scheduling. The complexity of the problem has a major role to play in this. Firstly (Esquivel *et al.,* 2002), (Kacem *et al.,* 2002b) and (Xia *et al.,* 2005) investigate the generation of multi-objective schedule in classical and flexible job shops. Secondly (Iima *et al.,* 1999) and (Itoh *et al.,* 1993) consider the minimisation of a function that concerns all the objectives study in classical job shop.

In this paper, a method of the Ant System and the Fuzzy Logic Controller (AS-FLC) is proposed for solving multi-objective FJSP. Thus, the formulation of FJSP is described in section 2. Then, in section 3, the approach to hybridising the ant system with the fuzzy model for controlling the ant system research mechanism is defined. The last section will be devoted to the presentation of some results and some conclusions relating to this research work.

2. Problem Formulation

A flexible job shop production workshop is characterised by a non linear production. In fact, the FJSP may be formulated as follows. Let us consider a set of *n* jobs which are carried out by *m* machines $\{M_1, M_2, \ldots, M_m\}$. Each job J_j consists of a sequence of n_j operations $O_{1,j}, O_{2,j}, \ldots, O_{n_j,j}$. Each routing has to be performed to achieve a job. The execution of each operation *i* of a job J_j, noted $O_{i,j}$ requires one resource or machine selected from a set of available machines. The assignment of the operation $O_{i,j}$ to the machine M_k entails the occupation of the latter one during a processing time called $pt_{i,j,k}$.

The problem is thus to determine both an assignment and a sequence of the operations on all machines that minimize some criteria. All machines $M_k \subseteq M$ are available at $t = 0$.

- A set of *J* independent jobs and each job consists of one fixed sequence of operations. There are no precedence constraints among operations of different jobs.
- Each job is characterised by the earliest starting time r_j and the latest finishing time d_j.
- Denote by $r_{i,j}$ the ready date of the operation $O_{i,j}$.
- A started operation can not be interrupted.
- Each machine can not perform more than one operation at the same time.
- Our objective is to find an operation ordering set satisfying a multi-objective function under problem constraints. In this paper, we consider the balance and compromise between the workload for all machines and the makespan C_{max} of jobs to be performed.

In this study, the following criteria had been minimized:
C_1: makespan or maximal completion time of the all jobs:

$$C\max = \max_{1 \le j \le n}(d_j), \quad (1)$$

C_2: critical machine workload or the maximum of workloads for all machines.:

$$W = \max_{1 \le j \le n}(w_k), \quad (2)$$

where, w_k characterise the workload of the machine M_k.
C_3: total workload of the machines (which represents the total working time of all machines):

$$W_T = \sum_{k=1}^{K} w_k \ , \quad (3)$$

The criteria C_2 and C_3 give a physical meaning to the FJSP, which referring to reducing total processing time and finding the workload compromise between the set of available machines.

3. The Ant System Optimisation and Fuzzy Controller

Initially, the ant colony optimisation (ACO) was described by Dorigo in his PhD thesis (Dorigo *et al.,* 1992) and was inspired by the ability and the organisation of real ant colony using external chemical pheromone trails acting as a means of communication. The main idea is that of a parallel search over several constructive computational solutions based on characteristics problem data and on a dynamic memory structure containing information on the quality of the previous obtained solutions. Generally, the behaviour of ant system optimisation mechanism depends on many unsure parameters, incomplete knowledge of the real ant system attitude and the imprecise information for identification of the relationship between the strategy choice of the parameters and the global behaviour of the ant system metaheuristics. In our context, the fuzzy sets theory are applied for adapting and tuning the ant system parameters to improve the searching ability of ant system in finding the global optimum taking into account of the criteria described in section 1.

3.1. Ant System Scheduling

The ant system approach was inspired by the behaviour of the real ants. The ants depose the chemical *pheromone* when they move in their environment; they are also able to detect and to follow pheromone trails.

In our case, the pheromone trail describes how the ant system builds the solution of the FJSP problem. The probability of choosing a branch at a certain time depends on the total amount of pheromone on the branch, which in turn is proportional to the number of ants that used the branch until that time. The probability $P^f_{i,j,k}$ that an ant will assign an operation $O_{i,j}$ to job J_j to an available machine M_k. Each of the ants builds a solution using a combination of the information provided by the pheromone trail $\tau_{i,j,k}$ and by the heuristic function defined by $\eta_{i,j,k} = pt_{i,j,k}$.

Formally, the probability of picking that an ant f^{th} will assign an operation $O_{i,j}$ of job J_j to the machine M_k is given in equation 4.

$$P^f_{i,j,k} = \begin{cases} \dfrac{(\tau_{i,j,k})^{\alpha} * (\eta_{i,j,k})^{-\beta}}{\sum\limits_{j \in D} (\tau_{i,j,k})^{\alpha} * (\eta_{i,j,k})^{-\beta}} & if \quad j \in D \\ 0 & if \quad j \notin D \end{cases} \tag{4}$$

In this equation, D denotes the set of available non-executed operations set and where α and β are parameters that control the relative importance of trail versus visibility. Therefore the transition probability is a trade-off between visibility and trail intensity at the given time.

Adapting α and β parameters automatically permits firstly to improve the ability of ant system in finding the global multi-objective solution and secondly to fine tuning them with respect the criteria defined in section 1.

The main idea is to use the fuzzy controller to compute new strategy parameter values of the ant system favourability probability $P^f_{i,j,k}$ taking into account the workload W and the total workload W_T.

3.2. Updating the Fheromone Trail

To allow the ants to share information about good solutions, the updating of the pheromone trail must be established. After each iteration of the ant system algorithm, equation 5 describes in detail the pheromone update used when all ants have completed their own scheduling solution denote S^f , that represent the ant solution. In order to guide the ant system towards good solutions, a mechanism is required to assess the quality of the best solution. The obvious choice would be to use the best makespan $L^{min} = C_{max}$ of all solutions given by a set of ants with minimisation of the criteria C_2 and C_3.

$$\Delta\tau^f_{i,j,k} = \begin{cases} \dfrac{L^{\min}}{L^f} & if \ i,j,k \in S^f \\ 0 & otherwise \end{cases} \tag{5}$$

Where L^f represent the makespean of the solution S^f given by the f^{th} ant.

After all the ants have completed their tours, the trails levels on all of the arcs need to be updated. The evaporation factor ρ ensures that pheromone is not accumulated infinitely and denotes the proportion of old pheromone that is carried over to the next iteration of the algorithm. Then for each edge the pheromone deposited by each ant that used this edge are added up, resulting in the following pheromone-level-update equation:

$$\tau_{i,j,k} = \rho.\tau_{i,j,k} + \sum_{f=1}^{NA} \Delta\tau_{i,j,k}^{f} \qquad (6)$$

where NA defines the number of ants to use in the colony.

3.3. Tabu Search Optimisation

A simple tabu search was also implemented for this optimisation FJSP problem. The proposed is to allow the ants to build their solutions as described in section 3.1 and then the resulting solutions are taken to a superior solution by the tabu search mechanism (Liouane *et al, 2007*).

Each of these ant solutions is then used in the pheromone update stage. The tabu search is performed on every ant system solution, every iteration, so it needs to be fairly fast. In the case of the FJSP problem, the method is to pick the machine responsible for the C_{max} and check if any operations $O_{i,j}$ could be swapped between other machines which would result in a lower makespan.

Following their concept, the tabu search considers one problem machine at a time and attempts to swap one operation from the problem machine with any other (non-problem) machine in the solution (non-problem operations). Then the ants are used to generate promising scheduling production solutions and the tabu search algorithm is used to try to improve these solutions.

3.4. Hybridation of the Ant System and the Fuzzy Logic Controller

Based on the fuzzy logic, the fuzzy characterisation of the workload and the total workload the fuzzy logic controller of the ant system appears very useful when the processes are too complex for analysis using conventional techniques and where the available information is interpreted qualitatively and approximately.

Applying fuzzy logic for adapting such parameters regularly does not only enrich the searching ability of the ant system mechanism in finding the very good solution but also permits fine tuning them taking into account the criteria (C_2 and C_3) described in section 2.

The basic fuzzy logic controller concept is introduced to automatically adjust the two α and β parameters at the different stages in the ant system scheduling mechanism and change the exploration strategy of the research space.

3.4.1. Criteria Aggregation by Fuzzy Logic Controller

In this section, the multi-objective FJSP is considered in which two criteria defined in section 2 (C_2, C_3), they are combined by aggregation of the criteria based on the fuzzy inference. For this, the fuzzy controller is used to aggregate the objectives. The decision-maker has to preset the objective criteria and the upper bound values are calculated for every criterion. According to the fuzzy logic, the fuzzy controller is based on four principal components:

- The knowledge base: shaping the expertise base of the control parameters.
- The fuzzification interface: that transforms the crisp parameters values into fuzzy data.
- The inference process: that represents the criteria aggregation based on the knowledge base.
- The defuzzification interface: that transforms the aggregation results into the values parameters action.

3.4.2. Lower Bounds Criteria and Fuzzification

A lower bound C_i^b for each criterion value is determined by (Kacem *et al.,* 2002b) (Saad et *al.,* 2006). So, the following relations are demonstrated:

$$C_i(x) \geq C_i^b \;, \quad \forall\; x \in S, i = 1,\ldots,n_c$$

where S represents the set of feasible solutions, n_c is the number of criteria and C_i is the criterion of the solution x, and:

$$C_1^b = \max_{1 \le j \le n}\left(r_j + \sum_{i=1}^{n_j} \gamma_{i,j} \right) \tag{7}$$

where $\gamma_{i,j} = \min_k (pt_{i,j,k})$.

The lower bound of C_2^b and C_3^b are determined by the following relations:

$$C_2^b = E\left(\frac{\sum_{j=1}^{n} \sum_{i=1}^{n_j} \gamma_{i,j}}{m} \right) \tag{8}$$

where $E(x)$: round the elements of x to the nearest integer.

$$C_3^b = \sum_{j=1}^{n} \sum_{i=1}^{n_j} \gamma_{i,j} \tag{9}$$

The fuzzy characterisation of the criteria is given by the membership function $\mu c_i(x)$ shown in Figure 1. According to the criteria objective, at the *kth* iteration of the ant system algorithm, the quality of solutions is evaluated. The upper bounds of the C_2^h and C_3^h represent the highest feasible solution obtained by the ant system scheduling mechanism at the *kth* iteration. The lower bound is calculated by (Kacem et at 2002a).

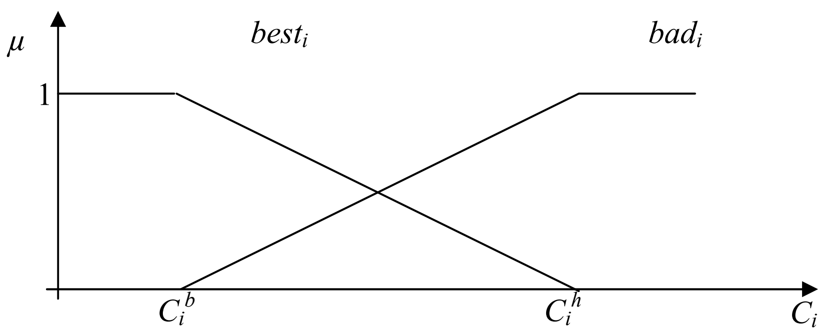

Figure 1. Fuzzy characterisation of the criteria

The two considered fuzzy subsets are the subset of the best solutions according to the criterion C_i, noted $best_i$ and the subset of the bad solutions according to the criterion C_i, noted bad_i, and membership functions is given in figure 1. Then, the quality of each solution x is characterized by $\mu_i^{best}(C_i(x))$ and $\mu_i^{bad}(C_i(x))$.

3.4.3. Control Ant System Parameters

Based on a number of experiments and opinion of the expertise in scheduling domain, the fuzzy decision rules were constructed for controlling the evolution of the parameters (α and β) related to the ant system strategy research solutions. The ant system strategy is given by the following basis rules:

- **If** $C_2(x)$ **is** $best_2$ **and** C_3 (x) **is** $best_3$ **then** α decrease **and** β increase, for the next iteration of ant system algorithm
- **If** $C_2(x)$ **is** bad_2 **and** C_3 (x) **is** bad_3 **then** α increase **and** β decrease, for the next iteration of ant system algorithm

In order to implement and facilitate the fuzzy logic controller, the control ant system vector are noted with $\mathrm{U} = [\alpha \quad \beta]^T$, $\mathrm{U} \in \Re^2$ and the decision strategy matrices $\mathrm{A} \in \Re^{2\times 2}$ that represent the expert decision for increasing or decreasing parameters (α and β). The equivalent basis rules are given by:

- **If** $C_2(x)$ **is** *best₂* **and** C_3 **is** $best_3$ **then** $\mathrm{U}^+ = A_1 \times U$
- **If** $C_2(x)$ **is** bad_2 **and** C_3 **is** bad_3 then $\mathrm{U}^+ = A_2 \times U$

In our case, the U^+ represent the control ant system vector for the next iteration,

$\mathrm{A}_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$, $\mathrm{A}_1 = \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix}$ represent respectively decision strategy of first and second rule.

3.4.4. Defuzzification and the Fuzzy Inference System

Based on the theory of the fuzzy logic, the fuzzy controller concerns the non linear system represented by a set of fuzzy rules of which the consequent part are linear equations. The non linearity is transformed in weighted sum of these linear state equations.

According the form of basis rules and the fuzzification phase described previously, the final output of the fuzzy logic controller is inferred as follows:

$$\mathrm{U}^+ = \frac{\min\limits_{i=2,3}\left(\mu_i^{best}(C_i(x))\right)A_1 + \min\limits_{i=2,3}\left(\mu_i^{bad}(C_i(x))\right)A_2}{\min\limits_{i=2,3}\left(\mu_i^{best}(C_i(x))\right) + \min\limits_{i=2,3}\left(\mu_i^{bad}(C_i(x))\right)}U \tag{10}$$

3.4.5. The Set Up Parameter Values

The set up parameter values used in the ant system scheduling algorithms and the fuzzy logic controller are often very important in getting good results, however the appropriate values are very often entirely problem dependent (Dorigo *et al.* 2002), and cannot always be derived from features of the problem itself. Generally, these parameters are defined by the expertise in the production domain:

- the control vector $U = [\alpha, \beta]^T$ precise how the ant system mechanism exploits the fuzzy controller for adapting the compromise between the pheromone trail and the heuristic information,
- the α determines the degree to which pheromone trail is used as the ants build their solution. The lower value, the less 'attention' the ants pay to the pheromone trail, but the higher values implicate the ants then perform too little exploration,
- β determines the extent to which heuristic information is used by the ants,
- the matrices A_1 and A_2 perform how the fuzzy controller increases or decreases the control vector for adapting multi-objective optimisation problems,
- τ_0 is the value to which the pheromone trail values are initialised. Initially the value of the parameter should be moderately high to encourage initial exploration, while the pheromone evaporation procedure will gradually stabilise the pheromone trail,
- ρ is the pheromone evaporation parameter and is always set to be in the range $[0 < \rho < 1]$. It defines how quickly the ants 'forget' past solutions. A higher value makes for a more aggressive search; it tests a value of around 0.5-0.75 to find good solutions,
- NA defines the number of ants to use in the colony, a low value speeds the algorithm up because less search is done, a high value slows the search down, as more ants run before each pheromone update is performed. A value of 10 appeared to be a good compromise between execution speed and the quality of the solution achieved.

It is interesting to note that for each value of parameters the ant system scheduling meta-heuristics yields a good solution. Moreover, its convergence speed depends essentially on the number of used ants *NA*.

3.4.6. Building a Solution Steps

The main steps in the strategy of the FJSP by ant system and fuzzy logic controller algorithm are given below:

- Initialise parameters $NA, \alpha, \beta, \tau_0, \rho$.
- Create an initial solution and an empty tabu list of a given size.
  In order to generate feasible and diverse solutions, initial ants are represented by solutions issued from the heuristic rules (SPT, DL, FIFO, etc) and a random method. Heuristics are used to approximate the optima as near as possible.
- **Repeat the following steps until the termination criteria are met:**
  - Find *new solution* by ant system procedure scheduling given in section 3.1.
  - Evaluate the quality of the new solution.
  - If a *new solution* is improved then the current *best solution* becomes *new solution*
  - *else* If *no new solution* was improved then apply the tabu search optimisation given in section 3.3.
  - Add *solution* to the tabu list, if the tabu list is full then delete the oldest entry in the list.
  - Apply the updating pheromone trail procedure given in section 3.2
  - Compute the upper bound of criteria C_2^h and C_3^h given in section 3.4.2
  - Apply the fuzzy controller and compute U^+ by the FLC model
- **END Repeat**

4. Benchmark and Simulation Results

All results of the ant system and fuzzy logic controller optimisation (AS-FLC) are presented for 1000 iterations with 10 the number of ants, and each run was performed 10 times. The algorithms have been coded in VB language and tested using a P4 Pentium processor 2.4 GHz and Windows XP system.

To illustrate the effectiveness and performance of the algorithm proposed in this paper, four representative benchmark FJSP instances, represented by problem *nxm*, based on practical data have been selected to compute. These benchmark instances are all taken from of (Kacem *et al.,* 2002a), (Xia *et al.,* 2005), and (Saad *et al.,* 2006). The benchmark concerns the *(8x8), (10x7), (10x10)* and *(15x10)* instances problems. The different results obtained by proposed approach, Ant System and Fuzzy Logic Controller (AS-FLC), is presented and compared with the other methods in table 3.

The different results obtained by our approach are compared with the other methods:

- "AL+CGA" refers to (Kacem *et al.,* 2002a) : the Pareto optimality approach based on the hybridation of evolutionary algorithms and fuzzy logic.
- "PSO-SA" refers to (Xia *et al.,* 2005): the hybridation of particle swarm optimisation and the simulated annealing methaheuristics.
- "MOGA" refers to (Saad *et al.,* 2006): the Choquet Integral for Criteria Aggregation by the genetic algorithms.
- "classic GA" refers to classical genetic algorithm.

The simulation conditions are presented in table 2 and the parameters values of NBA, τ_0 and ρ are presented in table 1.

	n	*m*	*NA*	τ_0	ρ
Instance1	8	8	10	0.1	0.2
Instance 2	10	7	10	0.1	0.2
Instance 3	10	10	10	0.05	0.1
Instance 4.1	15	10	10	0.05	0.25
Instance 4.2	15	10	10	0.05	0.25

Table 1: Parameters values.

- **Instance 1** *(8 jobs/27 operations/8 machines):* $r_j = 0, \forall j$
- **Instance 2** *(10 jobs/29 operations/7 machines):* $r_1 = 2$, $r_2 = 4$, $r_3 = 9$, $r_4 = 6$, $r_5 = 7$, $r_6 = 5$, $r_7 = 7$, $r_8 = 4$, $r_9 = 1$ and $r_{10} = 0$.
- **Instance 3** *(10 jobs/30 operations/10 machines):* $r_j = 0, \forall j$.
- **Instance 4.1** *(15 jobs/56 operations/10 machines)*: $r_1 = 5$, $r_2 = 3$, $r_3 = 6$, $r_4 = 4$, $r_5 = 9$, $r_6 = 7$, $r_7 = 1$, $r_8 = 2$, $r_9 = 8$, $r_{10} = 0$, $r_{11} = 14$, $r_{12} = 13$, $r_{13} = 11$, $r_{14} = 12$, and $r_{15} = 5$.
- **Instance 4.2** *(15 jobs/56 operations/10 machines):* $r_j = 0, \forall j$

Table 2: Simulation conditions**.**

nxm		C_i^b	classic GA	AL+CGA	PSO+SA	MOGA	AS+FLC
8x8	C_1	12	16	16	15	16	***15***
	C_2	10		13	12	13	***13***
	C_3	73	77	75	75	75	***73***
10x7	C_1	11		15		11	***11***
	C_2	9		11		10	***11***
	C_3	60		61		64	***61***
10x10	C_1	7	7	7	7	7	***7***
	C_2	5	7	5	6	5	***5***
	C_3	41	53	45	44	44	***43***
15x10 (1)	C_1	23		23		23	***23***
	C_2	10		11		11	***11***
	C_3	91		95		99	***91***
15x10 (2)	C_1	11			12		***12***
	C_2	10			11		***12***
	C_3	91			91		***91***

Table 3: Comparison of results.

In the majority of cases, the obtained results by AS+FLC indicate that this approach can yield better results contrary to other methods such as Genetic Algorithm and fuzzy logic (Kacem *et al*, 2002), Particle Swarm Optimization and Simulated Annealing (Xia *et al,* 2005), the Genetic Algorithm by Choquet Integral (Saad *et al*, 2006)... Concerning the FJSP instances, the results obtained by the AS+FLC show that the given solutions are generally acceptable and satisfactory. The values of the different objective functions show the efficiency of the suggested approach, table 3. Moreover, the proposed method enables us to obtain good results in a polynomial computation time.

*	O1	O2	O3	O4
J1	M51 :[0,3]	M5 :[3,6]	M6 :[9,11]	
J2	M3 :[0,3]	M4 :[7,9]	M7 :[9,10]	M5 :[10,14]
J3	M7 :[0,2]	M4 :[9,13]	M1 :[13,14]	
J4	M2 :[0,1]	M6 :[1,6]	M2 :[14,16]	
J5	M1 :[0,3]	M4 :[3,7]	M6 :[7,9]	M7 :[10,13]
J6	M3 :[3,4]	M8 :[4,8]	M2 :[9,14]	
J7	M3 :[4,6]	M8 :[8,13]	M4 :[13,16]	
J8	M1 :[3,5]	M2 :[5,9]	M8 :[13,14]	M5 :[14,15]

Table 4: Optimisation solution of problem 8 x 8 ($Cmax = 15$, $W_T = 73$, $W = 13$)

*	O1	O2	O3
J1	M1 :[0,1]	M3 :[1,2]	M4 :[2,3]
J2	M1 :[2,4]	M10 :[4,5]	M10 :[5,7]
J3	M10 :[3,4]	M8 :[4,5]	M7 :[5,6]
J4	M7 :[1,2]	M3 :[3,6]	M4 :[6,7]
J5	M9 :[0,2]	M9 :[2,3]	M4 :[3,4]
J6	M6 :[0,2]	M9 :[3,5]	M9 :[5,6]
J7	M1 :[1,2]	M8 :[2,4]	M6 :[4,5]
J8	M5 :[0,2]	M2 :[2,5]	M2 :[5,7]
J9	M3 :[2,3]	M7 :[4,5]	M6 :[5,6]
J10	M6 :[2,3]	M7 :[3,4]	M4 :[4,6]

Table 5: Optimisation solution of problem 10 x 10 ($Cmax = 7$, $W_T = 43$, $W = 5$)

*	O1	O2	O3	O4
J1	M1 :[5,6]	M2 :[6,7]	M3 :[11,12]	M9 :[18,20]
J2	M4 :[3,4]	M3 :[4,6]	M10 :[16,17]	M4 :[17,18]
J3	M7 :[6,7]	M9 :[10,11]	M2 :[11,13]	M4 :[13,14]
J4	M5 :[4,5]	M5 :[5,6]	M6 :[12,13]	M10 :[19,20]
J5	M9 :[9,10]	M6 :[10,12]	M2 :[13,15]	M4 :[15,17]
J6	M2 :[5,6]	M1 :[14,15]		
J7	M1 :[1,2]	M2 :[7,8]		
J8	M7 :[2,3]	M4 :[4,6]	M4 :[6,8]	M1 :[15,16]
J9	M10 :[8,9]	M3 :[9,11]	M7 :[15,19]	M8 :[20,22]
J10	M10 :[0,1]	M8 :[1,3]	M10 :[13,14]	M2 :[15,17]
J11	M7 :[14,15]	M10 :[15,16]	M3 :[16,18]	M7 :[19,20]
J12	M8 :[13,15]	M5 :[15,19]	M6 :[19,21]	M7 :[21,23]
J13	M10:[11,13]	M9 :[13,16]	M10 :[17,19]	M2 :[19,21]
J14	M1 :[12,14]	M9 :[16,18]	M8 :[18,20]	M5 :[21,23]
J15	M1 :[6,8]	M6 :[8,10]	M3 :[12,14]	M5 :[19,21]

Table 6: Optimisation solution of problem 15 x 10(1) (*Cmax* = 23, W_T = 91, *W* = 11)

*	O1	O2	O3	O4
J1	M1 :[0,1]	M2 :[3,4]	M3 :[5,6]	M9 :[7,11]
J2	M4 :[0,1]	M3 :[1,3]	M10 :[7,8]	M4 :[8,9]
J3	M7 :[1,2]	M9 :[2,3]	M2 :[4,6]	M4 :[7,8]
J4	M5 :[0,1]	M5 :[1,2]	M6 :[5,6]	M10 :[10,11]
J5	M9 :[0,1]	M6 :[1,3]	M7 :[3,5]	M4 :[5,7]
J6	M2 :[0,1]	M1 :[6,7]		
J7	M1 :[5,6]	M2 :[6,7]		
J8	M7 :[0,1]	M8 :[2,3]	M4 :[3,5]	M1 :[7,8]
J9	M10 :[1,2]	M3 :[3,5]	M7 :[5,9]	M8 :[9,11]
J10	M10 :[0,1]	M8 :[3,5]	M10 :[6,7]	M2 :[7,9]
J11	M7 :[2,3]	M10 :[5,6]	M3 :[8,10]	M7 :[11,12]
J12	M8 :[0,2]	M5 :[2,6]	M6 :[6,8]	M7 :[9,11]
J13	M2:[1,3]	M10 :[3,5]	M10 :[8,10]	M2 :[10,12]
J14	M1 :[3,5]	M9 :[5,7]	M8 :[7,9]	M5 :[10,12]
J15	M1 :[1,3]	M6 :[3,5]	M3 :[6,8]	M5 :[8,10]

Table 7: Optimisation solution of problem 15 x 10(2) (*Cmax* = 12, W_T = 91, *W* = 12)

*	O1	O2	O3
J1	M1 :[1,2]	M5 :[5,6]	M7 :[6,9]
J2	M7 :[2,5]	M1 :[5,7]	
J3	M7 :[5,6]	M3 :[7,8]	M5 :[5,10]
J4	M1 :[3,5]	M4 :[9,10]	M5 :[10,11]
J5	M2 :[5,6]	M1 :[7,9]	M4 :[10,11]
J6	M3 :[2,6]	M3 :[6,7]	M3 :[8,10]
J7	M7 :[0,2]	M6 :[2,6]	M6 :[7,8]
J8	M4 :[0,1]	M4 :[1,9]	M2 :[9,11]
J9	M5 :[1,5]	M2 :[6,8]	M7 :[9,11]
J10	M2 :[1,5]	M6 :[6,7]	M1 :[9,10]

Table 8: Optimisation solution of problem 10 x 7 (*Cmax* = 11, W_T = 61, *W* = 11)

5. Conclusion

In this paper, a new approach based on the combination of the ant system with the fuzzy controller techniques for solving flexible job-shop scheduling problems, is presented. The results for the reformulated problems show that the ant system with the fuzzy controller techniques can find an optimised solution for different problems that can be adapted to deal with the FJSP problem. The detail concepts of implementation for the multi-objective FJSP are presented and their performances are compared using computational benchmarks.

The numerical simulation results have shown that the ant system algorithm with the fuzzy logic controller can find good quality of solutions for many different problems, furthermore for multi-objective optimisation FJSP.

The performances of the new approach are evaluated and compared with the results obtained from other methods. The obtained results show the effectiveness of the proposed method. Ant system algorithms and the fuzzy logic controller techniques described are very effective and they alone can outperform all the alternative techniques. The comparison study, based on benchmark simulation, shows that our AS-FLC algorithms significantly surpass previous approaches in term of both quality and time execution.

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