Localization Algorithm of Time Disturbances in Tolerant Multi-product Job-shops

Nabil Jerbi [1,2], Simon Collart Dutilleul [1], Etienne Craye [1], Mohamed Benrejeb [2]

[1]Ecole Centrale de Lille

Laboratoire d'Automatique, Génie Informatique et Signal

Cité Scientifique, BP 48, 59651 Villeneuve d'Ascq, France

nabil.jerbi@isetso.rnu.tn, simon.collart_dutilleul@ec-lille.fr, etienne.craye@ec-lille.fr

[2]Ecole Nationale d'Ingénieurs de Tunis

Unité de recherche LARA-Automatique

BP 37, Le Belvédère, 1002 Tunis, Tunisie ,

mohamed.benrejeb@enit.rnu.tn

Abstract: This paper deals with time disturbances localization in critical time manufacturing job-shops. In such systems, operation times are included between a minimum and a maximum value. Controlled P-time Petri nets are used for modeling. Some definitions and a series of lemmas are quoted in order to build a theory dealing with such problem. They are illustrated step by step on examples of a given workshop. A new algorithm built upon the lemmas results is provided in order to localize time disturbances occurrence.

Keywords: P-time Petri net, time constraints, tolerant systems, time disturbance, localization.

Nabil Jerbi was born in Tunis, Tunisia, in 1970. He obtained the Engineer degree in electrical engineering from the Ecole Nationale d'Ingénieurs de Tunis (ENIT), in 1994. Also, he received the Aggregation Certificate from the Ecole Supérieure des Sciences et Techniques de Tunis and the Master degree in automatic and signal treatment from ENIT, in 2001 and 2003, respectively. He is currently a Technologue at the Institut Supérieur des Etudes Technologiques de Sousse. He is working toward Ph.D. degree in automatic and computer science within the framework of LAGIS/EC-Lille and LARA/ENIT cooperation. His research interests include robustness and supervision of multi-product job-shops with time constraints.

Simon Collart Dutilleul obtained a Ph.D. degree in Electronics, Electrotechnics and Automatic Control from Université de Savoie in 1997. He is currently an Assistant Professor at Ecole Centrale de Lille, and a member of the research team on Discrete Event Systems in the LAGIS laboratory (Laboratory of Control Engineering, Computer Science & Signal), where he studies specifically time constrained systems.

Pr. Étienne Craye was born in Roubaix (France) in 1961; he obtained in 1984 the Engineer Diploma of the "Institut Industriel du Nord" (French "Grande Ecole") and the same year his Master Degree in Computer Sciences. He obtained a Ph.D. in Automatic control for Manufacturing and Discrete Events systems in 1989 and his "Habilitation à Diriger des Recherches" in 1994. He is now, since 1995, Professor at the Ecole Centrale de Lille and in the same time the Director of this institution. Pr. Craye is currently working on Monitoring and Supervision of Fault Tolerant Systems. Specially, reconfiguration and working mode management are today studied by taken into account on one-hand failures and on the other hand the flexibilities of the system architecture. The objective is to be able to go on with the production and not to reconsider the objectives.

Pr. Mohamed Benrejeb was born in Tunisia in 1950. He obtained the Diploma of "Ingénieur IDN" (French "Grande Ecole") in 1973, the Master degree of Automatic Control in 1974, the Ph.D. in Automatic Control of the University of Lille in 1976 and the DSc of the same University in 1980. He is currently a full Professor at the Ecole Nationale d'Ingénieurs de Tunis and an invited Professor at the Ecole Centrale de Lille. His research interests are in the area of analysis and synthesis of complex systems based on classical and non conventional approaches.

1. Introduction

Critical time manufacturing job-shops use to be tolerant in order to maintain product quality when there are time disturbances [1], [2]. Nevertheless, it may happen that the temporal abnormal functioning ensues from wear of tools, irregularity of machines, etc. The observability of such occurrence is an important aspect of the maintaining task [3], [4]. As the rejection of disturbances may hide them, the localization problem is really difficult in robust systems [5].

This paper begins by modeling the workshop under consideration. Controlled P-time Petri nets are used for this purpose. A decomposition of the P-time Petri net into four sets is done. Afterward, the problem of localization of time disturbances in critical time manufacturing systems is tackled. Some definitions and lemmas are quoted in order to build a theory dealing with such problem. They are illustrated step by step on examples of a given workshop. Finally, an algorithm providing a localization of time disturbances is established.

2. Modeling

2.1. Controlled P-time Petri net

The formal definition of a P-time Petri net is given by a pair < R; I >, where [6]:

- R is a marked Petri net,
- IS: $P \rightarrow (Q^{+} \cup 0)$ X $(Q^{+} \cup +\infty)$

  $p_i \rightarrow IS_i=[a_i, b_i]$ with $0 \leq a_i \leq b_i$.

IS_i defines the static interval of staying time of a mark in the place p_i belonging to the set of places P (Q^+ is the set of positive rational numbers). A mark in the place p_i is taken into account in transition validation when it has stayed in p_i at least a duration a_i and no longer than b_i. After the duration b_i the token will be dead.

Using [7], controlled P-time Petri net is defined as a quadruplet Rpc=(Rp, φ, U, U_0) such that:

- Rp is a P-time Petri net which describes the opened loop system,
- φ is an application from the set of places (P) toward the set of operations (Γ): $\varphi: P \rightarrow \Gamma$,
- U is the external control of the set of transitions (T) built on the predicates using the occurrence of internal or external observable events of the system: $U: T \rightarrow \{0, 1\}$,
- U_0 is the initial value of the predicate vector.

Let us denote by:

- T_O : the set of observable transitions,
- T_{UO} : the set of non observable transitions,
- T_S : the set of synchronization transitions,
- T_{NS} : the set of non synchronization transitions,
- T_P : the set of parallelism transitions,
- t_i (resp. t_i) : the output (resp. the input) places of the transition t_i,
- p_i (resp. p_i) : the output (resp. the input) transitions of the place p_i,
- q_{ie} : the expected sojourn time of the token in the place p_i,
- $St_e(n)$: the n^{nd} expected firing instant of the transition t,
- $St(n)$: the n^{nd} effective firing instant of the transition t.

2.2. Functional decomposition

A workshop in repetitive functioning mode is modeled by a Strongly Connected Event Graph (SCEG) [8]. Performances of a SCEG running in mono-periodic functioning mode are proved to be the same as when using the K-periodic functioning [8]. Consequently, a mono-periodic functioning is used in order to decrease the complexity of the supervisory problem [9]. In this case, for each transition t, $St_e(n+1)=St_e(n)+\pi_0$ where π_0 is the period of the periodic functioning of the given discrete event system. In this paper, the scheduling task is supposed to be done. Therefore, the SCEG corresponding to the system is provided. Moreover, the setting of transitions firing instants is fixed too. Then, the problem of time disturbances localization will be studied in the following.

As the sojourn times in places have not the same functional signification when they are included in the sequential process of a product or when they are associated to a free resource, a decomposition of the P-time Petri net model into four sets is made using [7]. The assumption of multi-product job-shops without assembling tasks as it was established in [10] is used:

- R_U is the set of places representing the used machines,
- R_N corresponds to the set of places representing the free machines which are shared between manufacturing circuits,
- $Trans_C$ is the set of places representing the loaded transport resources,
- $Trans_{NC}$ is the set of places representing the unloaded transport resources (or the interconnected buffers).

Figure 1, shows a P-time Petri net (G) modeling a system composed by two sequential processes GO_1 and GO_2 with two shared machines (M_1, M_2), where: $R_U=\{p_2, p_4, p_{11}, p_{13}, p_{15}\}$, $R_N=\{p_6, p_7, p_8, p_9\}$, $Trans_C=\{p_1, p_3, p_{10}, p_{12}, p_{14}\}$, $Trans_{NC}=\{p_5, p_{16}\}$, $GO_1=(t_{12}, p_{10}, t_6, p_{11}, t_7, p_{12}, t_8, p_{13}, t_9, p_{14}, t_{10}, p_{15}, t_{11})$ and $GO_2=(t_5, p_1, t_1, p_2, t_2, p_3, t_3, p_4, t_4)$.

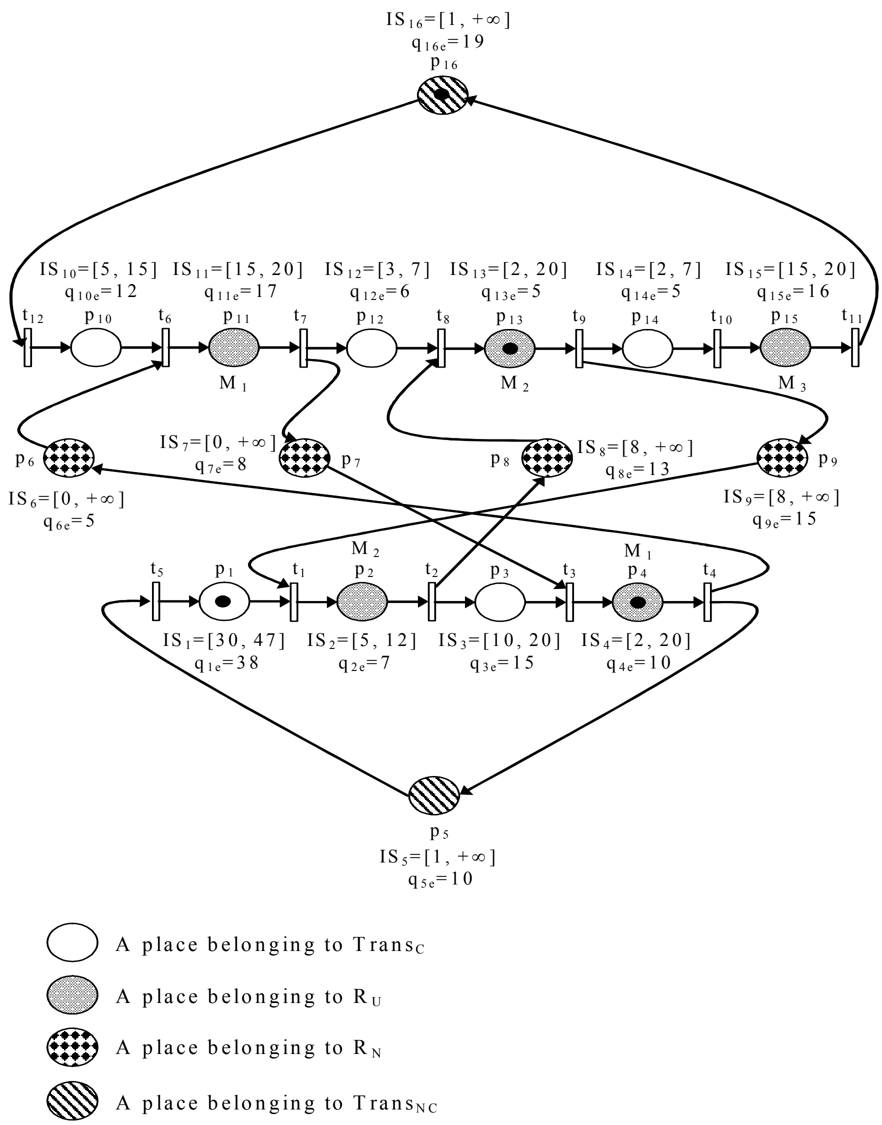

Figure 1. An Hillion Like Model with Functional Decomposition

The intervals (IS_i) and the expected staying times (q_{ie}) associated to the places (p_i) are: IS_1=[30, 50], q_{1e}=38, IS_2=[5, 12], q_{2e}=7, IS_3=[10, 20], q_{3e}=15, IS_4=[5, 20], q_{4e}=10, IS_5=[1, +∞], q_{5e}=10, IS_6=[0, +∞], q_{6e}=5, IS_7=[0, +∞], q_{7e}=8, IS_8=[8, +∞], q_{8e}=13, IS_9=[8, +∞], q_{9e}=15, IS_{10}=[5, 15], q_{10e}=12, IS_{11}=[15, 20], q_{11e}=17, IS_{12}=[3, 7], q_{12e}=6, IS_{13}=[2, 20], q_{13e}=5, IS_{14}=[2, 7], q_{14e}=5, IS_{15}=[15, 20], q_{15e}=16, IS_{16}=[1, +∞] and q_{16e}=19.

The initial expected firing instants of each transition are: $St_{1e}(1)=15$, $St_{2e}(1)=22$, $St_{3e}(1)=37$, $St_{4e}(1)=7$, $St_{5e}(1)=17$, $St_{6e}(1)=12$, $St_{7e}(1)=29$, $St_{8e}(1)=35$, $St_{9e}(1)=0$, $St_{10e}(1)=5$, $St_{11e}(1)=21$ and $St_{12e}(1)=0$.

The repetitive functioning mode is characterized by the period π_0=40.

Definition 1: A mono-synchronized subpath is a path containing one and only one synchronization transition which is its last node.

Definition 2: An elementary mono-synchronized subpath is a mono-synchronized subpath beginning with a place p such as p is a synchronization transition.

In Figure 1, there are eight elementary mono-synchronized subpaths constituting a partition of G: $Lp_1=(p_{13}, t_9, p_{14}, t_{10}, p_{15}, t_{11}, p_{16}, t_{12}, p_{10}, t_6)$, $Lp_2=(p_{13}, t_9, p_9, t_1)$, $Lp_3=(p_2, t_2, p_3, t_3)$, $Lp_4=(p_2, t_2, p_8, t_8)$, $Lp_5=(p_4, t_4, p_5, t_5, p_1, t_1)$, $Lp_6=(p_4, t_4, p_6, t_6)$, $Lp_7=(p_{11}, t_7, p_7, t_3)$ and $Lp_8=(p_{11}, t_7, p_{12}, t_8)$.

Property 1: A place p_{mp} belonging to a sequential process represents a shared machine if and only if $p_{mp} \in T_P$ or $p_{mp} \in T_S$.

Property 2: The first node of an elementary mono-synchronized subpath is a place belonging to R_U and representing a shared machine.

3. Time Disturbances Localization

3.1. Definitions and lemmas

Let us remember some definitions.

Definition 3: A time disturbance is detectable if, when it occurs, there exists at least one transition $t \in T_O$ such as $St(n) \neq St_e(n)$.

Definition 4: A time disturbance is quantifiable if its value can be analytically known.

Definition 5: A time disturbance is localizable when its occurrence node can be identified.

Definition 6: A time disturbance is partially localizable when its occurrence node location can be proved to belong to a given subset of P.

Definition 7: A time disturbance is observable when it is detectable, quantifiable and localizable.

Definition 8: The time passive rejection capacity interval of a path Lp is RC(Lp)=[Ca(Lp), Cr(Lp)] where:

$$Ca(Lp) = \sum_{p_i \in Lp \cap (R_N \cup Trans_{NC})} (q_{ie} - b_i), \qquad (1)$$

$$Cr(Lp) = \sum_{p_i \in Lp \cap (R_N \cup Trans_{NC})} (q_{ie} - a_i). \qquad (2)$$

Ca(Lp) (resp. Cr(Lp)) is called the time passive rejection capacity for an advance (resp. a delay) time disturbance occurrence.

Considering the path $Lp=(p_{12}, t_8, p_{13}, t_9, p_9, t_1)$, $RC(Lp)=[-\infty, 7]$ ($IS_9=[8, +\infty]$, $q_{9e}=15$).

Definition 9: Let δ a time disturbance and SN a set of nodes belonging to a P-time Petri net.

$\delta \in SN$ (resp. $\delta \notin SN$) means that the occurrence of δ is (resp. is not) in a node of SN.

Used notations

- C_{se} is the set of elementary mono-synchronized subpaths.
- IN(Lp) is the first node of the path Lp.
- OUT(Lp) is the last node of the path Lp.
- Lp(t*,t) is the oriented subpath of Lp beginning with t* and ending with t.
- $M_{n-1}(Lp(t^*,t))$ is the number of tokens in Lp(t*,t) after the completion of the cycle (n−1).
- Given a time disturbance δ, $\delta r_t(n)$ is the resulting residue quantified at the transition t which is fired at St(n).
- EC(IN(Lp),t) is the set of oriented paths connecting the node IN(Lp) of the path Lp to the transition t.
- $H(IN(Lp),t) = \min_{L_i \in [EC(IN(Lp),t) \setminus Lp(IN(Lp),t)]}(Cr(L_i)) + \delta r_t(n)$.
- $H'(IN(Lp),t) = \min_{L_i \in EC(IN(Lp),t)}(Cr(L_i)) + \delta r_t(n)$.

Figure 2 Shows an elementary mono-synchronized subpath $Lp_1=(p_{13}, t_9, p_{14}, t_{10}, p_{15}, t_{11}, p_{16}, t_{12}, p_{10}, t_6)$ with different notations.

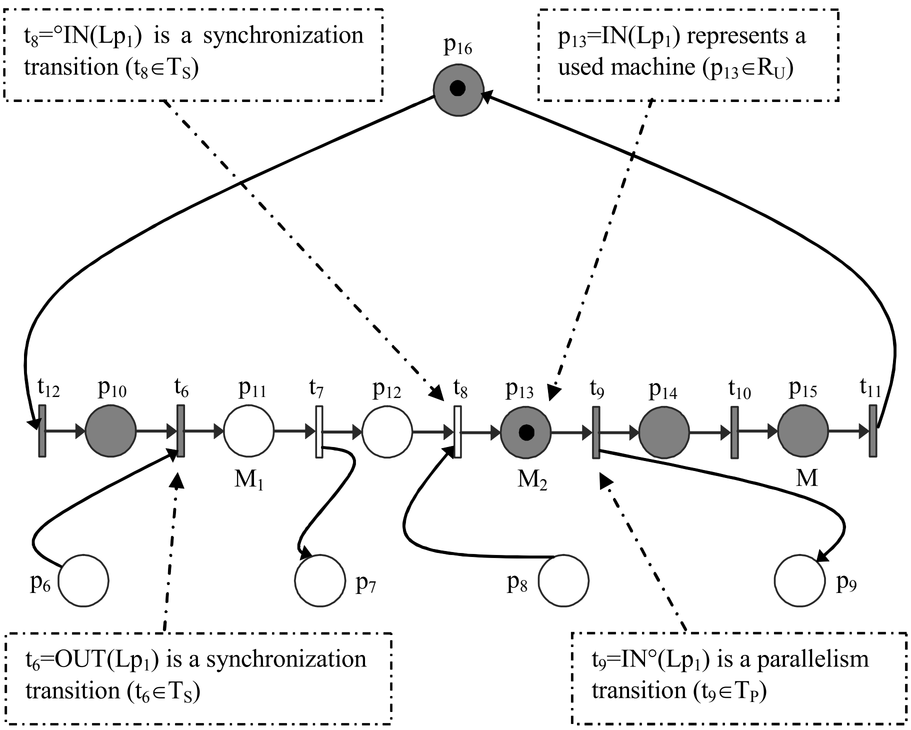

Figure 2. An Elementary Mono-Synchronized Subpath with Different Notations

Lemma 1: Let $Lp \in C_{se}$, $t \in (Lp \cap T_O \cap T_{NS})$, $t^* \in (Lp \cap T_O)$, and δ a time disturbance having a residue $\delta r_t(n) \neq 0$ quantified at t. The following results are established [11]:

$$\delta r_{t^*}(n - M_{n-1}(Lp(t^*,t))) = 0 \quad \Rightarrow \quad \delta \in [Lp(t^*,t) \setminus \{t^*\}], \tag{3}$$

$$\delta r_{t^*}(n - M_{n-1}(Lp(t^*,t))) \neq 0 \quad \Rightarrow \quad \delta \notin [Lp(t^*,t) \setminus \{t^*\}]. \tag{4}$$

Proof: The time disturbance is assumed to be a single one. The number of tokens in Lp(t*,t) after the cycle (n−1) is equal to $M_{n-1}(Lp(t^*,t))$. This means that the token crossing the transition t at St(n) with a residue $\delta r_t(n)$ has crossed the transition t* at $St^*(n-M_{n-1}(Lp(t^*,t)))$. When a disturbance is detected at a downstream transition t and is not detected at t*, it is generated between these two transitions. Otherwise, its initial occurrence is outside of Lp(t*,t).

It is to remark that the only oriented path connecting t* to t is Lp(t*,t). This is due to the fact that Lp is an elementary mono-synchronized subpath and t is not a synchronization transition.

Example 1

Let $Lp=(p_{13}, t_9, p_{14}, t_{10}, p_{15}, t_{11}, p_{16}, t_{12}, p_{10}, t_6)$, $t_{12}\in(Lp\cap T_O\cap T_{NS})$, $t_{10}\in(Lp\cap T_O)$, $(\delta r_{t12}(n)\neq 0)$ and $M_{n-1}(Lp(t_{10},t_{12}))=1$ (Figure 3).

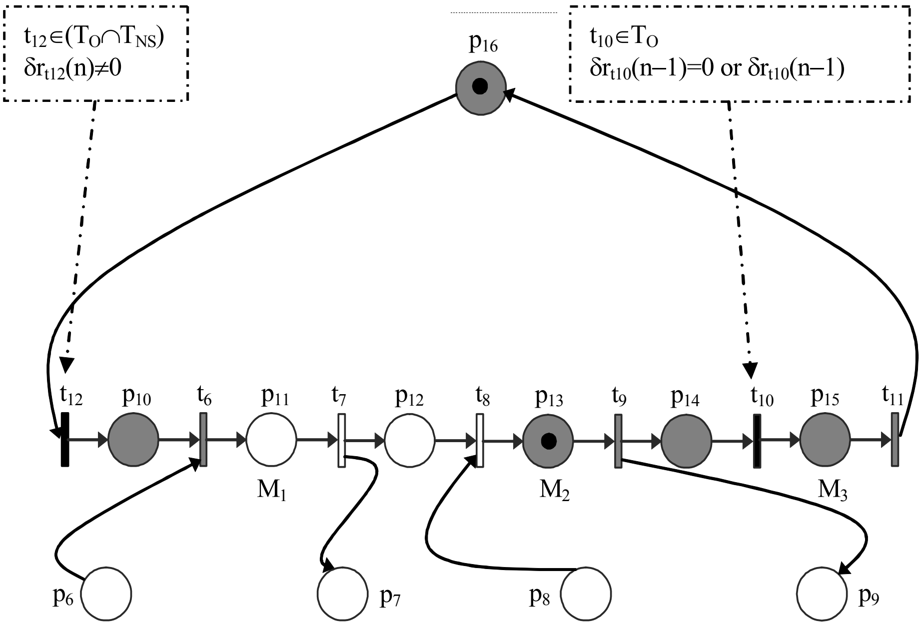

Figure 3. An Elementary Mono-Synchronized Subpath with Two Observable Transitions

Firs case: If $(\delta r_{t10}(n-1)=0)$, the disturbance δ has not crossed the transition t_{10}. Hence, (3) is applied and $\delta\in\{p_{15}, t_{11}, p_{16}, t_{12}\}$. In fact, the token in p_{16} has forcibly crossed the transition t_{10} at the cycle (n−1) at $St_e(n-1)$ with a null residue.

Second case: If $(\delta r_{t10}(n-1)\neq 0)$, (4) is applied and $\delta\notin\{p_{15}, t_{11}, p_{16}, t_{12}\}$.

Lemma 2: Let $Lp\in C_{se}$, $t\in(Lp\cap T_O)$, $tp\in(Lp\cap T_p)$, $I_{Lp}=\{L_i\in C_{se}/\ OUT(L_i)=IN(Lp)\}$ and δ a delay time disturbance having a residue $\delta r_t(n)>0$ quantified at t. The following assertion is true [11]:

$$\delta r_{tp}(n-M_{n-1}(Lp(tp,t)))<H'(tp,t) \quad\Rightarrow\quad \delta\notin\left(\bigcup_{L_i\in I_{Lp}}\{L_i\setminus\{IN(L_i), IN^\circ(L_i)\}\}\right)\cup\{tp, tp\}. \tag{5}$$

Proof: It is to remark that tp=IN(Lp) since Lp is an elementary mono-synchronized subpath. This transition is also supposed to be observable directly or indirectly.

I_{Lp} is the set of mono-synchronized subpaths having IN(Lp) as the shared synchronization transition. A path $L_i\in I_{Lp}$ involves $OUT(L_i)=IN(Lp)=tp$.

Moreover, $[L_i\setminus\{IN(L_i), IN(L_i)\}]$ is a subpath of L_i which does not contain any parallelism transition. $[L_i\setminus\{IN(L_i), IN(L_i)\}]\cup\{tp, tp\}$ has a single parallelism transition that is tp. EC(tp,t) is the set of oriented paths connecting the transition tp to the transition t. We prove (5) by contradiction.

If δ is in $[L_i \backslash \{IN(L_i), IN(L_i)\}] \cup \{tp, tp\}$, the paths belonging to EC(tp, t) are the only ones making it possible to convey the disturbance δ to the transition t. Hence, it exists a path $L_k \in EC(tp, t)$ by which δ is arrived at the transition t. In this case, $\delta r_{tp}(n-M_{n-1}(Lp(tp,t)))=[Cr(L_k)+\delta r_t(n)]$. Or $[Cr(L_k)+\delta r_t(n)] \geq H'(tp,t)$, thus (5) is true.

Example 2

Let $Lp=(p_{13}, t_9, p_{14}, t_{10}, p_{15}, t_{11}, p_{16}, t_{12}, p_{10}, t_6)$, $t_6 \in (Lp \cap T_O)$, $t_9 \in (Lp \cap T_P)$, $\delta r_{t6}(n)=2$, $\delta r_{t9}(n-1)=8$ and $M_{n-1}(Lp(t_9,t_6))=1$ (Figure 4).

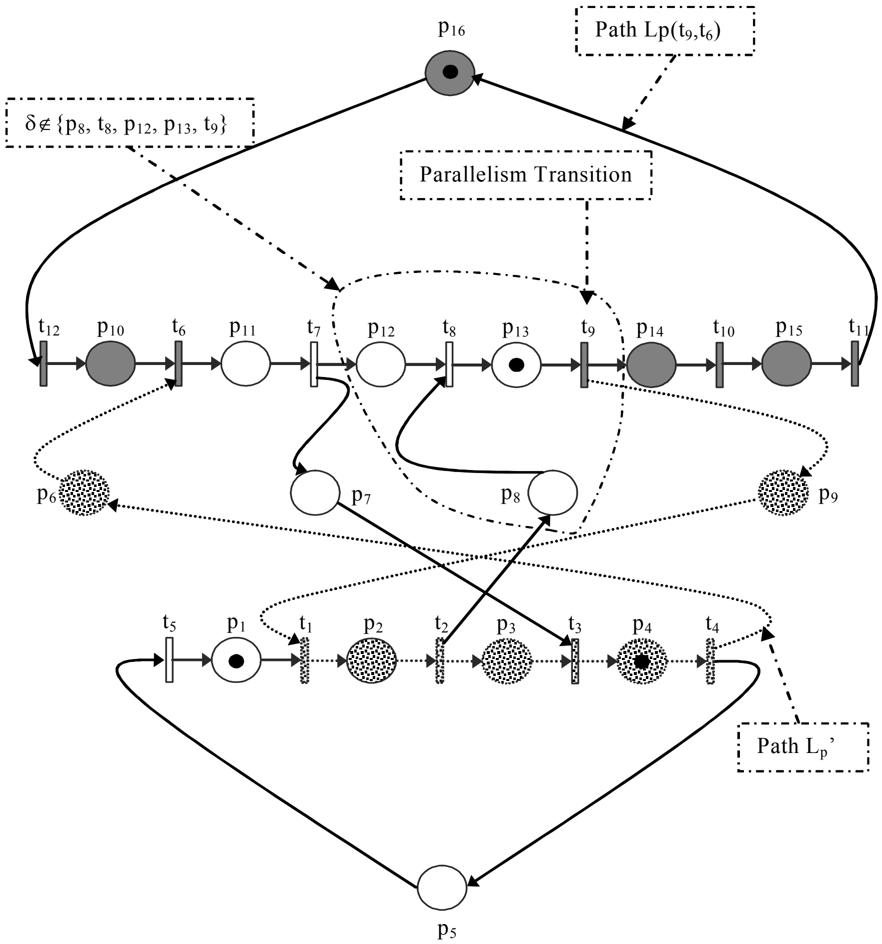

Figure 4. Illustration of Lemma 2 on the Considered Workshop

$EC(t_9,t_6)=\{Lp(t_9,t_6), Lp'=(t_9, p_9, t_1, p_2, t_2, p_3, t_3, p_4, t_4, p_6, t_6)\}$, $Cr(Lp(t_9,t_6))=18$, $Cr(Lp')=12$, $IN(Lp)=p_{13}=t_8$, $H'(t_9,t_6)=\min(20, 14)=14$, $I_{Lp}=\{L_1=(p_2, t_2, p_8, t_8), L_2=(p_{11}, t_7, p_{12}, t_8)\}$, $L_1 \backslash \{p_2, t_2\}=(p_8, t_8)$ and $L_2 \backslash \{p_{11}, t_7\}=(p_{12}, t_8)$.

Obviously, if $\delta \in \{p_8, t_8, p_{12}, p_{13}, t_9\}$, a residue $\delta r_{t9}(n-1)=14$ must be observed at t_9 to have a residue $\delta r_{t6}(n)=2$ at t_6. Therefore, the disturbance δ does not belong to the set: $\{p_8, t_8, p_{12}, p_{13}, t_9\}$.

Lemma 3: Let $Lp \in C_{se}$, $t \in (Lp \cap T_O \cap T_S)$, $t^* \in (Lp \cap T_O)$, and δ a delay time disturbance having a residue $\delta r_t(n)>0$ quantified at t. The following results are established [11]:

$$\delta r_{t*}(n - M_{n-1}(Lp(t^*,t))) = 0 \quad \Rightarrow \quad \delta \notin [Lp(IN(Lp),t^*) \backslash \{IN(Lp)\}], \tag{6}$$

$$\begin{cases} 0 \le Cr(Lp(IN(Lp),t^*)) < H(IN(Lp),t) \\ \delta r_{t^*}(n - M_{n-1}(Lp(t^*,t))) = 0 \end{cases} \Rightarrow$$

$$\begin{cases} \delta \notin \left[(Lp \setminus Lp(t^*,t)) \cup \{t^*\}\right] \\ \delta r_{IN^\circ(Lp)}(n - M_{n-1}(Lp(IN(Lp),t))) < H'(IN(Lp),t) \end{cases}, \quad (7)$$

$$\begin{cases} \delta r_{t^*}(n - M_{n-1}(Lp(t^*,t))) \neq 0 \\ \delta r_t(n) + Cr(Lp(t^*,t)) \neq \delta r_{t^*}(n - M_{n-1}(Lp(t^*,t))) \end{cases} \Rightarrow \delta \notin \left[Lp(IN(Lp), t) \setminus \{IN(Lp)\}\right]. \quad (8)$$

Proof: The assumption of a unique disturbance is made. The path Lp is an elementary mono-synchronized subpath verifying OUT(Lp)=t, since the synchronization transition t belongs to Lp. According to properties 1 and 2, IN(Lp) is the only parallelism transition of Lp.

Assertion (6):

The subpath $\Gamma=[(Lp(IN(Lp),t^*))\setminus\{IN(Lp)\}]$ does not contain any parallelism transition. There is only and only one path connecting each node $m\in\Gamma$ to the transition t that is Lp(m,t). Since $t^*\in Lp(m,t)$, the residue $(\delta r_{t^*}(n-M_{n-1}(Lp(t^*,t))=0))$ proves that the occurrence of the disturbance δ is outside of Γ.

Assertion (7):

The residue at the transition IN(Lp) must verify $\delta r_{IN(Lp)}(n-M_{n-1}(Lp(IN(Lp),t)))\le Cr(Lp(IN(Lp),t^*))$, else a residue $\delta r_{t^*}(n-M_{n-1}(Lp(t^*,t)))\neq 0$ will be observed.

Since $Cr(Lp(IN(Lp),t^*))<H(IN(Lp),t)$, therefore $\delta r_{IN(Lp)}(n-M_{n-1}(Lp(IN(Lp),t)))<H(IN(Lp),t)$.

$\delta\notin[Lp\setminus Lp(t^*,t)\cup\{t^*\}]$ means that $\delta\notin[\Gamma\cup\{IN(Lp), IN(Lp)\}]$. Since $(\delta r_{t^*}(n-M_{n-1}(Lp(t^*,t)))=0)$, (6) gives $\delta\notin\Gamma$.

Now, we try to prove by contradiction that $\delta\notin\{IN(Lp), IN(Lp)\}$. If $\delta\in\{IN(Lp), IN(Lp)\}$, necessarily there is a path $L_k\in[EC(IN(Lp),t)\setminus Lp(IN(Lp),t)]$ by which the disturbance δ is arrived at the transition t. Thus, the residue $\delta r_{IN(Lp)}(n-M_{n-1}(Lp(IN(Lp),t)))=[Cr(L_k)+\delta r_t(n)]$. By assumption, this is not possible since $[Cr(L_k)+\delta r_t(n)]\ge H(IN(Lp),t)$. Immediately, we have $\delta\notin\{IN(Lp), IN(Lp)\}$. What results in saying that $\delta\notin[(Lp\setminus Lp(t^*,t))\cup\{t^*\}]$.

The second result of (7) is proved as follows.

According to the definition of H'(IN(Lp), t) and knowing that $\delta r_t(n)>0$, $Cr(Lp(IN(Lp), t^*)) \le Cr(Lp(IN(Lp, t)))$ and $Cr(Lp(IN(Lp), t^*)) < H(IN(Lp), t)$, we conclude that $H'(IN(Lp), t) > Cr(Lp(IN(Lp), t^*))$. This involves that $H'(IN(Lp), t) \ge \delta r_{IN(Lp)}(n-M_{n-1}(Lp(IN(Lp), t)))$.

It is suitable to remark that this last result allows applying (5).

Assertion (8):

According to (4), $(\delta r_{t^*}(n-M_{n-1}(Lp(t^*,t))\neq 0))$ gives $\delta\notin[Lp(t^*,t)\setminus\{t^*\}]$. It remains to prove that $\delta\notin\Gamma=[(Lp(IN(Lp), t^*))\setminus\{IN(Lp)\}]$. We prove this by contradiction.

We suppose that $\delta\in\Gamma$. There is only and only one path connecting each node $m\in\Gamma$ to the transition t that is Lp(m, t). Since $t^*\in Lp(m, t)$, forcibly the residue at t* verifies $\delta r_{t^*}(n-M_{n-1}(Lp(t^*, t)))=[\delta r_t(n)+Cr(Lp(t^*, t))]$. For that reason, the disturbance $\delta\notin\Gamma$. Finally, we conclude that $\delta\notin[Lp(IN(Lp), t)\setminus\{IN(Lp)\}]$.

Example 3

Let $Lp=(p_{13}, t_9, p_{14}, t_{10}, p_{15}, t_{11}, p_{16}, t_{12}, p_{10}, t_6)$, $t_6\in(Lp\cap T_O\cap T_S)$, $t_{12}\in(Lp\cap T_O)$, $\delta r_{t6}(n)=2$ and $M_{n-1}(Lp(t_{12}, t_6))=0$.

If $\delta r_{t12}(n)=0$, (6) is applied and $\delta \notin \{p_{14}, t_{10}, p_{15}, t_{11}, p_{16}, t_{12}\}$.

Example 4

Let $Lp=(p_{13}, t_9, p_{14}, t_{10}, p_{15}, t_{11}, p_{16}, t_{12}, p_{10}, t_6)$, $t_6 \in (Lp \cap T_O \cap T_S)$, $t_{10} \in (Lp \cap T_O)$, $\delta r_{t6}(n)=2$ and $M_{n-1}(Lp(t_{10}, t_6))=1$.

First case: $(\delta r_{t10}(n-1)=0)$

$IN(Lp)=t_9$, $EC(t_9,t_6)=\{Lp(t_9,t_6), Lp'=(t_9, p_9, t_1, p_2, t_2, p_3, t_3, p_4, t_4, p_6, t_6)\}$, $Cr(Lp')=7+5=12$, $Cr(Lp(t_9,t_6))=18$, $Cr(Lp(t_9,t_6))+\delta r_{t6}(n)=20$, $H(t_9,t_6)=Cr(Lp')+\delta r_{t6}(n)=14$ and $H'(t_9,t_6)=\min(14, 20)=14$.

As $Cr(Lp(t_9,t_{10}))=0<H(t_9,t_6)$, (7) is applied and $\delta \notin \{p_{13}, t_9, p_{14}, t_{10}\}$. Besides, we have $\delta r_{t9}(n)<H'(t_9,t_6)$.

Using (5), we can conclude that $\delta \notin \{p_8, t_8, p_{12}, p_{13}, t_9\}$. Finally, we have $\delta \notin \{p_8, t_8, p_{12}, p_{13}, t_9, p_{14}, t_{10}\}$.

Second case: $(\delta r_{t10}(n-1)\neq 0)$

$[\delta r_{t6}(n)+Cr(Lp(t_{10},t_6))]=2+18=20$.

If $\delta r_{t10}(n-1)\neq 20$, (8) is applied and $\delta \notin \{p_{14}, t_{10}, p_{15}, t_{11}, p_{16}, t_{12}, p_{10}, t_6\}$. In fact, if $\delta \in \{p_{15}, t_{11}, p_{16}, t_{12}, p_{10}, t_6\}$ then $\delta r_{t10}(n-1)=0$. Furthermore, if $\delta \in \{p_{14}, t_{10}\}$ then $\delta r_{t10}(n-1)=20$, knowing that $Lp(p_{14}, t_6)$ is the only path connecting (p_{14}, t_{10}) to t_6.

Lemma 4: Let $Lp \in C_{se}$, $tp \in (Lp \cap T_P \cap T_{UO})$, $t \in (Lp \cap T_O)$, and $Cr(Lp(tp, t))$ the time passive rejection capacity of Lp between tp and t for delay occurrence.

Let us call DIF(tp) the set of paths beginning with tp.

Let us denote $DIF_n(tp)$ the restriction of DIF(tp) such that $\forall Lp' \in DIF_n(tp)$, $\forall t' \in Lp'$, we have $St'(n+m_{t'})<St(n)$, where $m_{t'}=M_{n-1}(Lp'(tp, t'))-M_{n-1}(Lp(tp, t))$.

Now, let $Lp' \in DIF_n(tp)$, $t^* \in (Lp' \cap T_O)$ and $Cr(Lp'(tp, t^*))$ the passive rejection capacity of Lp' between tp and t*. Given a delay time disturbance δ, the following results are true [11]:

$$\begin{cases} (t \notin T_S) \wedge (\delta r_t(n) > 0) \\ \delta r_t(n) + Cr(Lp(tp,t)) - Cr(Lp'(tp,t^*)) > 0 \\ \delta r_{t^*}(n + m_{t^*}) = 0 \end{cases} \Rightarrow \quad \delta \in [Lp(tp,t) \setminus \{tp\}], \tag{9}$$

$$\begin{cases} (t \notin T_S) \wedge (\delta r_t(n) > 0) \\ \delta r_{t^*}(n + m_{t^*}) \neq 0 \end{cases} \Rightarrow \quad \delta \notin [(Lp(tp,t) \cup Lp'(tp,t^*)) \setminus \{tp\}], \tag{10}$$

$$\begin{cases} (t \in T_S) \wedge (\delta r_t(n) > 0) \\ Cr(Lp'(tp,t^*)) < H'(tp,t) \\ \delta r_{t^*}(n + m_{t^*}) = 0 \end{cases} \Rightarrow \quad \begin{cases} \delta \notin \{{}^{\circ}tp, tp\} \\ \delta r_{tp}(n - M_{n-1}(Lp(tp,t))) < H'(tp,t) \end{cases}, \tag{11}$$

$$\begin{cases} (t \in T_S) \wedge (\delta r_t(n) > 0) \\ \delta r_{t^*}(n + m_{t^*}) \neq 0 \end{cases} \Rightarrow \quad \delta \notin [Lp(tp,t) \setminus \{tp\}]. \tag{12}$$

Proof: To be able to conclude on the localization of the disturbance δ at the instant St(n), it is necessary that the token which crossed tp at $Stp(n-M_{n-1}(Lp(tp, t)))$ must cross the transition t* before St(n). The condition $[St^*(n+m_{t^*})<St(n)]$ is putted to allow us to make a decision at St(n).

The fact that $m_{t*}>0$ means that the number of tokens in Lp'(tp, t*) is strictly higher than that in Lp(tp, t). In this case, it is not possible to conclude because the token which crossed tp at $Stp(n-M_{n-1}(Lp(tp, t)))$ has not crossed t* yet.

Assertions (9) and (10):

If $\delta \notin Lp(tp,t)$ and t is not a synchronization transition, the quantity $[\delta r_t(n)+Cr(Lp(tp,t))]$ is the residual effect of the disturbance δ at tp. When tp is a non observable parallelism transition, the following assertion may be used: if a disturbance modifies the tp firing instant, it must be seen downstream of tp. Consequently, when the value of the residual effect of the disturbance is greater than the rejection capacity of a given path, a residual variation has to be observed. Otherwise, the zero value of the residual disturbance can be only explained by the occurrence of another disturbance. By assumption, this last case is not possible. The disturbance has not passed through tp and (9) is true. When a residual value is not zero, the disturbance obviously occurs in the upstream of tp and (10) is true.

Let us point out that: if Cr(Lp'(tp, t*)) is greater than the residual value in tp of the supposed disturbance, it is not possible to conclude.

Assertion (11):

We suppose that $\delta \in \{tp, tp\}$. Therefore, it exists a path $L_k \in EC(tp, t)$ by which δ arrived at the transition t. In this case, $\delta r_{tp}(n-M_{n-1}(Lp(tp, t)))=[Cr(L_k)+\delta r_t(n)]$. Knowing that $[Cr(L_k)+\delta r_t(n)] \geq H'(tp, t)$ and $Cr(Lp'(tp, t^*))<H'(tp, t)$, a residue at t* different of zero must be observed ($\delta r_{t*}(n+m_{t*}) \neq 0$). This is in contradiction with the assumption of (11). Therefore, (11) is true.

It is suitable to remark that the second result of (11) allows applying (5).

Assertion (12):

The subpath Lp(tp, t)\{tp} does not contain any parallelism transition. It is evident that: if $\delta \in [Lp(tp, t) \backslash \{tp\}]$ then $\delta r_{t*}(n+m_{t*})=0$.

Example 5

Let $Lp=(p_{13}, t_9, p_{14}, t_{10}, p_{15}, t_{11}, p_{16}, t_{12}, p_{10}, t_6)$, $t_{11} \in (Lp \cap T_O \cap T_{NS})$, $t_9 \in (Lp \cap T_P \cap T_{UO})$, $Lp'=(t_9, p_9, t_1)$, $t_1 \in (Lp' \cap T_O)$, $M_{n-1}(Lp(t_9, t_{11}))=0$ and $M_{n-1}(Lp(t_9, t_1))=0$.

$Lp' \in DIF_n(t_9)$ since $St_{9e}(1)=0$, $St_{11e}(1)=21$ and $St_{1e}(1)=15$. $Cr(Lp(t_9, t_{11}))=0$, $Cr(Lp'(t_9, t_1))=Cr(Lp')=8$ and $m_{t1}=0$.

First case: $\delta r_{t11}(n)=10>Cr(Lp')$

If $\delta r_{t1}(n)=0$, the conditions of (9) are satisfied and $\delta \in (Lp(t_9, t_{11}) \backslash \{t_9\})=\{p_{14}, t_{10}, p_{15}, p_{11}\}$. Otherwise, a residue $\delta r_{t1}(n)=10-Cr(Lp')=2$ must be observed at t_1 before the firing of the transition t_{11}.

If $\delta r_{t1}(n)=2$, the conditions of (10) are satisfied and $\delta \notin \{p_9, t_1, p_{14}, t_{10}, p_{15}, p_{11}\}$.

Second case: $\delta r_{t11}(n)=6<Cr(Lp')$

If $\delta r_{t1}(n)=0$, we can not conclude because the passive rejection capacity Cr(Lp') is greater than the residual effect of δ at the input place of Lp'. The considered time disturbance is not detectable at t_1.

Example 6

Let $Lp=(p_{13}, t_9, p_9, t_1)$, $t_1 \in (Lp \cap T_O \cap T_S)$, $t_9 \in (Lp \cap T_P \cap T_{UO})$, $Lp'=(t_9, p_{14}, t_{10})$, $t_{10} \in (Lp' \cap T_O)$, $\delta r_{t1}(n)=1$, $M_{n-1}(Lp(t_9, t_1))=0$ and $M_{n-1}(Lp(t_9, t_{10}))=0$.

$Lp' \in DIF_n(t_9)$ since $St_{9e}(1)=0$, $St_{1e}(1)=15$ and $St_{10e}(1)=5$.

$m_{t10}=0$, $EC(t_9, t_1)=\{Lp(t_9, t_1), Lp_1=(t_9, p_{14}, t_{10}, p_{15}, t_{11}, p_{16}, t_{12}, p_{10}, t_6, p_{11}, t_7, p_7, t_3, p_4, t_4, p_5, t_5, p_1, t_1)\}$, $Cr(Lp(t_9, t_1))=7$, $Cr(Lp_1))=18+8+9=35$, $H'(t_9, t_1)=7+1=8$ and $Cr(Lp'(t_9, t_{10}))=Cr(Lp')=0<H'(t_9, t_1)$.

If $\delta r_{t10}(n)=0$, the conditions of (11) are satisfied. The results are $\delta \notin \{p_{13}, t_9\}$ and $\delta r_{t9}(n)<H'(t_9,t_1)$.

Using (5), we can conclude that $\delta \notin \{p_8, t_8, p_{12}, p_{13}, t_9\}$. Finally, δ does not belong to the set: $\{p_8, t_8, p_{12}, p_{13}, t_9\}$.

If $\delta r_{t10}(n)\neq 0$, the conditions of (12) are satisfied. The result is $\delta \notin \{p_9, t_1\}$.

Lemma 5: Let $t \in T_O$, $t^* \in T_O$ and $Lp \in EC(t^*, t)$. If $(\delta r_t(n)\neq 0$ and $\delta r_{t^*}(n-M_{n-1}(Lp))\neq 0)$ then $\delta \notin [EC(t^*,t)\backslash\{t^*\}]$.

Proof: If the disturbance $\delta \in [EC(t^*,t)\backslash\{t^*\}]$, necessarily the token subject of this disturbance has crossed the transition t* at the instant $St^*(n-M_{n-1}(Lp))$ with a null residue: $\delta r_{t^*}(n-M_{n-1}(Lp))=0$.

3.2. Localization algorithm of time disturbances

The above lemmas will be used to build a localization algorithm of a delay time disturbance δ which residue $\delta r_{t0}(n)\neq 0$ is quantified at the transition t_0. Despite the fact that no proof of the algorithm is provided, it is easy to check that all the algorithm conditions ensue from the lemmas results. The algorithm is now presented.

```
Et0={Lpj∈Cse/ t0∈Lpj}
tdo=t0
F1(Lpj, tdo)
  {For each Lpj∈Et0 Do
     {t*=(°°tdo∩ Lpj)
      While t*∈Lpj Do
           {If t*∈TO Then
                 {If tdo∈TS then apply lemma 1
                 Else apply lemma 3
                   If formula (3) is applied Then Stop
                   If it is possible, apply lemma 2
                 tdo=t*
               }
             Else
               {If t*∈TP Then
                    {Apply lemma 4
                     If formula (9) is applied Then Stop
                     If it is possible, apply lemma 2
                   }
               }
             t*=t*
           }
```

$E_{t*}=\{Lp'_j \in C_{se} / t* \in Lp_j\}$

For each $Lp'_j \in E_{t*}$ Do $F_2(Lp'_j, t_{do}, t*)$

}

}

$F_2(Lp'_j, t_{do}, t*)$

{If $t* \in T_O$) Then

{If $\delta r_{t*}=0$ Then

{If $t_{do} \in T_S$ Then

{$\delta \notin [\bigcup_{L_i \in I_{t*}} \{L_i(IN(L_i),t*)\}]$ with $I_{t*}=\{L_i \in C_{se} / t* \in L_i\}$

Partial Stop

}

Else

{If there is no synchronization transition in $EC(t*,t_{do})$ other than t* Then

{$\delta \in EC(t*,t_{do}) \setminus \{t*\}$ according to Lemma 2

Stop

}

Else

{Lemma 2 gives $\delta \notin [\bigcup_{L_i \in I_{t*}} \{L_i(IN^{\circ\circ}(L_i),t*)\}]$

Partial Stop

}

}

}

Else

{If $0<\delta r_{t*}<H'(t*,t_{do})$ Then

{$\delta \notin [\bigcup_{L_i \in I_{t*}} \{L_i(IN^{\circ\circ}(L_i),t*)\}]\ )] \cup EC(t*,t_{do})$

Partial Stop

}

If $\delta r_{t*}=H'(t*,t_{do})$ Then

{$\delta \notin EC(t*,t_{do}) \setminus \{t*\}$ according to Lemma 5

$t_{do}=t*$

For each $Lp_j \in E_{tdo}$ Do $F_1(Lp_j, t_{do})$

}

}

}

Else

{For each $Lp_j \in E_{t*}$ Do

{$t*=(t* \cap Lp_j)$

If $\delta r_{t*}=0$ Then

$\{\delta \notin [\bigcup_{L_i \in I_{t*}} \{L_i(IN(L_i),t*)\}]$

Partial Stop

}

Else

{If $0<\delta r_{t*}<H'(t*,t_{do})$ Then

$\{\delta \notin [\bigcup_{L_i \in I_{t*}} \{L_i(IN(L_i),t*)\}] \cup EC(t*,t_{do})$

Partial Stop

}

If $\delta r_{t*}=H'(t*,t_{do})$ Then

$\{\delta \notin [EC(t*,t_{do}) \backslash \{t*\}]$ according to Lemma 5

$t_{do}=t*$

For each $Lp_j \in E_{tdo}$ Do $F_1(Lp_j, t_{do})$

}

}

}

}

}

In order to localize the time disturbance occurrence, mono-synchronized subpaths are tested one by one. The algorithm converges since their number is limited.

4. Conclusion

This paper deals with time disturbances localization in critical time manufacturing job-shops. In such systems, operation times are included between a minimum and a maximum value. Controlled P-time Petri nets are used for modeling. Some definitions and a series of lemmas are quoted in order to build a theory dealing with such problem. They are illustrated step by step on examples of a given workshop. A new algorithm built upon the lemmas results is provided in order to localize time disturbances occurrence.

It should be mentioned that the problem of time disturbances localization is really difficult. The established algorithm gives in the general case only a partial localization. Thus, it is without surprise we note that the instrumentation and the positioning of sensors are key problems for the workshops with time constraints.

The localization of time disturbances gives information concerning the state of the production line. It allows a functional diagnosis.

The complexity and the optimality of the given algorithm must be studied in future works. Also, the considered topology needs to be extended to cover the field of multi-product job-shops with assembling tasks.

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