A Multi-objective Model for the Location of the Emergency Facilities and the Selection of the Rescue Path Based on the Restoration of the Damaged Edges

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Abstract: In this paper, an equivalence theorem is given, that is, in a weighted graph, the maximum value of the minimum edge weights of all minimal edge cut sets between two points is equal to the minimum value of the maximum edge weights of all paths between the two points. On this basis, the recovery time between two points in the network is defined from the perspective of the minimal edge cut set and the path, respectively, and the equivalence of the two definitions is shown. Two models are established to optimize the partial recovery of network functions, by taking the damaged road network after the earthquake as the scene, based on the definition of the network recovery time and the total network time. Model 1 includes three objective functions, which are the sum of the failure levels between all pairs of the demand points and the facilities serving them, the total network time and the total cost. Different priorities were set for solving these three objectives. Since such a hierarchical model is not conducive to the optimization of the objective functions with lower priorities, model 1 is modified appropriately. The sum of the failure levels is combined with the total network time into an integrated objective function by weighting, and the minimization of this new function is taken as the highest-priority objective and the minimization of the total cost is taken as the secondary-priority objective, in order to achieve the balance between the sum of the failure levels and the total network time. Thus, model 2 is obtained. A case study is used to test the two models and verify their effectiveness.

Keywords: Equivalence theorem, Total network time, Network recovery time, Reliability level.

1. Introduction

Earthquakes occur frequently on earth, causing huge casualties. The strongest and most damaging earthquake in Romania was in March 1977. The earthquake with a magnitude M_W 7.4 at a hypocentral depth of 94 km occurred in the Vrancea region, situated 170 Km from Bucharest, the capital city. Only in Bucharest, the earthquake caused severe damage to 33,000 buildings while more than 1,400 people lost their lives (Lang et al., 2012). On May 12, 2008, a 8.0-magnitude earthquake occurred in Wenchuan County, Sichuan Province, China. Statistics showed that the earthquake killed about 69,000 people and injured more than 370,000 people (Peng et al., 2011).

Earthquakes represent a particular subclass of a more general one of natural disasters. Decision Support Systems (DSS) are helpful tools for the management of disasters (van de Walle & Turoff, 2008; Quansah, Engel & Rochon, 2010). Various models and multicriteria methods have been proposed for decision-making in disaster management (Zhou et al., 2018; Manyaga, Nilufer & Hajaoui, 2020). In particular, in case of earthquakes, timely post disaster rescue operations could not only reduce greater casualties, but also avoid greater losses to property. Post disaster rescue includes the repair of the damaged road network, the selection of the emergency facilities, the evacuation of the disaster victims, the transportation of the rescue materials and so on (Hu, Yang & Xu, 2014). Therefore, it is necessary to establish models to be included as computerized modules in appropriate DSS to support decision makers in managing critical events such as earthquakes (Ren, Xu & Gu, 2016; Wu et al., 2020; Cimellaro, Arcidiacono & Reinhorn, 2021). This paper briefly reviews the specialized literature related to these aspects. The *emergency shelter location* problem belongs to the network location problem domain in which many scholars have carried out in-depth research on it. Because the location of emergency shelters is affected by various factors, the problems related to the location of emergency shelters are often represented by a multi-objective model. One factor that is usually taken into account is the evacuation distance/time, which determines whether the affected residents can be evacuated to the safe emergency refuge facilities quickly and timely. Therefore, minimizing the evacuation distance/time is usually one of the objectives of the emergency refuge facility location model (Hu, Yang & Xu, 2014; Wang, Xu & Filip, 2022). While some researchers have considered only the minimization of the total evacuation distance (Yenice & Samanlioglu, 2020), other researchers considered the minimization of both

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the total evacuation distance and the individual evacuation distance (Xu et al., 2018). Since strong earthquakes could cause the damage of the road and the disruption of the traffic, route restoration was considered to be one of the most important priorities in disaster relief (Caunhye, Aydin & Duzgun, 2020). Recovery time is an important aspect of the route restoration research. The shorter the recovery time, the faster the transportation of materials and the transfer of personnel. Therefore, some location models take the recovery time as a research object (Zhang, Wang & Nicholson, 2017). Based on the background that the road network is damaged to a certain extent after the earthquake, this paper studies how to restore some functions of the road network by repairing some roads to provide services for each demand point. The total time is taken as a main research objective. Here, it includes both the recovery time of the damaged road and the travel time of the path. In the present model, the minimization of the total network time is taken as one of the objectives.

In addition to the evacuation time and the recovery time, whether the demand is completely met is also one of the important factors affecting the location of emergency refuge facilities. From the perspective of the demand side, some scholars considered minimizing the uncovered demand (Trivedi & Singh, 2017). From the perspective of the supply side, some scholars considered whether the current emergency shelter is enough to provide services for all demand points (Tsai & Yeh, 2016). In this article, a candidate set of facilities is provided, and emergency refuge facilities are selected with the aim to realize the full coverage of demand points. The construction of the emergency refuge facilities involves government investment. From the perspective of the rescue stage, Boostani et al. (2021) proposed that the costs should include both the cost of the preparation stage and the cost of the response stage. Since the budget is always limited, the minimization of the cost is often one of the objectives of the emergency location model. Thus, it can be seen that one often needs to weigh between cost and other factors. This paper considers not only the location of facilities, but also repairing the transportation network. Therefore, the total cost involved in this paper includes the cost of the selected repaired roads and the cost of the facility location. The minimization of the total cost is an objective of the model proposed in this paper.

Fairness is an important consideration in facility location network design. However, there is no general or unified definition of fairness. Some scholars defined the equity as “fairness, impartiality, or equality of services” (Huang, Smilowitz & Balcik, 2011; Savas, 1978). Balcik et al. (2010) argued that, in general, equity was related to fairness and justice, and to the distribution of resources and benefits. In order to maximize the equity, different methods have been used. In the early network location problem, the nearest transfer principle was often used to achieve fairness. One of the most used methods to achieve equity is the so-called minimax method (Sabouhi et al., 2019; Javadian, Modarres & Bozorgi, 2017). Contrary to the minimax method, some scholars have used the maximin method to obtain the maximum fairness (Tzeng, Cheng & Huang, 2007). This paper proposes a procedure to select a path between each demand point and the corresponding facility point with the objective of minimizing the maximum total time between all pairs of the demand points and the facility points serving them. This objective is set to ensure fairness in emergency rescue.

Network reliability is an important aspect of network research. However, so far, researchers have not reached a consensus on the definition of network reliability (Huang, 2020). The concept of network reliability was first proposed by Lee (1955) for communication networks, and the reliability index based on connectivity was used for the first time. Early research on network reliability focused on network connectivity, and primarily concerns with the enumeration of cuts or paths, in which the reliability problem without flow in a binary-state network was firstly discussed by Aggarwal et al. (1975). In fact, the network reliability based on connectivity mainly focuses on the structure of the graph itself. The maximum flow minimum cut theorem of Ford-Fulkerson (Ford & Fulkerson, 1956; Gale, 1957) reveals an important attribute of the weighted graph and provides a theoretical basis for further research on network reliability. Scholars have done a lot of research on the reliability of the network with flow (Lee, 1980; Lin & Chen, 2017). Lee (1980) extended the reliability problem to a network with flow, and he argued that a network was good when a certain amount of flow could be transmitted from the input node to the output node. Xue (1985) extended the reliability problem within a binary-state network to a multistate network, which was also called the stochastic-flow network.

When facing disasters such as earthquakes, the reliability of road network is very important. A reliable road network could greatly resist the damage of disasters. Some scholars have studied the impact of the disaster on links by random failures (Peeta, Salman & Viswanath, 2010; Yucel, Salman & Arsik, 2018). Yu (2020) argued that the interruption of the link was actually caused by the damage that the link sustains beyond its tolerance, and, on this basis, he defined the reachability guarantee between the two points. Based on the concept of damage tolerance of edge in Yu's paper, Wang et al. (2022) defined the reliability level between two points in the network and the network reliability, and established a multi-objective model to optimize the network reliability level. This paper continues to use the concept of the reliability level between two points in the network (Wang, Xu & Filip, 2022), and discusses how to maximize the sum of the reliability levels between all pairs of the demand points and the facilities serving them by selectively repairing some damaged roads. The modeling idea and method of this paper could play a certain auxiliary role in constructing the decision support system meant to work in emergency situation (Filip, Zamfirescu & Ciurea, 2017; Filip, 2020).

The rest of the paper is organized as follows: Section 2 presents an equivalence theorem, and defines the network recovery time and the total network time. In Section 3, the problem is described and the formulation of the model is introduced. Then, in Section 4, the previous multi-objective model is applied to a case study. The results of the solution are analyzed and the effectiveness of the models is verified. Conclusions are given in Section 5.

2. Methodology

An equivalence theorem based on a lemma is presented in this section. By means of this theorem, the recovery time of the network between two points is defined from the perspective of the minimal edge cut set and the path, respectively, and the equivalence of the two definitions is demonstrated. Then, the network recovery time and the total network time are defined from the perspective of the path.

2.1 Equivalence Theorem

The problem is defined on the undirected weighted connected graph $\hat{G}\left(\hat{N},\hat{A}\right)$, where $\hat{N}$ denotes the point set, and $\hat{A}$ represents the edge set.

2.1.1 Lemma and the Proof of the Equivalence Theorem

Lemma: In the connected graph $\hat{G}$, there is at least one path between the two points $\hat{X}$ and $\hat{Y}$, if and only if at least one edge remains uncut for each minimal edge cut set between the two points (Wang, Xu & Filip, 2022).

Equivalence theorem: As shown in Figure 1, assume that $\hat{X}$ and $\hat{Y}$ are two points in $\hat{G}$, $\hat{C}_{\hat{X},\hat{Y}}$ is the set of the minimal edge cut sets between $\hat{X}$ and $\hat{Y}$, $\hat{C}_{\hat{X},\hat{Y}}=\left\{c^1_{\hat{X},\hat{Y}},c^2_{\hat{X},\hat{Y}},\cdots,c^a_{\hat{X},\hat{Y}},\cdots,c^{\left|\hat{X},\hat{Y}\right|_{\hat{C}}}_{\hat{X},\hat{Y}}\right\}$, in which $c^a_{\hat{X},\hat{Y}}$ is the *a-th* minimal edge cut set between $\hat{X}$ and $\hat{Y}$, $\left|\hat{X},\hat{Y}\right|_{\hat{C}}$ is the total number of the minimal edge cut set between $\hat{X}$ and $\hat{Y}$, $c^a_{\hat{X},\hat{Y}}=\left\{l^a_1,l^a_2,\cdots,l^a_b,\cdots,l^a_{q_a}\right\}$, in which l^a_b is the *b-th* edge of the minimal edge cut set $c^a_{\hat{X},\hat{Y}}$, q_a is the total number of edges in the minimal edge cut set $c^a_{\hat{X},\hat{Y}}$, $t_r\left(l^a_b\right)$ is the weight of the edge l^a_b. Assume that $\hat{P}_{\hat{X},\hat{Y}}$ is the set of all paths between $\hat{X}$ and $\hat{Y}$, $\hat{P}_{\hat{X},\hat{Y}}=\left\{p^1_{\hat{X},\hat{Y}},p^2_{\hat{X},\hat{Y}},\cdots,p^u_{\hat{X},\hat{Y}},\cdots,p^{\left|\hat{X},\hat{Y}\right|_{\hat{P}}}_{\hat{X},\hat{Y}}\right\}$, in which $p^u_{\hat{X},\hat{Y}}$ is the *u-th* path between $\hat{X}$ and $\hat{Y}$, $\left|\hat{X},\hat{Y}\right|_{\hat{P}}$ is the total number of paths between $\hat{X}$ and $\hat{Y}$, $p^u_{\hat{X},\hat{Y}}=\left\{l^u_1,l^u_2,\cdots,l^u_v,\cdots,l^u_{k_u}\right\}$, in which l^u_v is the *v-th* edge of the path $p^u_{\hat{X},\hat{Y}}$, k_u is the total number of edges on the path $p^u_{\hat{X},\hat{Y}}$, $t_r\left(l^u_v\right)$ is the weight of the edge l^u_v. Then, $\max\limits_{c^a_{\hat{X},\hat{Y}}\in\hat{C}_{\hat{X},\hat{Y}}}\min\limits_{l^a_b\in c^a_{\hat{X},\hat{Y}}} t_r\left(l^a_b\right)=\min\limits_{p^a_{\hat{X},\hat{Y}}\in\hat{P}_{\hat{X},\hat{Y}}}\max\limits_{l^u_v\in p^u_{\hat{X},\hat{Y}}} t_r\left(l^u_v\right)\cdot$

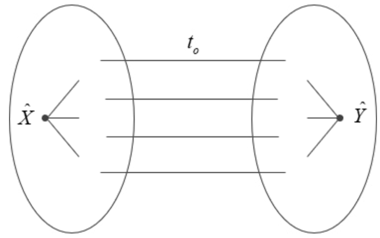

Figure 1. A connected weighted graph with the two points $\hat{X}$ and $\hat{Y}$ (Wang, Xu & Filip, 2022)

Proof 1: In the connected graph $\hat{G}$, starting from the edge with the maximum weight, the edges in $\hat{G}$ are cut off successively according to the non-increasing order of edge weight, until when an edge with weight t_o is cut off, the points $\hat{X}$ and $\hat{Y}$ are then divided into two components and the process ends. Thus, it can be asserted that $t_o=\max\limits_{c^a_{\hat{X},\hat{Y}}\in\hat{C}_{\hat{X},\hat{Y}}}\min\limits_{l^a_b\in c^a_{\hat{X},\hat{Y}}} t_r\left(l^a_b\right)$ and $t_o=\min\limits_{p^a_{\hat{X},\hat{Y}}\in\hat{P}_{\hat{X},\hat{Y}}}\max\limits_{l^u_v\in p^u_{\hat{X},\hat{Y}}} t_r\left(l^u_v\right)$.

Let l_o denote the special edge with weight t_o in the above operation. According to the operation above, when the edge l_o is cut, the cut edge forms an edge cut set between $\hat{X}$ and $\hat{Y}$. This edge cut set is not necessarily a minimal edge cut set of the graph $\hat{G}$. If the edge cut set is not a minimal edge cut set, then the redundant edges are removed and a minimal edge cut set is formed. This minimal edge cut set must contain the edge l_o. Once the edge l_o is removed from this edge cut set, all the other cut edges cannot form a minimal edge cut set. Obviously, t_o is the minimum weight of this minimal edge cut set. The edges of the graph are cut in non-increasing order of weight, and, since other minimal edge cut sets have not been formed, the minimum value of edge weight in any other minimal edge cut set does not exceed t_o. From this, $t_o = \max_{c^a_{\hat{X},\hat{Y}} \in \hat{C}_{\hat{X},\hat{Y}}} \min_{l^a_b \in c^a_{\hat{X},\hat{Y}}} t_r\left(l^a_b\right)$ is obtained.

On the other hand, the above operation process could be regarded as cutting the edges of the paths in the non-increasing order of weight. Before cutting off the edge l_o, there is no minimal edge cut set between $\hat{X}$ and $\hat{Y}$. According to the Lemma, there is at least one $\hat{X}$-$\hat{Y}$ path at this time. When the edge l_o is deleted, all $\hat{X}$-$\hat{Y}$ paths are broken. Obviously, t_o is not only the maximum edge weight on the last broken path, but also not greater than the maximum edge weight of every other $\hat{X}$-$\hat{Y}$ path. Therefore $t_o = \min_{p^a_{\hat{X},\hat{Y}} \in \hat{P}_{\hat{X},\hat{Y}}} \max_{l^u_v \in p^u_{\hat{X},\hat{Y}}} t_r\left(l^u_v\right)$.

From the above discussion, it can be concluded that $\max_{c^a_{\hat{X},\hat{Y}} \in \hat{C}_{\hat{X},\hat{Y}}} \min_{l^a_b \in c^a_{\hat{X},\hat{Y}}} t_r\left(l^a_b\right) = \min_{p^a_{\hat{X},\hat{Y}} \in \hat{P}_{\hat{X},\hat{Y}}} \max_{l^u_v \in p^u_{\hat{X},\hat{Y}}} t_r\left(l^u_v\right)$.

Proof 2: Cut all edges of the graph $\hat{G}$. Starting from the edge with the minimum weight, restore the edges in $\hat{G}$ in non-decreasing order of weight. When restoring the edge with weight t_o, then the points $\hat{X}$ and $\hat{Y}$ appear in the same connected component and the process stops. Now, it can be asserted that $t_o = \max_{c^a_{\hat{X},\hat{Y}} \in \hat{C}_{\hat{X},\hat{Y}}} \min_{l^a_b \in c^a_{\hat{X},\hat{Y}}} t_r\left(l^a_b\right)$.

Let l_o denote the special edge with weight t_o in the above operation. When l_o is not restored, there is no connection between $\hat{X}$ and $\hat{Y}$. According to the lemma, there is at least one minimal edge cut set between $\hat{X}$ and $\hat{Y}$, in which no edge is restored. When restoring the edge l_o, $\hat{X}$ and $\hat{Y}$ appear in the same connected component. Then, at least one edge of each minimal edge cut set between $\hat{X}$ and $\hat{Y}$ in the graph $\hat{G}$ is restored, and the minimum edge weight of each minimal edge cut set between $\hat{X}$ and $\hat{Y}$ does not exceed t_o, where t_o is the minimum weight of the minimal edge cut set in which only l_o is restored.

On the other hand, according to the above operation, when the edge l_o is restored, the path between $\hat{X}$ and $\hat{Y}$ is formed. t_o is the maximum edge weight of the recovered paths between $\hat{X}$ and $\hat{Y}$. There is no other path with maximum edge weight less than t_o. If other edges will be restored to form a new path between $\hat{X}$ and $\hat{Y}$, then t_o will not be greater than the maximum edge weight on these new paths. Therefore, $t_o = \min_{p^a_{\hat{X},\hat{Y}} \in \hat{P}_{\hat{X},\hat{Y}}} \max_{l^u_v \in p^u_{\hat{X},\hat{Y}}} t_r\left(l^u_v\right)$.

From the above discussion, it can be concluded that $\max_{c^a_{\hat{X},\hat{Y}} \in \hat{C}_{\hat{X},\hat{Y}}} \min_{l^a_b \in c^a_{\hat{X},\hat{Y}}} t_r\left(l^a_b\right) = \min_{p^a_{\hat{X},\hat{Y}} \in \hat{P}_{\hat{X},\hat{Y}}} \max_{l^u_v \in p^u_{\hat{X},\hat{Y}}} t_r\left(l^u_v\right)$.

2.1.2 Application Scenarios of the Equivalence Theorem

The edge weight in the equivalence theorem can have different meanings in different situations. For example, when a network needs to be constructed to connect two points, and the edge weight represents the time required for the construction of this edge, then, t_o in the theorem represents the shortest construction time required for the connection of these two points under the condition that all edges start construction at the same time. For another example, it is assumed that two points of a regional road network cannot be connected due to disasters such as earthquake, and that the edge weight represents the recovery time of each edge of the damaged road network (when an edge is not damaged, the recovery time of this edge is 0). Then, t_o in the theorem represents the shortest recovery time required for the connection between these two points under the condition that the damaged edges start to repair at the same time. This paper discusses this situation accurately.

2.2. Network Recovery Time

Firstly, the minimal edge cut set is used to define the recovery times involved in the network. In the following definitions, the symbols specified in the equivalence theorem are used and it is assumed that all the damaged edges in the network begin to recover at the same time.

Definition 1. Let $t_r\left(l^a_b\right)$ be the recovery time of the edge l^a_b of the minimal edge cut set $c^a_{\hat{X},\hat{Y}}$, then $\min_{l^a_b \in c^a_{\hat{X},\hat{Y}}} t_r\left(l^a_b\right)$ is called the recovery time of $c^a_{\hat{X},\hat{Y}}$. Let $t_r\left(c^a_{\hat{X},\hat{Y}}\right)$ be the recovery time of $c^a_{\hat{X},\hat{Y}}$, then $t_r\left(c^a_{\hat{X},\hat{Y}}\right) = \min_{l^a_b \in c^a_{\hat{X},\hat{Y}}} t_r\left(l^a_b\right)$.

When the time taken does not reach the minimum recovery time $t_r\left(c^a_{\hat{X},\hat{Y}}\right)$ of the minimal edge cut set

$c^a_{\hat{X},\hat{Y}}$, it can be said that the minimal edge cut set has not been recovered; otherwise, this minimal edge cut set is restored. Therefore, when all edges of a minimal edge cut set are damaged, the restoration of the minimal edge cut set here does not mean that all edges in this minimal edge cut set are restored, but that the edge with the minimum recovery time in this minimal edge cut set is restored.

Definition 2. Let $t_r\left(l^a_b\right)$ be the recovery time of the edge l^a_b of the minimal edge cut set $c^a_{\hat{X},\hat{Y}}$, then $\max\limits_{c^a_{\hat{X},\hat{Y}} \in \hat{C}_{\hat{X},\hat{Y}}} \min\limits_{l^a_b \in c^a_{\hat{X},\hat{Y}}} t_r\left(l^a_b\right)$ is called the recovery time of the network between $\hat{X}$ and $\hat{Y}$. Let $t_r\left(\hat{C}_{\hat{X},\hat{Y}}\right)$ be the recovery time of the network between $\hat{X}$ and $\hat{Y}$, then $t_r\left(\hat{C}_{\hat{X},\hat{Y}}\right) = \max\limits_{c^a_{\hat{X},\hat{Y}} \in \hat{C}_{\hat{X},\hat{Y}}} \min\limits_{l^a_b \in c^a_{\hat{X},\hat{Y}}} t_r\left(l^a_b\right)$.

The points in $\hat{G}$ are divided into two categories: one is the demand point, represented by i, and the other is the facility point, represented by j. Let the demand point set be $\hat{I}$ ($i \in \hat{I}$) and the facility point set be $\hat{J}$ ($j \in \hat{J}$), here $\hat{I} \bigcup \hat{J} = \hat{N}$. Next, it is demonstrated how to use the minimal edge cut set to calculate the recovery time between two points in the network. Let $\hat{C}_{i,j}$ be the set of the minimal edge cut set between the demand point i and the facility point j, $\hat{C}_{i,j} = \left\{c^1_{i,j}, c^2_{i,j}, \cdots, c^a_{i,j}, \cdots, c^{|i,j|_{\hat{C}}}_{i,j}\right\}$, in which $c^a_{i,j}$ is the *a-th* minimal edge cut set between i and j, $|i,j|_{\hat{C}}$ is the total number of the minimal edge cut set between i and j. $c^a_{i,j} = \left\{l^a_1, l^a_2, \cdots, l^a_b, \cdots, l^a_{q_a}\right\}$, in which l^a_b is the *b-th* edge of $c^a_{i,j}$, q_a is the total number of edges of $c^a_{i,j}$. In $c^a_{i,j}$, the recovery time of every edge is $t_r\left(l^a_1\right)$, $t_r\left(l^a_2\right)$,..., $t_r\left(l^a_b\right)$,..., $t_r\left(l^a_{q_a}\right)$, respectively. The recovery times of all edges in $c^a_{i,j}$ are ranked in a non-decreasing order, and it is assumed that $t_r\left(l^a_1\right) \le t_r\left(l^a_2\right) \le \cdots \le t_r\left(l^a_b\right) \le \cdots \le t_r\left(l^a_{q_a}\right)$, in which the minimum recovery time is $t_r\left(l^a_1\right)$. For all minimum edge cut sets between i and j, the recovery time of each minimal edge cut set is $t_r\left(l^1_1\right)$, $t_r\left(l^2_1\right)$,..., $t_r\left(l^a_1\right)$,..., $t_r\left(l^{|i,j|_{\hat{C}}}_1\right)$, respectively. Let $t_r\left(\hat{C}_{i,j}\right)$ be the recovery time of the network between i and j, according to Definition 2, then $t_r\left(\hat{C}_{i,j}\right) = \max\left\{t_r\left(l^1_1\right), t_r\left(l^2_1\right), \cdots, t_r\left(l^a_1\right), \cdots, t_r\left(l^{|i,j|_{\hat{C}}}_1\right)\right\}$.

Next, the recovery times in the network are defined from the perspective of the path:

Definition 3. Let $t_r\left(l^u_v\right)$ be the recovery time of the edge l^u_v on the path $p^u_{\hat{X},\hat{Y}}$, then $\max\limits_{l^u_v \in p^u_{\hat{X},\hat{Y}}} t_r\left(l^u_v\right)$ is called the recovery time of $p^u_{\hat{X},\hat{Y}}$. Let $t_r\left(p^u_{\hat{X},\hat{Y}}\right)$ be the recovery time of $p^u_{\hat{X},\hat{Y}}$, then $t_r\left(p^u_{\hat{X},\hat{Y}}\right) = \max\limits_{l^u_v \in p^u_{\hat{X},\hat{Y}}} t_r\left(l^u_v\right)$.

According to Definition 3, the recovery time of a path refers to the maximum recovery time of all damaged edges on this path. If any edge of a path cannot pass normally, then the path cannot pass normally. Therefore, the normal travel of a path requires all edges of the path to pass normally. If all damaged edges of a path are repaired at the same time, the maximum recovery time of all damaged edges on a path is considered the recovery time of this path.

Definition 4. Let $t_r\left(p^u_{\hat{X},\hat{Y}}\right)$ be the recovery time of the path $p^u_{\hat{X},\hat{Y}}$, then $\min\limits_{p^u_{\hat{X},\hat{Y}} \in \hat{P}_{\hat{X},\hat{Y}}} t_r\left(p^u_{\hat{X},\hat{Y}}\right)$ is called the recovery time of the network between the two points $\hat{X}$ and $\hat{Y}$. Let $t_r\left(\hat{P}_{\hat{X},\hat{Y}}\right)$ be the recovery time of the network between $\hat{X}$ and $\hat{Y}$, then $t_r\left(\hat{P}_{\hat{X},\hat{Y}}\right) = \min\limits_{p^u_{\hat{X},\hat{Y}} \in \hat{P}_{\hat{X},\hat{Y}}} t_r\left(p^u_{\hat{X},\hat{Y}}\right)$.

Next, it is illustrated how to use the path to calculate the recovery time between two points. Let $\hat{P}_{i,j}$ be the set of all paths between the demand point i and the facility point j, $\hat{P}_{i,j} = \left\{p^1_{i,j}, p^2_{i,j}, \cdots, p^u_{i,j}, \cdots, p^{|i,j|_{\hat{P}}}_{i,j}\right\}$, in which $p^u_{i,j}$ is the *u-th* path between i and j, $|i,j|_{\hat{P}}$ is the total number of paths between i and j, $p^u_{i,j} = \left\{l^u_1, l^u_2, \cdots, l^u_v, \cdots, l^u_{k_u}\right\}$, in which k_u is the number of edges of $p^u_{i,j}$. For $p^u_{i,j}$, if the recovery time of every edge is $t_r\left(l^u_1\right)$, $t_r\left(l^u_2\right)$,..., $t_r\left(l^u_v\right)$,..., $t_r\left(l^u_{k_u}\right)$, respectively. The recovery times of all edges on the path $p^u_{i,j}$ are ranked in a non-decreasing order, and it is assumed that $t_r\left(l^u_1\right) \le t_r\left(l^u_2\right) \le \cdots \le t_r\left(l^u_v\right) \le \cdots \le t_r\left(l^u_{k_u}\right)$, in which the maximum recovery time is $t_r\left(l^u_{k_u}\right)$. For all paths between i and j, the recovery time of every path is $t_r\left(l^1_{k_1}\right)$, $t_r\left(l^2_{k_2}\right)$,..., $t_r\left(l^u_{k_u}\right)$,..., $t_r\left(l^{|i,j|_{\hat{P}}}_{k_{|i,j|_{\hat{P}}}}\right)$, respectively. Let $t_r\left(\hat{P}_{i,j}\right)$ be the recovery time of the network between i and j, then $t_r\left(\hat{P}_{i,j}\right) = \min\left\{t_r\left(l^1_{k_1}\right), t_r\left(l^2_{k_2}\right), \cdots, t_r\left(l^u_{k_u}\right), \cdots, t_r\left(l^{|i,j|_{\hat{P}}}_{k_{|i,j|_{\hat{P}}}}\right)\right\}$.

According to the equivalence theorem, $t_r\left(\hat{C}_{i,j}\right) = t_r\left(\hat{P}_{i,j}\right)$. Later, $t_r(i,j)$ is used to represent the recovery time between i and j.

Definition 5. Let $t_r\left(\hat{G}\right)$ be the maximum recovery time between all pairs of points, i.e., $t_r\left(\hat{G}\right) = \max\limits_{\hat{X} \in \hat{A}, \hat{Y} \in \hat{A}} t_r\left(\hat{P}_{\hat{X},\hat{Y}}\right)$, then $t_r\left(\hat{G}\right)$ is called the network recovery time.

When the time taken exceeds $t_r\left(\hat{G}\right)$, all pairs of points in $\hat{G}$ are connected. When the time taken does not exceed $t_r\left(\hat{G}\right)$, at least one pair of points is not connected.

2.3 Total Network Time

Let $t_s\left(l^u_v\right)$ be the travel time of the edge l^u_v, $t_s\left(p^u_{\hat{X},\hat{Y}}\right)$ be the travel time of the path $p^u_{\hat{X},\hat{Y}}$, i.e., the sum of the travel times of all edges on the path $p^u_{\hat{X},\hat{Y}}$, then $t_s\left(p^u_{\hat{X},\hat{Y}}\right)=\sum_{v=1}^{k_u} t_s\left(l^u_v\right)$. Let $t_s\left(\hat{P}_{\hat{X},\hat{Y}}\right)$ be the minimum travel time among all the paths between $\hat{X}$ and $\hat{Y}$ in the network, then $t_s\left(\hat{P}_{\hat{X},\hat{Y}}\right)=\min_{p^u_{\hat{X},\hat{Y}}\in\hat{P}_{\hat{X},\hat{Y}}} t_s\left(p^u_{\hat{X},\hat{Y}}\right)$. Let $t_s\left(\hat{G}\right)$ be the network travel time of $\hat{G}$, i.e., the maximum value among all travel times of all pairs of points, then $t_s\left(\hat{G}\right)=\max_{\hat{X}\in\hat{A},\hat{Y}\in\hat{A}} t_s\left(\hat{P}_{\hat{X},\hat{Y}}\right)$ (Vahdani et al., 2018).

Definition 6. The sum of the recovery time and the travel time of the path is called the total time of this path. Let $t\left(\hat{P}_{\hat{X},\hat{Y}}\right)$ be the recovery time of the path $p^u_{\hat{X},\hat{Y}}$, $t_s\left(p^u_{\hat{X},\hat{Y}}\right)$ be the travel time of $p^u_{\hat{X},\hat{Y}}$, $t\left(p^u_{\hat{X},\hat{Y}}\right)$ be the total time of $p^u_{\hat{X},\hat{Y}}$, then $t\left(p^u_{\hat{X},\hat{Y}}\right)=t_r\left(p^u_{\hat{X},\hat{Y}}\right)+t_s\left(p^u_{\hat{X},\hat{Y}}\right)$.

Definition 7. The minimum total time of all paths between $\hat{X}$ and $\hat{Y}$ is called the total time between $\hat{X}$ and $\hat{Y}$. Let $t\left(\hat{P}_{\hat{X},\hat{Y}}\right)$ be the total time between $\hat{X}$ and $\hat{Y}$, then $t\left(\hat{P}_{\hat{X},\hat{Y}}\right)=\min_{p^u_{\hat{X},\hat{Y}}\in\hat{P}_{\hat{X},\hat{Y}}} t\left(p^u_{\hat{X},\hat{Y}}\right)$.

Definition 8. $\max_{\hat{X}\in\hat{A},\hat{Y}\in\hat{A}} t\left(\hat{P}_{\hat{X},\hat{Y}}\right)$ is called the total network time and is denoted by t, then $t=\max_{\hat{X}\in\hat{A},\hat{Y}\in\hat{A}} t\left(\hat{P}_{\hat{X},\hat{Y}}\right)=\max_{\hat{X}\in\hat{A},\hat{Y}\in\hat{A}}\min_{p^u_{\hat{X},\hat{Y}}\in\hat{P}_{\hat{X},\hat{Y}}} t\left(p^u_{\hat{X},\hat{Y}}\right)$.

2.4 Reliability Levels in the Network

Wang et al. (2022) found that the reliability level between two points defined by the minimal edge cut set and by the path are equivalent in a weighted connected graph $\hat{G}$. This paper uses the definition of the reliability level between two points by the path. Yu (2020) and Wang et al. (2022) called the maximum ability of an edge to withstand disasters as the reliability level of the edge. Let $r(l^u_v)$ be the reliability level of the edge l^u_v. If the damage of the disaster exceeds $r(l^u_v)$, the edge l^u_v will be interrupted, otherwise, the edge could still pass normally. They argued that the reliability level of a path is the minimum reliability level of all edges on this path (Wang, Xu & Filip, 2022; Yu, 2020). Let $r\left(p^u_{\hat{X},\hat{Y}}\right)$ be the reliability level of the path $p^u_{\hat{X},\hat{Y}}$, then $r\left(p^u_{\hat{X},\hat{Y}}\right)=\min_{l^u_v\in p^u_{\hat{X},\hat{Y}}} r\left(l^u_v\right)$. If the damage of the disaster exceeds $r\left(p^u_{\hat{X},\hat{Y}}\right)$, the path $p^u_{\hat{X},\hat{Y}}$ will be interrupted, otherwise, the path $p^u_{\hat{X},\hat{Y}}$ will not be interrupted. It could be seen that this value represents the maximum ability of the path $p^u_{\hat{X},\hat{Y}}$ to withstand disasters. Wang et al. (2022) called the maximum reliability level of all paths between two points in network $\hat{G}$ as the reliability level between these two points. Let $\hat{R}\left(\hat{P}_{\hat{X},\hat{Y}}\right)$ be the reliability level between two points $\hat{X}$ and $\hat{Y}$, then $\hat{R}\left(\hat{P}_{\hat{X},\hat{Y}}\right)=\max_{p^u_{\hat{X},\hat{Y}}\in\hat{P}_{\hat{X},\hat{Y}}} r\left(p^u_{\hat{X},\hat{Y}}\right)$. If the damage does not exceed $\hat{R}\left(\hat{P}_{\hat{X},\hat{Y}}\right)$, then the two points $\hat{X}$ and $\hat{Y}$ are connected, otherwise, they are not connected.

3. Models

3.1 Problem Description

This paper studies how to choose the damaged roads to repair and shorten the network recovery time under a certain budget. It is assumed that all damaged edges selected for repair start construction at the same time, which could reduce the network recovery time. After the damaged edges selected for restoration are completely repaired, at least one path is connected between each demand point and the facility serving it, but only one path between the demand point and the facility point serving it is selected to provide the service.

3.2 The Formulation of the Objective Functions

With regard to the network recovery time and the network travel time, a special situation is taken into consideration: the network can be put into use only after all damaged edges are completely repaired, that is, after the repair of the whole network is completed, personnel begin to be evacuated and rescue materials begin to be transported. In this case, the sum of the network recovery time $t_r\left(\hat{G}\right)$ and the network travel time $t_s\left(\hat{G}\right)$ refers to the minimum time required from the beginning of recovery to the arrival of personnel and disaster relief materials at the last destination, that is, the time required to complete a rescue operation. At this time, the sum of $t_r\left(\hat{G}\right)$ and $t_s\left(\hat{G}\right)$ can be minimized as an objective of the model. This objective could be expressed as $\min\ (t_r\left(\hat{G}\right)+t_s\left(\hat{G}\right))$. In fact, generally speaking, the paths between some or all demand points and facility points are formed before the whole construction is completed. Therefore, the rescue action for some demand points does not need to be carried out after the construction of the whole network is completed. In addition, in an emergency, not all damaged edges in the network need to be repaired, as only some damaged edges of the network should be restored to ensure that each demand point could be served. In this way, the damaged edges whose recovery times are equal to the network recovery time are not necessarily selected for recovery. Therefore, it is assumed that after the selected path between a demand point and the corresponding facility is formed, this path could be put into use. In this case, the sum of the recovery time and the travel time of the path needs to be calculated, that is, the total time of this path. Let $t_r\left(p^u_{i,j_i}\right)$ and $t_s\left(p^u_{i,j_i}\right)$ be the recovery time and the travel time of the *u-th* path p^u_{i,j_i} between the demand point i and its corresponding facility j_i, respectively. Let $t\left(p^u_{i,j_i}\right)$ be the sum of $t_r\left(p^u_{i,j_i}\right)$ and $t_s\left(p^u_{i,j_i}\right)$, then $t\left(p^u_{i,j_i}\right)=t_r\left(p^u_{i,j_i}\right)+t_s\left(p^u_{i,j_i}\right)$. Let $t\left(\hat{P}_{i,j_i}\right)$ be the total time between the demand point i and its corresponding facility j_i, then $t\left(\hat{P}_{i,j_i}\right)=\min_{p^u_{i,j_i}\in\hat{P}_{i,j_i}} t\left(p^u_{i,j_i}\right)$.

In the model, when the minimization of the total network time is not the top priority objective, the selection of the path between the demand point and its corresponding facility should be subject to other higher priority objectives. Thus, the selected path cannot be determined according to the above formula. This paper assumes that only one path is selected between each pair of demand point and the corresponding facility. Therefore, a binary variable Y^u_{ij} is set to denote whether the *u-th* path between the demand point i and its corresponding facility j_i is selected. Therefore, the above formula is modified to $t\left(\hat{P}_{i,j_i}\right)=\sum_{p^u_{i,j_i}\in\hat{P}_{i,j_i}} t\left(p^u_{i,j_i}\right)Y^u_{ij}$. Then $t=\max_{i\in\hat{I},j_i\in\hat{J}} t\left(\hat{P}_{i,j_i}\right)$.

Since $t_r\left(\hat{P}_{i,j_i}\right)\le t_r\left(\hat{G}\right)$, $t_s\left(\hat{P}_{i,j_i}\right)\le t_s\left(\hat{G}\right)$, then, $t_r\left(\hat{P}_{i,j_i}\right)+t_s\left(\hat{P}_{i,j_i}\right)\le t_r\left(\hat{G}\right)+t_s\left(\hat{G}\right)$, i.e., $t\left(\hat{P}_{i,j_i}\right)\le t_r\left(\hat{G}\right)+t_s\left(\hat{G}\right)$. This means that the sum of the recovery time and the travel time between any demand point i and its corresponding facility point j_i does not exceed the sum of the network recovery time and the network travel time. From this, $t\le t_r\left(\hat{G}\right)+t_s\left(\hat{G}\right)$ is obtained. If the selected path between each demand point and the corresponding facility starts to pass after repair, the minimum time required for the whole network from repair to the arrival of the personnel and the disaster relief materials at the last destination will not exceed the sum of the network recovery time and the network travel time. In this way, on the whole, all demand points will be rescued in a timelier manner, which is of more practical significance. Therefore, this paper actually takes the minimization of the total network time t instead of min $(t_r\left(\hat{G}\right)+t_s\left(\hat{G}\right))$ as an objective of the model. In the proposed model, only partial damaged edges are considered to be restored. Therefore, the network here does not include the edges that are not repaired. Since there may be aftershocks after the disaster, the reliability level is an important factor affecting the success of the rescue operation. This paper takes the maximization of the sum of the reliability levels between all demand points and the facilities serving them as an objective of the model. This objective is as follows: $\max\hat{R}=\sum_{i\in\hat{I}}\sum_{j\in\hat{J}}\hat{R}\left(\hat{P}_{i,j}\right)Y_{ij}$, in which, Y_{ij} is a binary variable of whether the facility j provides services for the demand point i. Now, we will make appropriate changes to this objective function. Let $\hat{F}\left(\hat{P}_{i,j}\right)=1-\hat{R}\left(\hat{P}_{i,j}\right)$, $\hat{F}\left(\hat{P}_{i,j}\right)$ is called the failure level between i and j, then

$$\hat{R}=\sum_{i\in\hat{I}}\sum_{j\in\hat{J}}\hat{R}\left(\hat{P}_{i,j}\right)Y_{ij}=\sum_{i\in\hat{I}}\sum_{j\in\hat{J}}\left(1-\hat{F}\left(\hat{P}_{i,j}\right)\right)Y_{ij}$$
$$=\sum_{i\in\hat{I}}\sum_{j\in\hat{J}}Y_{ij}-\sum_{i\in\hat{I}}\sum_{j\in\hat{J}}\hat{F}\left(\hat{P}_{i,j}\right)Y_{ij}$$

where $\sum_{i\in\hat{I}}\sum_{j\in\hat{J}}Y_{ij}$ is the total number of the selected facilities, which is a constant value.

Let $f=\sum_{i\in\hat{I}}\sum_{j\in\hat{J}}\hat{F}\left(\hat{P}_{i,j}\right)Y_{ij}$, so, $\max\hat{R}$ is equivalent to $\min f$. In addition, in the post disaster emergency action, the use of the emergency budget funds needs to be taken into consideration. In this paper, the emergency budget funds are used in two situations: one is the repair of the damaged roads, and the other is the location of the facilities. Let e ($e\in\hat{E}$) be the damaged edge, $\hat{E}$ be the set of the damaged edges, and Y_e be the binary variable of whether the damaged edge e is recovered, b_e be the repair cost of the damaged edge e, b_j be the location cost of the facility j, and Y_j be the binary variable of whether facility j is selected. The sum of the repair cost of the damaged edges and the location cost of the facilities is called the total cost and it is denoted by $\hat{B}$, then $\hat{B}=\sum_{e\in\hat{E}}b_eY_e+\sum_{j\in\hat{J}}b_jY_j$. Let $\hat{B}_1=\sum_{e\in\hat{E}}b_eY_e$, $\hat{B}_2=\sum_{j\in\hat{J}}b_jY_j$, then $\hat{B}=\hat{B}_1+\hat{B}_2$. The total cost $\hat{B}$ is minimized as an objective of the model.

3.3 Models

Assumptions:

1. The facilities in the network are absolutely reliable;

2. The demand point set and the candidate facility point set in the network are known;

3. The upper limit of the reliability level of each edge in the network is 1;

4. The reliability level of the damaged edge selected for repair in the network reaches the level before damage;

5. Each demand point can be assigned only to one facility, and each facility can serve multiple demand points.

Sets:

$\hat{I}$: The set of the demand points;
$\hat{J}$: The set of the candidate facilities;
$\hat{E}$: The broken edge set in network.

Parameters:

b_e : The recovery cost of the damaged edge e, $e \in \hat{E}$;

b_j : The location cost of the candidate facility j, $j \in \hat{J}$;

t : The total time of the network $\hat{G}$;

f : The sum of the failure levels between all pairs of demand points and the corresponding facilities;

$\hat{B}$: The total cost;

$\hat{B}_0$: The total budget funds;

$\hat{B}_1$: The restoration cost of the damaged edges;

$\hat{B}_2$: The location cost of the selected facilities;

M_f : The total number of the selected facilities;

$|i,j|_{\hat{P}}$: The total number of the paths between the two points i and j.

$r\left(p_{i,j}^u\right)$: The reliability level of the path $p_{i,j}^u$;

$\hat{R}\left(\hat{P}_{i,j}\right)$: The reliability level between the point i and the point j;

$t\left(p_{i,j}^u\right)$: The total time of the path $p_{i,j}^u$;

$t\left(\hat{P}_{i,j}\right)$: The total time between the point i and the point j;

Decision variables:

$$Y_j = \begin{cases} 1 \textit{ if the candidate facility } j \textit{ is selected} \\ 0 \textit{ otherwise} \end{cases}, \ j \in \hat{J}.$$

$$Y_{ij} = \begin{cases} 1 \textit{ if the candidate facility } j \textit{ is allocated to provide} \\ \quad \textit{service for the demand point } i \\ 0 \textit{ otherwise} \end{cases}, \ i \in \hat{I}, \ j \in \hat{J}.$$

$$Y_{ij}^u = \begin{cases} 1 \textit{ if the } u\textit{-th path between the demand point} \\ \quad i \textit{ and the candidate facility } j \textit{ is selected} \\ 0 \textit{ otherwise} \end{cases}, \ i \in \hat{I}, \ j \in \hat{J},$$

$u \in \left[1, |i,j|_{\hat{P}}\right]$.

$$Y_e = \begin{cases} 1 \textit{ if the damaged edge } e \textit{ is selected to be recovered} \\ 0 \textit{ otherwise} \end{cases},$$

$e \in \hat{E}$.

Model 1:

$$\min t \tag{1}$$

$$\min f \tag{2}$$

$$\min \hat{B} \tag{3}$$

Subject to

$$\sum_{j \in \hat{J}} Y_j = M_f \tag{4}$$

$$Y_{ij} \le Y_j, \ \forall i \in \hat{I}, \ \forall j \in \hat{J} \tag{5}$$

$$\sum_{j \in \hat{J}} Y_{ij} = 1, \ \forall i \in \hat{I} \tag{6}$$

$$Y_{ij} = \sum_{u=1}^{|i,j|_{\hat{P}}} Y_{ij}^u, \ \forall i \in \hat{I}, \ \forall j \in \hat{J}, \ \forall u \in \left[1, |i,j|_{\hat{P}}\right] \tag{7}$$

$$\hat{B} \le B_0 \tag{8}$$

$$Y_j, Y_{ij}, Y_{ij}^u \in \{0,1\}, \ \forall i \in \hat{I}, \ \forall j \in \hat{J}, \ \forall u \in \left[1, |i,j|_{\hat{P}}\right] \tag{9}$$

Equation (4) represents the constraint on the total number of the selected facilities. The constraint (5) indicates that the service could be provided for a demand point only after the facility is selected. (6) indicates that the total number of the facilities providing services for the demand point i is 1. (7) indicates that each demand point selects only one path to receive services. (8) indicates that the total cost spent cannot exceed the total budget funds. Y_j, Y_{ij} and Y_{ij}^u in the constraint (9) are all 0-1 variables.

In model 1, model 1.1 and model 1.2, formula (1), (2) and (3) are called objective (1), (2) and (3), respectively. Two *hierarchical optimization* methods are used to take into account the above three objectives. The first hierarchical method is as follows: the minimization of f is considered the highest priority, the minimization of t is considered the secondary priority and the minimization of $\hat{B}$ is considered the lowest priority. At this time, the model is called model 1.1. The second hierarchical method is as follows: the minimization of t is considered the highest priority, the minimization of f is considered the secondary priority and the minimization of $\hat{B}$ is considered the lowest priority. At this time, the model is called model 1.2. When it is necessary to balance the objectives (1) and (2), model 1 is appropriately modified, by integrating these objectives in only one objective. Since the size and dimension of the objectives (1) and (2) are different, they are standardized first. The standardization process is as follows (Grodzevich & Romanko, 2006):

Under a total budget constraint, objectives (2) and (3) are used in model 1.1 to obtain the maximum value $f_{\max}$ and the minimum value $f_{\min}$ of f, and objectives (1) and (3) are used in model 1.2 to obtain the maximum value $t_{\max}$ and the minimum value $t_{\min}$ of t. The normalized values of t and f are:

$$\bar{t} = \left(t - t_{\min}\right) / \left(t_{\max} - t_{\min}\right) \tag{10}$$

$$\bar{f} = (f - f_{\min})/(f_{\max} - f_{\min}) \tag{11}$$

Let ω be the weight of $\bar{f}$, then the objectives (2) and (1) could be integrated as an objective $\min \omega\bar{f} + (1-\omega)\bar{t}$.

Let $g = \omega\bar{f} + (1-\omega)\bar{t}$, then the above goal could be expressed as: $\min g$. Combining the above objective $\min g$ and the objective (3), model 2 is obtained.

Model 2:

$$\min g \tag{12}$$

$$\min \hat{B} \tag{13}$$

The constraints of this model are the same as those of model 1. When solving it, objective (12) is considered to have a higher priority, while objective (13) is considered to have a lower priority.

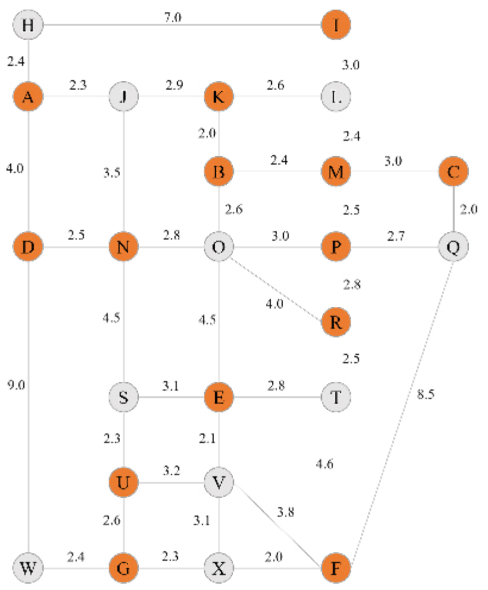

Figure 2. The road network

4. Case Study

4.1 Data

In this part, a case study is used to test the models. The Sioux Falls Network (Yu, 2020; Poorzahedy & Rouhani, 2007), a well-known transportation network, is used to test the proposed models, as shown in Figure 2.

The orange points in Figure 2 represent the demand points, while the other points represent the candidate facility points. Table 1 shows the reliability level and the travel time of each edge, and whether the edge is damaged. The travel time of each edge has been marked next to each edge. Table 2 shows the recovery time and the recovery cost of every damaged edge, and Table 3 shows the location cost of each facility.

Table 1. The reliability level and the travel time of each edge, and whether the edge is damaged

Edge	RE	TTE	WED	Edge	RE	TTE	WED	Edge	RE	TTE	WED	Edge	RE	TTE	WED
AH	0.55	2.4	No	EO	0.89	4.5	Yes	GX	0.71	2.3	Yes	OP	0.90	3.0	No
AJ	0.75	2.3	Yes	ES	0.77	3.1	No	HI	0.63	7.0	No	OR	0.84	4.0	No
AD	0.65	4.0	No	ET	0.79	2.8	No	IL	0.75	3.0	Yes	PQ	0.73	2.7	No
BK	0.78	2.0	No	EV	0.84	2.1	No	JK	0.76	2.9	No	PR	0.82	2.8	No
BM	0.82	2.4	No	FQ	0.66	8.5	No	JN	0.81	3.5	No	RT	0.79	2.5	Yes
BO	0.88	2.6	No	FT	0.73	4.6	No	KL	0.70	2.6	Yes	SU	0.81	2.3	No
CM	0.85	3.0	Yes	FV	0.66	3.8	Yes	LM	0.78	2.4	Yes	UV	0.73	3.2	No
CQ	0.73	2.0	Yes	FX	0.77	2.0	No	MP	0.89	2.5	No	VX	0.58	3.1	Yes
DN	0.87	2.5	No	GU	0.80	2.6	Yes	NO	0.89	2.8	Yes	/	/	/	/
DW	0.62	9.0	No	GW	0.62	2.4	No	NS	0.78	4.5	Yes	/	/	/	/

Legend: RE (The reliability of edge); TTE (the travel time of edge); WED (whether the edge is damaged).

Table 2. The recovery time and the recovery cost of every damaged edge

Edge	The recovery time	The recovery cost	Edge	The recovery time	The recovery cost
AJ	3.0	30	IL	2.5	20
CM	4.0	35	KL	3.5	31
CQ	2.5	28	LM	2.0	21
EO	1.6	15	NO	1.1	10
FV	4.0	36	NS	3.0	20
GU	2.0	18	RT	2.5	18
GX	3.0	26	VX	4.0	38

Table 3. The location cost of each facility

Facility	The location cost	Facility	The location cost
H	290	S	165
J	180	T	80
L	130	V	94
O	160	W	285
Q	205	X	115

4.2 Solutions and Analysis

4.2.1 Solutions and Analysis of Model 1.1 and Model 1.2

The time radius is assumed to be 20 (only if the total time between a facility and a demand point is lower than 20, this facility could be used as a candidate facility for this demand point). The model is solved when

$\hat{B}_0 = 200, 250, 300, 350, 400, 450, 500, 550, 600, 650, 700$,

respectively. Tables 4 and 5 display the solutions of model 1.1, Tables 6 and 7 illustrate the solutions of model 1.2, and Tables from 8 to 12 show the solutions of model 2. In Table 10, when the values of ω are 0 and 1, the solutions of the first objective (12) of the model 2 are all 0, thus they are not listed in the table.

From Table 4, it can be seen that, when $\hat{B}_0$ increases continuously, f decreases from 3.29 to 2.33, showing an obvious regularity. In general, t shows a non-increasing trend with the increase of $\hat{B}_0$. However, from $\hat{B}_0 = 200$ to $\hat{B}_0 = 250$, t increases from 18.3 to 19.5, which will be explained as follows. Table 5 shows part of the solutions obtained by solving model 1.1 when $\hat{B}_0 = 200$ and

Table 4. The solutions of model 1.1

$\hat{B}_0$	f	t	$\hat{B}$	$\hat{B}_1$	$\hat{B}_2$	SF
200	3.29	18.3	195	115	80	T
250	2.73	19.5	243	149	94	V
300	2.52	15.2	299	139	160	O
350	2.42	15.2	329	169	160	O
400	2.42	15.2	329	169	160	O
450	2.38	15.2	444	169	275	OX
500	2.37	12.9	474	149	325	OS
550	2.37	12.9	474	149	325	OS
600	2.33	12.9	589	149	440	OSX
650	2.33	12.9	589	149	440	OSX
700	2.33	12.5	698	128	570	LOSX

Legend: SF (the selected faciliy).

Table 5. Part of the solutions of model 1.1 when $\hat{B}_0 = 200$ and $\hat{B}_0 = 250$

$\hat{B}_0 = 200$			$\hat{B}_0 = 250$		
Paths	FLP	TTP	Paths	FLP	TTP
AD-DN-NO-OR-RT	0.35	18.3	AJ-JN-NO-OE-EV	0.25	18.2
BO-OP-PR-RT	0.21	13.4	BO-OE-EV	0.16	10.8
CQ-QP-PO-OR-RT	0.27	16.7	CM-MP-PO-OE-EV	0.16	19.1
DN-NO-OP-PR-RT	0.21	16.1	DN-NO-OE-EV	0.16	13.5
ET	0.21	2.8	EV	0.16	2.1
FT	0.27	4.6	FT-TE-EV	0.27	9.5
GU-US-SE-ET	0.23	12.8	GU-US-SE-EV	0.23	12.1
IL-LM-MP-PR-RT	0.25	15.7	IL-LM-MB-BO-OE-EV	0.25	19.5
KB-BO-OP-PR-RT	0.22	15.4	KB-BO-OE-EV	0.22	12.8
MP-PO-OR-RT	0.21	14.5	MP-PO-OE-EV	0.16	13.7
NO-OB-BM-MP-PR-RT	0.21	18.1	NO-OE-EV	0.16	11.0
PO-OR-RT	0.21	12.0	PO-OE-EV	0.16	11.2
RT	0.21	5.0	RO-OE-EV	0.16	12.2
US-SE-ET	0.23	8.2	US-SE-EV	0.23	7.5

$\hat{B}_0 = 250$, including the selected path, the failure level and the total time of each selected path between the demand point and the facility serving it, and the selected facilities. From Table 5 it can be seen that under these two budgets, the selected path of each demand point has changed due to the change of the selected facilities. Among them, the selected path of the demand point I changes from IL-LM-MP-PR-RT to IL-LM-MB-BO-OE-EV. Accordingly, t changes from 15.7 to 19.5. In this process, f is reduced from 3.29 to 2.73, which leads to a great improvement of $\hat{R}$. Therefore, in the process of increasing $\hat{B}_0$ from 200 to 250, $\hat{R}$ is improved by sacrificing t. Therefore, it could be found that the minimization of f, as the highest priority, plays a decisive role in model 1.1.

In addition, in general, with the increase of $\hat{B}_0$, the selected facilities change and the number of selected facilities increases. The change of the selected facilities and the increase in the number of selected facilities ensure the realization of higher priority objectives.

From Table 6, it can be seen that, with the increase of $\hat{B}_0$, t shows a non-increasing trend in general. When $\hat{B}_0 = 200$, t is 18.3; when $\hat{B}_0 = 700$, t is 9.4, the increase of $\hat{B}_0$ realizes the great optimization of t. The change trend of f is consistent with that of t within a certain range of $\hat{B}_0$. For example, when $\hat{B}_0$ increases from 200 to 300, t decreases from 18.3 to 14.4, and f decreases from 3.29 to 2.57. However, it can also be seen from Table 6 that, in individual cases, the decrease of t leads to the increase of f. For example, when $\hat{B}_0$ increases from 350 to 400, t decreases from 12.9 to 12.5, f increases from 2.64 to 3.28. This situation will be analyzed next. Table 7 lists part of the solutions of model 1.2 when $\hat{B}_0 = 350$ and $\hat{B}_0 = 400$. From Table 7, it can be observed that under the requirement of minimizing t, when $\hat{B}_0$ increases from 350 to 400, the selected facilities change from

Table 6. The solutions of model 1.2

$\hat{B}_0$	t	f	$\hat{B}$	$\hat{B}_1$	$\hat{B}_2$	**SF**
200	18.3	3.29	195	115	80	T
250	18.1	3.13	232	152	80	T
300	14.4	2.57	299	139	160	O
350	12.9	2.64	344	104	240	OT
400	12.5	3.28	393	133	260	JT
450	12.5	3.16	438	178	260	JT
500	10.0	3.23	498	94	404	JLV
550	9.7	3.13	528	124	404	JLV
600	9.7	3.13	528	124	404	JLV
650	9.4	3.11	626	142	484	JLTV
700	9.4	3.05	697	142	555	JLST

Legend: SF (selected facility).

Table 7. Part of the solutions of model 1.2 when $\hat{B}_0 = 350$ and $\hat{B}_0 = 400$

$\hat{B}_0 = 350$			$\hat{B}_0 = 400$		
Paths	TTP	FLP	Paths	TTP	FLP
AD-DN-NO	10.4	0.35	AD-DN-NJ	10.0	0.35
BO	2.6	0.12	BO-ON-NJ	10.0	0.19
CM-MP-PO	12.5	0.15	CQ-QP-PR-RT	12.5	0.27
DN-NO	6.4	0.13	DN-NJ	6.0	0.19
ET	2.8	0.21	ET	2.8	0.21
FT	4.6	0.27	FT	4.6	0.27
GU-US-SE-ET	12.8	0.23	GX-XF-FT	11.9	0.29
IL-LM-MB-BO	12.9	0.25	IL-LK-KJ	12.0	0.30
KB-BO	4.6	0.22	KB-BO-ON-NJ	11.9	0.22
MP-PO	5.5	0.11	MB-BO-ON-NJ	12.4	0.19
NO	3.9	0.11	NJ	3.5	0.19
PO	3.0	0.1	PO-ON-NJ	10.4	0.19
RO	4.0	0.16	RO-ON-NJ	11.4	0.19
US-SE-ET	8.2	0.23	US-SE-ET	8.2	0.23

Legend: TTP (the total time of the path); FLP (the failure level of the path).

O and T to J and T, most of the paths selected by the demand points change, and the value of t decreases from 12.9, which is the total time of the path selected by the demand point I when $\hat{B}_0 = 350$, to 12.5, which is the total time of the path selected by the demand point C when $\hat{B}_0 = 400$. However, when $\hat{B}_0 = 400$, the failure level of the path selected by each demand point is greater than or equal to that of the path selected by each demand point when $\hat{B}_0 = 350$. Therefore, the value of f when $\hat{B}_0 = 400$ is greater than that when $\hat{B}_0 = 350$. This shows that when the minimization of t is the highest priority objective, the minimization of t will lead to the increase of f some times. From Table 6, it can also be seen that, when $\hat{B}_0$ increases from 400 to 450, t remains unchanged, but f decreases. It is because that the increased capital is not enough to optimize t, but it could be used to realize the optimization of f.

4.2.2 Solutions and Analysis of Model 2

The value of f obtained by solving model 2 is turned into the value of $\hat{R}$, as shown in Table 8. Table 9 lists the value of t obtained by solving model 2. From Tables 8 and 9, it can be seen that under the same $\hat{B}_0$, the values of $\hat{R}$ and t tend to increase with the increase of ω, that is, the greater the weight of $\hat{R}$, the more conducive to the optimization of $\hat{R}$ and the less conducive to the optimization of t. When $\omega = 0$, the weight of $\hat{R}$ is the smallest and the weight of t is the highest, and t obtains the optimal value and $\hat{R}$ obtains the worst value. At this time, the optimized value of t is equal to the value of t under the same $\hat{B}_0$ in Table 6, while the value of f is not equal to the value of f under the same $\hat{B}_0$ in Table 6. In fact, when $\omega = 0$, model 2 actually takes the minimization of t as the highest priority and the minimization of $\hat{B}$ as the second priority, while the minimization of f is not a factor which affects the solution. In model 1.2, the minimization of t is the highest priority, f is the secondary priority, and the minimization of $\hat{B}$ is the lowest priority. Therefore, under the same $\hat{B}_0$, the value of t in model 2 when $\omega = 0$ is equal to the value of t in model 1.2. Since f in model 2 is not optimized when $\omega = 0$, its objective function value is lower than that in model 1.2. When $\omega = 1$, a similar analysis can be made.

Table 8. The solutions of $\hat{R}$ in model 2

ω	$\hat{B}_0$										
	200	250	300	350	400	450	500	550	600	650	700
0.0	10.61	10.57	10.85	10.76	10.52	10.56	10.73	10.67	10.76	10.67	10.73
0.1	10.71	10.87	11.43	11.36	10.72	10.84	10.77	10.87	10.87	10.89	11.40
0.2	10.71	10.87	11.43	11.36	11.56	10.84	10.77	10.87	10.87	10.89	11.40
0.3	10.71	10.87	11.43	11.36	11.56	11.57	11.30	10.87	10.87	10.89	11.40
0.4	10.71	10.87	11.43	11.36	11.56	11.57	11.30	11.41	11.46	10.89	11.40
0.5	10.71	10.87	11.43	11.36	11.56	11.57	11.30	11.41	11.46	11.46	11.40
0.6	10.71	10.87	11.47	11.36	11.56	11.57	11.30	11.41	11.46	11.46	11.40
0.7	10.71	10.87	11.47	11.36	11.56	11.57	11.30	11.41	11.46	11.46	11.40
0.8	10.71	11.27	11.47	11.57	11.56	11.57	11.63	11.41	11.46	11.60	11.50
0.9	10.71	11.27	11.47	11.57	11.56	11.57	11.63	11.63	11.67	11.67	11.67
1.0	10.71	11.27	11.48	11.58	11.58	11.62	11.63	11.63	11.67	11.67	11.67

Table 9. The solutions of t in model 2

ω	$\hat{B}_0$										
	200	250	300	350	400	450	500	550	600	650	700
0.0	18.3	18.1	14.4	12.9	12.5	12.5	10.0	9.7	9.7	9.4	9.4
0.1	18.3	18.1	14.4	12.9	12.5	12.5	10.0	9.7	9.7	9.4	9.5
0.2	18.3	18.1	14.4	12.9	12.9	12.5	10.0	9.7	9.7	9.4	9.5
0.3	18.3	18.1	14.4	12.9	12.9	12.9	10.4	9.7	9.7	9.4	9.5
0.4	18.3	18.1	14.4	12.9	12.9	12.9	10.4	10.5	10.5	9.4	9.5
0.5	18.3	18.1	14.4	12.9	12.9	12.9	10.4	10.5	10.5	10.5	9.5
0.6	18.3	18.1	14.5	12.9	12.9	12.9	10.4	10.5	10.5	10.5	9.5
0.7	18.3	18.1	14.5	12.9	12.9	12.9	10.4	10.5	10.5	10.5	9.5
0.8	18.3	19.5	14.5	14.5	12.9	12.9	12.9	10.5	10.5	12	10.4
0.9	18.3	19.5	14.5	14.5	12.9	12.9	12.9	12.9	12.9	12.9	12.5
1.0	19.9	19.5	17.5	18.1	18.1	17.2	19.2	13.4	19.2	17.2	13.7

In addition, from Table 8, it can be observed that, when $\hat{B}_0$ is the same, as the value of ω changes from 0 to 0.1, $\hat{R}$ generally increases significantly. When $\omega = 0$, under each $\hat{B}_0$, f obtains the worst value and t obtains the best value. When $\omega = 0.1$, although the weight of f is very small, it becomes a factor which affects the first objective value of model 2. Therefore, the value of f is optimized, which shows a significant decrease compared with the value of f when $\omega = 0$, that is, $\hat{R}$ increases significantly, while t remains unchanged or does not change significantly. In Table 9, when ω changes from 0.9 to 1, the change of t could be analyzed similarly.

From Table 10 it can be observed that when $\hat{B}_0 = 200$, the first objective value of the model 2 is 0. Since there are many edges needing to be restored, the restoration occupies a large number of budgets. Therefore, when $\hat{B}_0$ is not sufficient, the budgets for selecting facilities are limited, and only the facilities with lower location cost can be selected. In this case, the location costs of the facilities T and V are lower, which are 80 and 94, respectively. In fact, when $\hat{B}_0 = 200$, the facility T is selected. So, the total number of the path between all demand points and T is limited, which makes the values of f and t unchanged (3.29 and 18.3, respectively) besides when $\omega = 0$ and $\omega = 1$. In (10) and (11), $t_{\min} = 18.3$, $f_{\min} = 3.29$, therefore, after standardization, the values of f and t are both 0. In addition, from Table 10, it can be seen that, when $\hat{B}_0 \geq 250$, the first objective value of model 2 first increases and then decreases under each value of $\hat{B}_0$. $\hat{B}_0 = 300$ is taken as an example to illustrate this process. When $\hat{B}_0 = 300$, $t_{\max} = 20$, $t_{\min} = 14.4$, $f_{\max} = 5.15$, $f_{\min} = 2.52$, the standardized functions of t and f are:

$$\bar{t} = \frac{t - t_{\min}}{t_{\max} - t_{\min}} = \frac{t - 14.4}{5.6},$$

$$\overline{f} = \frac{f - f_{\min}}{f_{\max} - f_{\min}} = \frac{f - 2.52}{2.63}$$

When $\omega = 0.1, 0.2, 0.3, 0.4, 0.5$, $t = 14.4$, $\bar{t} = 0$, $f = 2.57$, $\overline{f} = 0.019$. As ω increases from 0.1 to 0.5, the value of the first objective function of model 2 increases $0.1\overline{f} = 0.1 \times 0.019 = 0.0019$ for every 0.1 increase of ω. When $\omega = 0.6$, there is a path change, i.e., the selected path of the demand point G changes from GU-UV-VE-EO to GU-US-SE-EO, leading to reducing the value of the first objective function of model 2. The failure level

Table 10. The solutions of the first objective function of model 2

ω	$\hat{B}_0$										
	200	250	300	350	400	450	500	550	600	650	700
0.1	0	0.0215	0.0019	0.0079	0.0257	0.0214	0.0225	0.0195	0.0203	0.0198	0.0153
0.2	0	0.0430	0.0038	0.0158	0.0439	0.0429	0.0450	0.0390	0.0406	0.0396	0.0213
0.3	0	0.0645	0.0057	0.0237	0.0391	0.0415	0.0539	0.0585	0.0609	0.0594	0.0272
0.4	0	0.0860	0.0076	0.0317	0.0344	0.0375	0.0586	0.0692	0.0679	0.0792	0.0331
0.5	0	0.1075	0.0095	0.0396	0.0297	0.0335	0.0632	0.0670	0.0655	0.0785	0.0390
0.6	0	0.1290	0.0094	0.0475	0.0249	0.0296	0.0678	0.0649	0.0630	0.0735	0.0449
0.7	0	0.1505	0.0080	0.0554	0.0202	0.0256	0.0725	0.0628	0.0606	0.0684	0.0508
0.8	0	0.1474	0.0066	0.0479	0.0155	0.0217	0.0580	0.0607	0.0582	0.0633	0.0534
0.9	0	0.0737	0.0052	0.0258	0.0107	0.0177	0.0290	0.0311	0.0311	0.0330	0.0292

Table 11. The solutions of the total cost of model 2

ω	$\hat{B}_0$										
	200	250	300	350	400	450	500	550	600	650	700
0	195	225	272	337	383	383	498	528	528	626	626
0.1	195	232	299	344	393	438	498	528	528	626	698
0.2	195	232	299	344	389	438	498	528	528	626	698
0.3	195	232	299	344	389	409	475	528	528	626	698
0.4	195	232	299	344	389	409	475	546	574	626	698
0.5	195	232	299	344	389	409	475	546	574	574	698
0.6	195	232	299	344	389	409	475	546	574	574	698
0.7	195	232	299	344	389	409	475	546	574	574	698
0.8	195	243	299	329	389	409	474	546	574	583	698
0.9	195	243	299	329	389	409	474	474	589	589	698
1.0	195	243	299	329	329	444	474	474	589	589	589

Table 12. The solutions of the selected facilities of model 2

ω	$\hat{B}_0$										
	200	250	300	350	400	450	500	550	600	650	700
0	T	T	O	OT	JT	JT	JLV	JLV	JLV	JLTV	JLTV
0.1	T	T	O	OT	JT	JT	JLV	JLV	JLV	JLTV	JLOV
0.2	T	T	O	OT	OT	JT	JLV	JLV	JLV	JLTV	JLOV
0.3	T	T	O	OT	OT	OT	LOV	JLV	JLV	JLTV	JLOV
0.4	T	T	O	OT	OT	OT	LOV	LOS	LOS	JLTV	JLOV
0.5	T	T	O	OT	OT	OT	LOV	LOS	LOS	LOS	JLOV
0.6	T	T	O	OT	OT	OT	LOV	LOS	LOS	LOS	JLOV
0.7	T	T	O	OT	OT	OT	LOV	LOS	LOS	LOS	JLOV
0.8	T	V	O	O	OT	OT	OS	LOS	LOS	LOS	LOSX
0.9	T	V	O	O	OT	OT	OS	OS	OSX	OSX	LOSX
1.0	T	V	O	O	O	OX	OS	OS	OSX	OSX	OSX

between the demand point G and the selected facility O is reduced from 0.27 to 0.23, the value of f is reduced to 2.53, and the value of t changes from 14.4 to 14.5. Compared with $\omega = 0.5$, when $\omega = 0.6$, the increment of the first objective function value of model 2 is -0.0001. This is a negative increment, so the value of the first objective function of model 2 begins to decrease. When $\omega = 0.7$, the value of f is 2.53 and the value of t is 14.5. Compared with $\omega = 0.6$, there is no change. Thus, compared with $\omega = 0.6$, the increment of the first objective function value of model 2 is -0.0014. Therefore, when $\omega = 0.7$, the value of the first objective function of model 2 is 0.0080. When the values of ω are 0.8 and 0.9, the value of f and the value of t do not change, therefore, for every 0.1 increase of ω, the value of the first objective function of model 2 decreases by 0.0014.

4.2.3 Discussions

Since model 2 sets different weights for f and t, the two objective functions are well balanced. For example, when $\hat{B}_0 = 450$, $f = 2.38$ and $t = 15.2$, as seen in Table 4; when $\hat{B}_0 = 450$, $t = 12.5$ and $f = 3.16$, as seen in Table 6. As illustrated in Tables 8 and 9, when $\hat{B}_0 = 450$ and $\omega = 0.3$, the value of f in model 2 is 14-11.57 = 2.43 and the value of t is 12.9. Compared with model 1.1, model 2 obtains a worse value of f, but a better value of t; compared with model 1.2, model 2 obtains a worse value of t, but a better value of f. As another example, when $\hat{B}_0 = 550$, it can be seen that $f = 2.37$, $t = 12.9$ from Table 4 and that $t = 9.7$, $f = 3.13$ from Table 6. As seen in Tables 8 and 9, when $\hat{B}_0 = 550$ and $\omega = 0.6$, the value of f in model 2 is 14-11.41 = 2.59 and the value of t is 10.5. Compared with model 1.1, model 2 obtains a worse value of f and a better value of t. Compared with model 1.2, model 2 obtains a worse value of t and a better value of f.

From the above analysis, when special attention is paid to the reliability level, the solution of model 1.1 can be chosen. When paying special attention to the total network time, the solution of model 1.2 can be selected. When an appropriate trade-off between the reliability level and the total network time needs to be made, the solution of model 2 can be chosen. In addition, in model 2, under the same value of ω ($\omega \neq 0,1$), with the increase of the total budget, although the change of the sum of the reliability levels fluctuates, the decreasing trend of the total network time is obvious. Therefore, in order to obtain a smaller total network time, a higher value of the total budget should be chosen.

5. Conclusion

Using the graph theory, this paper gives an equivalence theorem, that is, in a weighted graph, the maximum value of the minimum edge weights of all minimal edge cut sets between two points is equal to the minimum value of the maximum edge weights of all paths between these two points, and lists the application scenarios of the theorem. Based on this theorem, the network recovery time and the total network time are defined from the perspective of the path. Models are established to optimize the total network time and the reliability level. Model 1 takes a) the minimization of the sum of the failure levels between all demand points and the corresponding facilities, b) the minimization of the total network time and c) the minimization of the total cost as the objectives, respectively. The model is solved by setting different priorities for these three objectives. Model 1.1 takes the minimization of the sum of the failure levels between all demand points and the facilities serving them as the highest-priority objective. When the total budget increases, the sum of the failure levels can be gradually optimized in model

1.1. Model 1.2 takes the minimization of the total network time as the highest-priority objective. When the total budget increases, the total network time is continuously optimized in model 1.2.

The hierarchical method for solving model 1 is not conducive to the optimization of the lower-level objective function, i.e., the lower priority objectives. Therefore, the sum of the failure levels and the total network time are standardized, and a new objective function is formed by weighting. The minimization of this new objective function is the first objective, that is, the highest priority objective, and the minimization of the total cost is the second priority, thus model 2 is established. Although model 2 is also hierarchical, it is more conducive to the balance between the sum of the failure levels and the total network time. By solving model 2, it could be found that, with the increase in the weight of the sum of the failure levels, its function value shows a non-increasing change, and gradually obtains a better value, while the function value of the network total time shows a non-decreasing change. Therefore, when the total budget is fixed, if a better value of the sum of the failure levels is chosen, then ω can take a higher value; if a better value of the total network time is chosen, then ω can take a lower value. Compared with model 1, model 2 realizes the trade-off between the sum of the failure levels and the total network time, leaving more choices for practical applications.

In the future, the method proposed in this paper would be applied to larger scale networks and the application of the modeling method in large-scale practical networks would be discussed. The factors influencing the location decision-making of the emergency facilities considered in this model are limited. Factors, such as the important degree of different demand points and the population capacities of different facilities, are not discussed in this study. Therefore, more influencing factors could be included in the decision-making model for future research.

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